AWP
-I (22)

By substitute the eqn(3) in eqn(2),

$$E = -\nabla V - \partial A/\partial t \longrightarrow (4)$$

which satisfies both the static and the time varying conditions

since, $\nabla \cdot D = \nabla \cdot (\varepsilon E) = \varepsilon \nabla \cdot E$

$$= \varepsilon \nabla \cdot (-\nabla V - \partial A/\partial t)$$

$$= \varepsilon(-\nabla \cdot \nabla V - \partial/\partial t (\nabla \cdot A)) = \rho$$

from the above relation, $\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\rho/\varepsilon \longrightarrow (5)$

The RHS of (5) leads to the following relations:

$$\nabla^2 V = -\rho/\varepsilon \quad \text{for static conditions} \longrightarrow 6(a)$$

$$\nabla^2 V = -\rho/\varepsilon - \partial/\partial t(\nabla \cdot A) \quad \text{for time varying conditions} \longrightarrow 6(b)$$

But $\nabla \times H = J + \partial D/\partial t$

$B = \mu H$ or $H = B/\mu$

The LHS of above eqn can be written as

$$LHS = (\nabla \times B)/\mu = (\nabla \times \nabla \times A)/\mu = (\nabla(\nabla \cdot A) - \nabla^2 A)/\mu \longrightarrow (7)$$

This relation uses the vector identity,

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \longrightarrow (8)$$

The RHS of above eqn can also be written as

$$RHS = J + \varepsilon \frac{\partial E}{\partial t} = J + \varepsilon \frac{\partial}{\partial t}(-\nabla V - \partial A/\partial t)$$

$$= J + \varepsilon(-\nabla(\partial V/\partial t) - \partial^2 A/\partial t^2)$$

$$= J - \varepsilon\left[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2\right] \longrightarrow (9)$$

on By equating LHS and RHS terms i.e., eqn (7) & (9), we get

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J - \mu\varepsilon\left[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2\right] \longrightarrow (10)$$

But we know that

$\nabla^2 A = -\mu J$ (in general) and $\nabla^2 A = 0$ for $J = 0$.

where as the term $\nabla \cdot A$ is yet to be defined.

→ As per the statement of Helmholtz theorem A vector field is completely defined only when both its curl and divergence are kno

①

Introduction :—

Antennas are our electronic eyes and ears on the work. They are our links with space. They are essential, integral part of our civilization.

Antennas have been around for a long time, millions of years as the organ of touch or feeling of animals, birds and insects. But in the last 100 years they have acquired a new significance as the connecting link between a radio system and the out-side world.

The first radio antennas were built by Heinrich Hertz a professor at the technical Institute in Germany. He assembled apparatus for complete radio system operating at meter wavelengths with an end-loaded dipole as the transmitting and a resonant square-loop antenna as a receiver

Antennas are the essential communication link for aircraft and ships. Antennas for cellular phones and all types of wireless devices link us to everyone and every-thing. With mankind's activities expanding into space, the need for antennas will grow to an unprecedented degree Antennas will provide the vital links to and from everything out there. The future of antennas reaches to stars also.

Antennas are 3-Dimensional and live in a world of beam area, steradians, square degrees and solid angle. Antennas have impedances (self and Mutual). They couple

AWP
U-I (21)

$J =$ conduction Current density, A/m^2.

$\rho \rightarrow$ free charge density, C/m^3.

These four eqns are completely general and apply to all electromagnetic phenomena in media, which are at rest with-respect to the co-ordinate system used, and are valid for non homogeneous, non-linear and even for non-isotropic media.

→ These four Maxwell eqns, there are three relations which are concerned with the characteristics of the medium in which the fields are situated. These relations are

$$D = \varepsilon E$$

$$B = \mu H$$

$$J = \sigma E$$

for homogeneous, ε, σ, μ are constant throughout the medium

Maxwells eqns in Integral form:—

Maxwells eqns can be converted into integral form by integrating them over an area and applying stokes theorem or by integrating throughout the volume and

$$\oint_L A \cdot dl = \iint_S (\nabla \times A) \cdot ds = \iint (\nabla \times A) \cdot e\, ds$$

Applying Divergence theorem,

$$\iint_S B \cdot ds = 0 \Rightarrow \boxed{\oint_S B \cdot ds = 0} \longrightarrow (1)$$

$$\oint_S D \cdot ds = \int_V \rho \cdot dV$$

$$\oint E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot ds$$

$$\oint H \cdot dl = \int_S \left[J_c + \frac{\partial D}{\partial t} \right] \cdot ds$$

for free space, $J_c = 0$.

MP
12-5 (1)

Introduction :—

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Antennas have impedances (self and Mutual). They couple

Power can be transferred to and from the antenna. However, almost care must be taken to ensure correct antenna matching else internal reflections will take place.

Impedance of an Isolated Antenna [used for receiving & Transmitting] :—

Consider, the two antennas are separated with wide separation as shown in figure below:

→ The current distribution is same in case of transmitting and receiving antenna. Let antenna no.1 is the transmitting antenna and antenna no.2 is the receiving antenna.

→ The self impedance (Z_{11}) of transmitting antenna is given by

$$E_1 = Z_{11} I_1 + Z_{12} I_2 .$$

Here,

Z_{11} = self impedance of antenna (1)

Z_{12} = Mutual impedance b/w the two antennas.

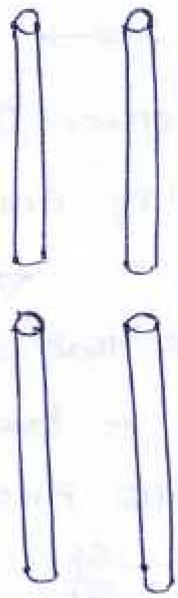


fig: Two antennas no.1 and no.2 with a wide separation.

Since the separation is more, mutual impedance (Z_{12}) is neglected

$$Z_{12} = 0$$

$$E_1 = Z_{11} I_1 + Z_{12} I_2$$

$$\therefore E_1 = Z_{11} I_1 \qquad [\because Z_{12} = 0]$$

$$\boxed{Z_{11} = \frac{E_1}{I_1}} \; **$$

The receiving antenna, under open circuit and short circuit conditions are as

AWP
U-I ②

→ The Electric charges are the Sources of the Electromagnetic fields. When these are time varying, then the Electromagnetic waves propagate away from the sources and the radiation takes place.

→ In general, the radiation can be considered as a process of transmitting energy. The radiation of the EM wave into the space is effectively done by using a conducting (or) dielectric structures called 'Antennas' or 'Radiators'.

→ In General, the antenna can be defined in no. of ways.

(1) A radio antenna may be defined as the structure associated with the region of transition between a guided wave and a free space or viceversa.
Guided Media and a free space or viceversa.

(2) A metallic device used for radiating or receiving the radio waves is called an 'Antenna'.

→ The system used for launching the EM waves is either by transmission line or by a waveguide. The antenna acts as a matching device between free space and the wave launching system.

→ Antenna having directional properties. It is the important component of a wireless communication system.

Basic Radiation Equation:—

→ The basic principle of an antenna is to produce radiations by accelerating or decelerating charge. These radiations are always perpendicular (at 90°) to the direction of

AWP
Q (9)

To prove the Reciprocity theorem, space b/w two antennas are replaced by linear, passive network:

From the figure,

Z_{11} = self impedance of antenna no. 1

Z_{22} = self impedance of antenna no. 2

Z_m = Mutual impedance = Z_{12} or Z_{21}

from Superposition theorem, by making E_{21} short.

By Applying KVL to loop 1 & loop 2

from loop 1 $\Rightarrow$ $\boxed{(Z_{11}+Z_m)I_1 - Z_m I_2 = E_{12}}$ $\longrightarrow$ (1)

from loop 2 $\Rightarrow$ $\boxed{(Z_{22}+Z_m)I_2 - Z_m I_1 = 0}$ $\longrightarrow$ (2) $[\because E_{21}=0]$

$$\boxed{I_2 = \frac{Z_m \quad I_1}{Z_{22}+Z_m}} \longrightarrow (3)$$

Substitute eqn (3) in eqn (1), then we get

$$(Z_{11}+Z_m)I_1 - \frac{Z_m^2 \ I_1}{Z_{22}+Z_m} = E_{12}$$

$$\Rightarrow I_1\left[(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2\right] = (Z_{22}+Z_m)E_{12}$$

$$I_1 = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m+\cancel{Z_m^2}-\cancel{Z_m^2}}$$

$$(4) \longleftarrow \boxed{\Rightarrow I_1 = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m} = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m}}$$

By substituting eqn (4) in eqn (3), we get

$$(5) \longleftarrow \boxed{I_2 = \frac{Z_m}{(\cancel{Z_{22}+Z_m})} * \frac{(\cancel{Z_{22}+Z_m})E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m} = \frac{Z_m E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m}}$$

Current I_1 in the meter can be obtained by symmetry.

Current I_1 in output port is

$$(6) \longleftarrow \boxed{I_1 = \frac{E_{21}Z_m}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})}}$$

AWP
U-I (3)

→ In the transmitting case, the radiated power is absorbed by Objects at a distance: trees, buildings, the ground, the sky and other antennas.

→ In the receiving case, passive radiation from distant objects or active radiation from other antennas raises the apparent temperature of Rr.

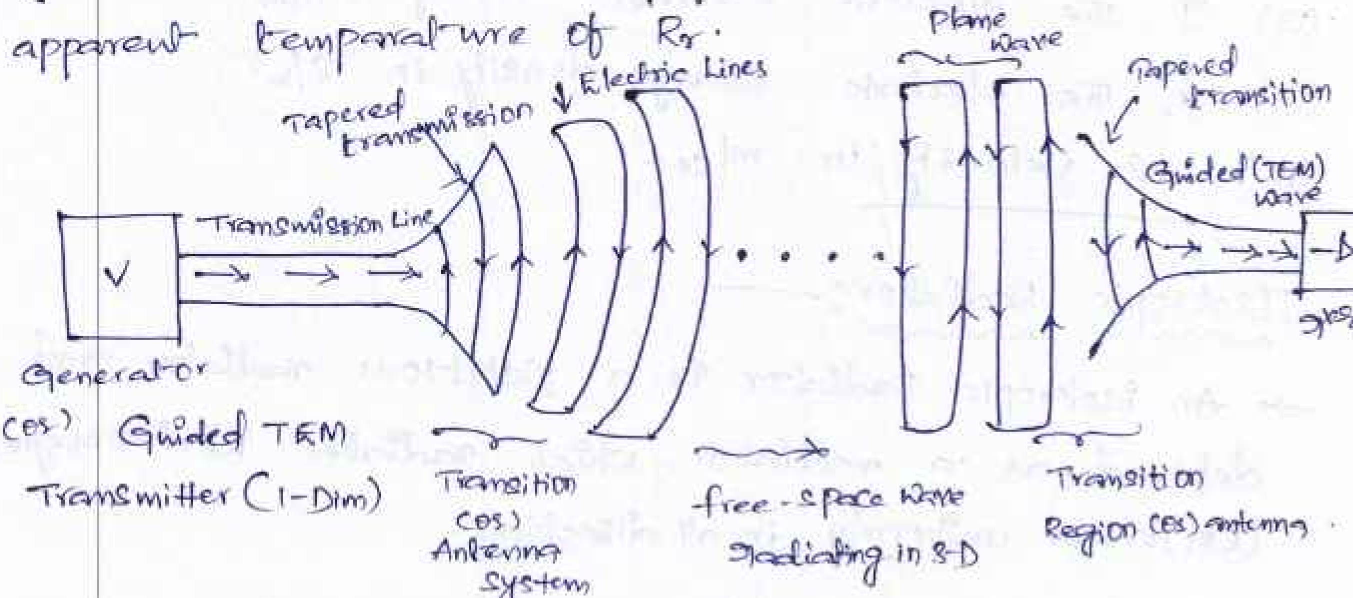


fig: wireless Communication link with transmitting antenna & Receiving Antenna

→ Antenna Theory is based on classification of Electromagnetic Theory as described by Maxwell's equation. Therefore a review of Electromagnetic phenomena is useful in order to have a proper understanding of Antenna Theory.

→ The important vector and scalar Quantities are:

(1) E, The electric field intensity in V/m.

(2) H, The Magnetic field intensity in A/m.

(3) D, The Electric displacement density in Col/m².

(4) B, The magnetic flux density in Wb/m².

(5) A, The magnetic vector potential in A/m².

(6) F, The Electric vector potential in V/m².

(7) P, The poynting vector in W/m².

AWP U-I (18)

T_{rs} = Receiver noise temp. at receiver terminals.

B_N = Band width.

→ The noise figure F, is related with effective noise Temp T_e as

***

$$F = 1 + \frac{T_e}{T_0} \text{ (or) } (F-1) = \frac{T_e}{T_0} \Rightarrow \boxed{T_e = (F-1)\,T_0}$$

where T_e = Effective noise temp^n, in °K.

$T_0 = 290°K$ $(273+17)°K$.

F = Noise figure (dimension less).

$$\boxed{F_{(dB)} = 10\log_{10} F}$$

Front-to-Back Ratio (FBR):

→ If we terminate an antenna, a travelling wave is produced. Thus the antennas that produce travelling waves are called as 'Aperiodic (or) Non-resonant (or) travelling waves antennas". The radiation pattern of such antennas consists of a front lobe, side lobes and a back lobe. Hence, the energy is radiated not only in one direction. Thus, front-to-back ratio is defined as the ratio of energy radiated in the forward direction to the energy radiated in the opposite direction.

$$\therefore \boxed{FBR = \frac{\text{Energy radiated in forward direction}}{\text{Energy radiated in backward direction}}}$$

→ The FBR depends on the following factors,

*> Antenna operating freq^n *> spacing b/w successive elements

*> parasitic elements electrical length.

*> Generally, higher values of FBR are desirable.

IMP
U-I (4)

Let us now imagine that an isotropic radiator is situated at the center of a sphere of radius (r). Then the energy (power) radiated from it, must pass over the surface area of the sphere (assume there is no obstruction to absorb the power) is $4\pi r^2$.

→ Poynting vector (or) power density $\bar{p}$ at any point on the sphere gives the "power radiated per unit area in any direction. Since radiated power from a isotropic source flows in radial lines, therefore for an isotropic radiator the magnitude of the poynting vector $\bar{p}$ is equal to the radial component only ($P_\theta = P_\phi = 0$).

$$|P| = P_r$$

Thus, if the poynting vector is known at all points on the sphere of radius 'r' from a point source in a loss less medium, the total power (W_t) radiated by the source is integral over the surface of the sphere of the radial component P_r of average poynting vector.

$$W_t = \iint \bar{P} \cdot ds = \iint P_r \cdot ds = P_r \iint ds \quad \{\theta = \phi = 0 \text{ for isotropic radiator}$$

$$W_t = P_r \cdot 4\pi r^2$$

$$\boxed{P_r = \frac{W_t}{4\pi r^2}} \quad W/m^2 \qquad |P| = P_r$$

$\iint ds \cos\phi \, ds = 4\pi r^2$ = area of surface

W_t → total power radiated, in watts.

P_r → Radial component of average power density Poynting vector, W/m^2.

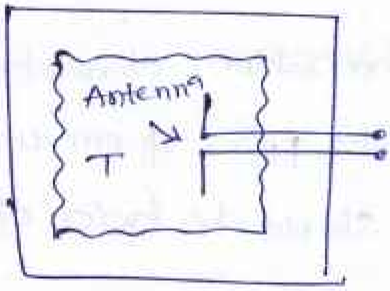
ds → Infinitesimal element of area of sphere of radius $r = r^2 \sin\theta \, d\theta \, d\phi$

r → radius of sphere in meters.

AWP. U-I (17)

Antenna Temperature :-

If the resistor 'R' is replaced by a lossless antenna of radiation resistance 'R' in an anechoic chamber at temp. 'T' the noise power per unit BW, available at the terminals is unchanged.



Antenna at ANECHOIC chamber at temp. 'T'.

→ If the power per unit Bandwidth $\frac{P}{B}$ is independent of frequ the total power 'P' is obtained by multiplying with the Band-width 'B' i.e.,

$P = KTB$ watts

$P \rightarrow$ Total power , $B \rightarrow$ Bandwidth in Hertz.

Let the antenna has an effective aperture Ae and that its beam is directed at a source of radiation which produces a power density per unit Bandwidth ~~or~~ flux density (S) at the antenna. The power received from the source is given by

$P = SA_eB$ watts

$S \rightarrow$ power density per unit BW in Watts/m² Hz.

$A_e \rightarrow$ Effective aperture.

→ power density per unit Bandwidth or flux density from the source at the antenna is

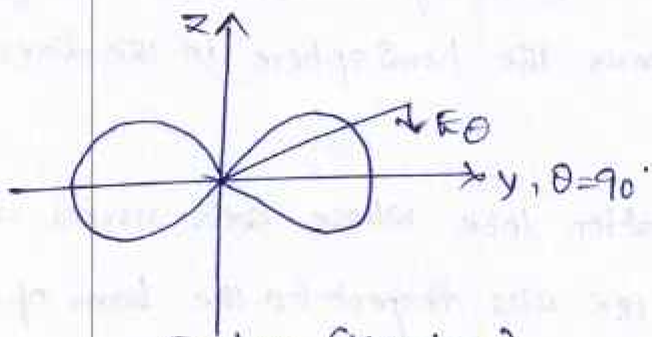
$\boxed{P = SA_eB = KTB}$

$\boxed{S = \frac{KT_A}{A_e}}$ W/m² · Hz $T_A \rightarrow$ Antenna temp. due to the source, in °K.

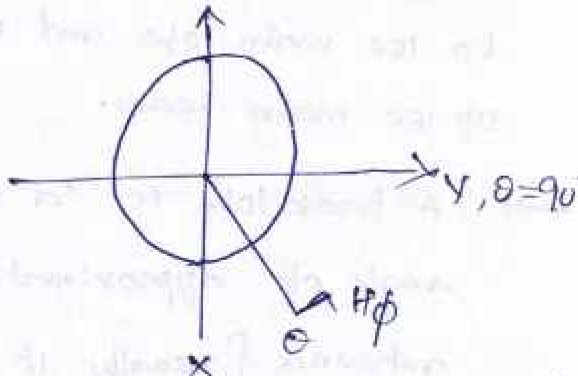
$\boxed{T_A = \frac{SA_e}{K}}$ °K.

E-plane pattern: It is 'the plane containing the Electric field vector and the direction of maximum radiation'.

H-plane pattern:— It is 'the plane containing the H-field vector and the direction of maximum radiation'.



E-plane (YZ-plane)



H-plane (XY-plane).

Radiation pattern Lobes:—

→ Various parts of a radiation pattern are referred to as 'Lobes'. Which may be Sub-classified into major (or) main, minor, side and back lobes.

→ A radiation lobe is a 'portion of the radiation pattern bounded by regions of relatively weak radiation intensity'.

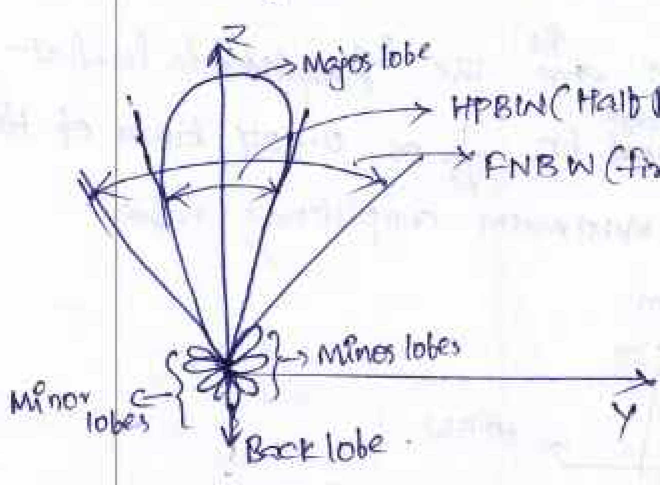


fig:- Radiation lobes & Beam widths of an antenna pattern

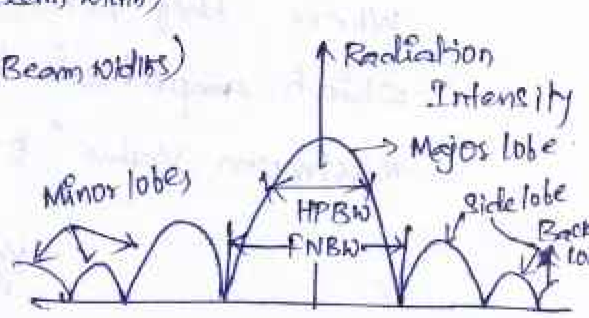


fig:- Linear plot of power pattern and its associated lobes and Beam widths.

figure 1(a) is the opened out two conductor transmission line. In this case, the input impedance is almost constant for $d \ll \lambda$ and $D \geq \lambda$ if it is extended far. Next, the curved conductors are made straight to form regular cones as shown in figure 1(b). This dipole provides maximum radiation in only one direction.

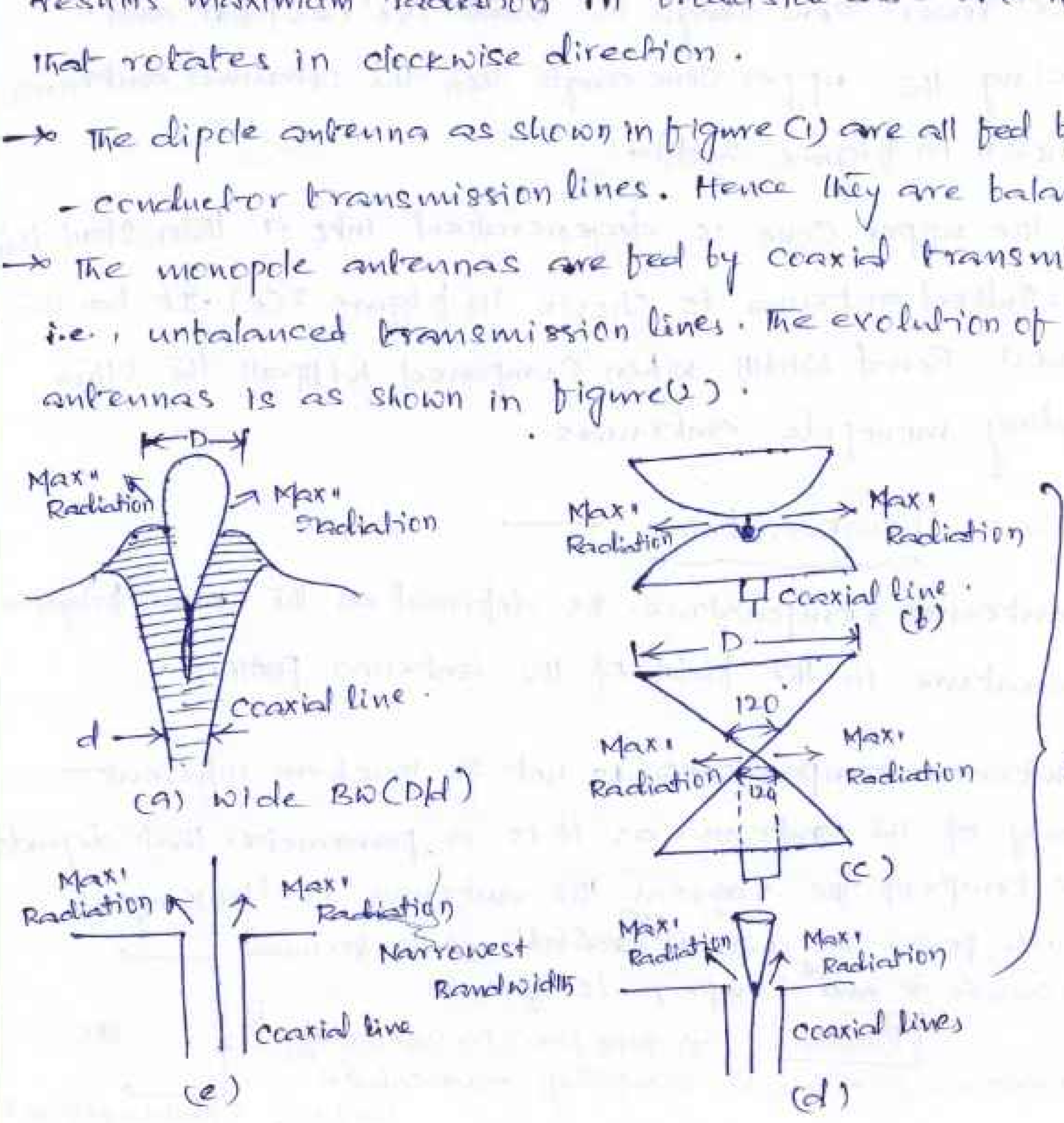
A biconical antenna is formed when the cones are arranged collinearly as shown in figure 1(c). Such a dipole antenna is omnidirectional in the horizontal plane.

Next, a thin cylindrical dipole antenna shown in figure 1(d) is formed by collapsing the cones of figure 1(c) into straight wires.

→ figure 1(e) shows a spiral antenna formed by two conductors that the sharply curved in the opposite direction. This antenna results maximum radiation in broadside case and has polarization that rotates in clockwise direction.

→ The dipole antenna as shown in figure (1) are all fed by two-conductor transmission lines. Hence they are balanced.

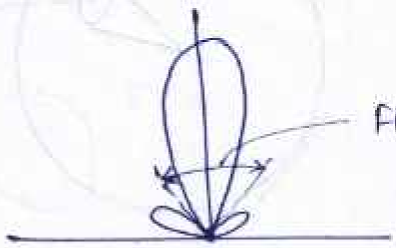
→ The monopole antennas are fed by coaxial transmission lines i.e., unbalanced transmission lines. The evolution of monopole antennas is as shown in figure(2).

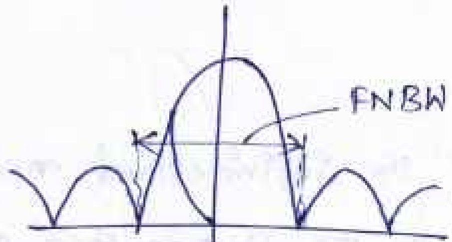


→ Beam width Between first Nulls (FNBW):—

FNBW defined as the Beam width between first nulls.

→ Null is defined as the direction in which the radiation is zero (or) direction in which antenna doesn't ~~radiation~~ radiate in that direction.





Radian & steradian:—

Radian:—

→ The measure of a plane angle is a 'Radian'.

One Radian is defined as the plane angle with its vertex at the center of a circle of radius 'r' that is subtended by an arc whose length is 'r' since the Circumference of a circle of radius 'r' is $C = 2\pi r$, there are 2π rad $(2\pi r/r)$ in a full circle i.e., 2π times of Arc's can cover the entire circle and the length of each arc is 'r' distance.

$\therefore$ 2π Arcs total distance $= 2\pi r$.

1 Arc distance $= r$

So 1 radian $= \frac{2\pi r}{r} = 2\pi$.

steradian:—

→ The measure of a solid angle is a steradian.

One steradian is defined as the solid angle with its vertex at the center of a sphere of radius i.e., subtended by a spherical

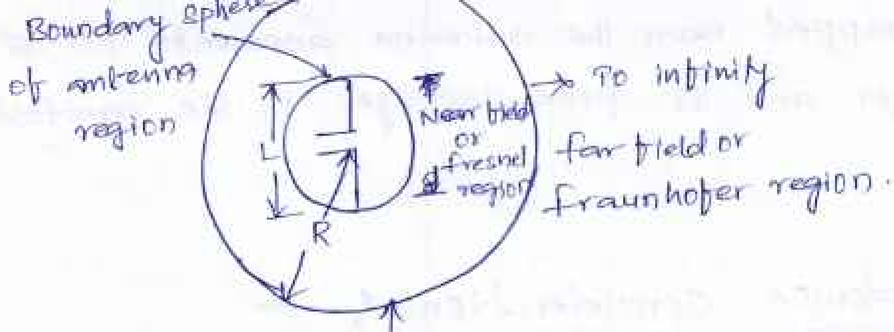


figure ①

→ This imaginary circle/sphere is called as the "fresnel-fraunhofer Boundary sphere.

field Patterns :-

→ The radiation field pattern for a dipole antenna is as shown in the figure below.

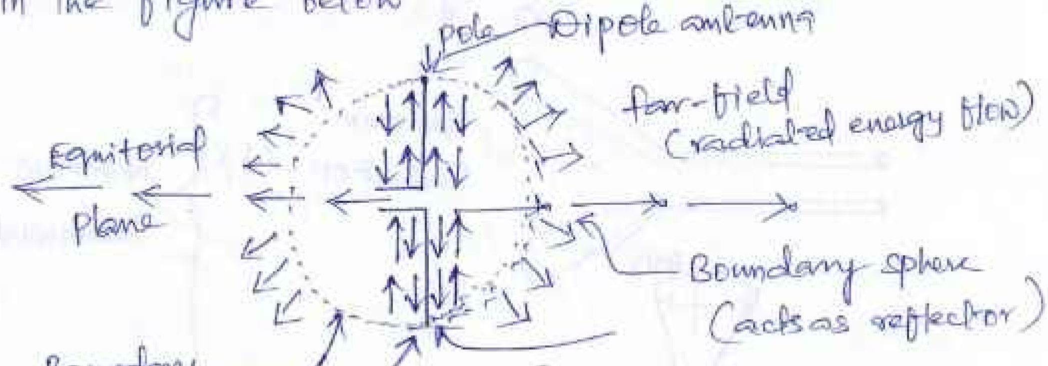


figure ②

** field components move radially out-wards from the boundary sphere in the far field zone, whereas in the near field region, the power flow is not completely radial.

** field Patterns shape depends on the distance in the near field regions whereas in far field regions it doesnot depend on the distance.

** The imaginary boundary sphere acts partially transparent and partially opaque.

** There is power leakage at the equitorial region where the sphere acts transparent and the waves escape out expanding in the perpendicular directions to the dipole as shown in figure ②.

** Near the poles, the sphere acts as a reflector (opaque) resulting in reciprocating energy flow near the antenna.

AWP
U-I (7)

Where P = instantaneous total power (W)

$\hat{n}$ = unit vector normal to the surface.

da = infinitesimal area of the closed surface (m^2).

W_{rad} = radiated power density

$W_{rad} = \frac{1}{2} Re[E \times H^*]$ W/m^2 → phasor notation

$W_{rad} = \hat{a}_r \cdot W_{avg}$

→ The average power radiated by an antenna (radiated power) can be written as

$$P_{rad} = P_{avg} = \oint\!\!\oint_S W_{rad} \cdot ds = \oint\!\!\oint_S W_{avg} \cdot \hat{n}\, da$$

$$= \frac{1}{2} \oint\!\!\oint_S Re(E \times H^*) ds$$

→ for an isotropic radiator, the total power radiated by it is given by

$$P_{rad} = \oint\!\!\oint_S W_0 \cdot ds = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} [\hat{a}_r W_0(r)] \cdot [\hat{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi]$$

$$\boxed{P_{rad} = 4\pi r^2 W_0(r)}$$

Since the power density is the power per unit area. but we know that Area of the sphere is $4\pi r^2$ if the radius of the sphere is 'r'. so the power density of an isotropic source given by

$$\boxed{W_0 = \hat{a}_r \cdot W_0(r) = \hat{a}_r \left[\frac{P_{rad}}{4\pi r^2}\right]} \quad W/m^2.$$

Which is uniformly distributed over the surface of sphere of radius 'r'.

I (14)

In oscillating dipole antenna consists of equal and opposite charges placed at two ends of the dipole (as shown in figure①). These charges oscillate up and down in harmonic motion when an electric field is applied and their separation varies with the change in period 'T'.

→ To illustrate the radiation from a dipole antenna, Let us assume a single electric field line and its variation with change in separation of charges at different time 't' as shown in figure(1)

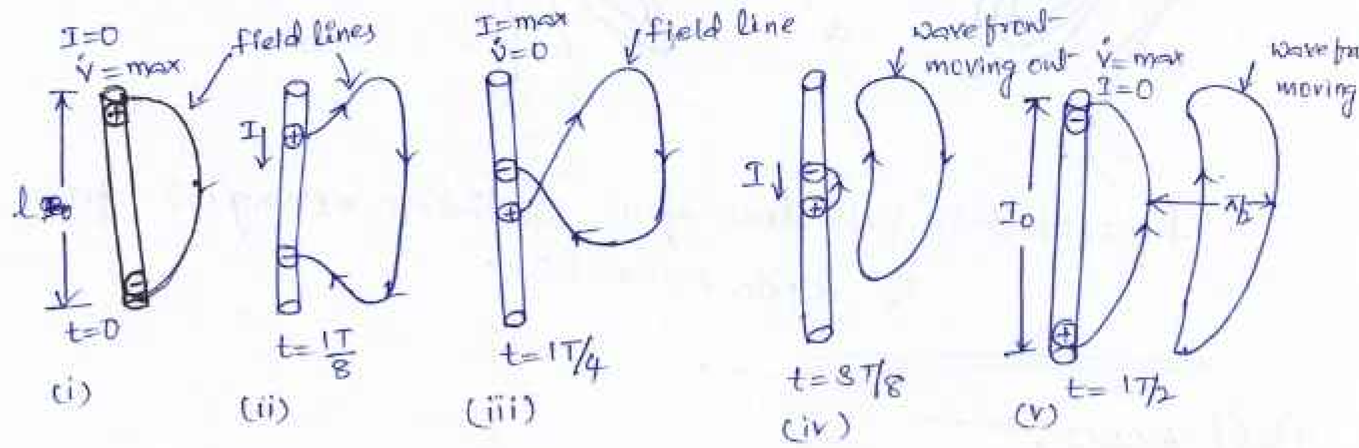


figure (1)

from the figure ①,

*→ At t=0, the separation is maximum, i.e., 'l'.

Acceleration of charges is maximum i.e., $\dot{v}$. and Current $I=0$.

The corresponding propagation of an Electric field line is shown in figure 1(i).

*→ At $t=T/8$, charges move towards each other and the field line variations are as shown in figure 1(ii).

*→ At $t=T/4$, charges arrive at the mid point and the field lines separate to form new field lines of opposite charge as shown in figure 1(iii). In this case, the charge acceleration, $\dot{v}=0$ & Current 'I' is maximum.

AWP U-I ⑧

Directivity :— (D)

Directivity of an antenna is defined as 'the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged in all directions'.

The average radiation intensity is equal to the total power radiated by the antenna devided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied.

→ The direction of a non-isotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

In Mathematically, D can be written as

$$\boxed{D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}}$$

→ Directivity is a measure that describes only the directional properties of the antenna.

→ If the direction is not specified, it implies the direction of maximum radiation intensity (max. directivity) expressed as

$$\boxed{D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}}$$

D = Directivity (Dimensionless)

D_0 = Maximum Directivity (Dimensionless)

U = Radiation intensity (Watts/unit solid angle)

U_{max} = Max. radiation intensity (Watts/unit solid angle)

U_0 = Radiation intensity of isotropic source [Watts/unit solid angle]

P_{rad} = total radiated power (Watt)

AWP U-I (13)

When the antenna is receiving with a load resistance R_L matched to the antenna radiation resistance R_r ($R_L = R_r$), as much power is reradiated from the antenna as is delivered to the load. This is the condition of maximum power transfer (antenna assumed lossless).

Effective height (he):—

Effective height is the ratio of induced voltage at the terminal of the receiving antenna under the open circuit condition to the incident Electric-field intensity or strength

→ the Effective height h_e (meters) of antenna is the parameter related to the aperture. Multiplying the effective height by the incident field E (V/mtr) of the same polarization gives the voltage V induced.

Thus, $\boxed{V = h_e E}$ → (1)

Accordingly, the effective height may be defined as the ratio of the induced voltage to the incident field are

$$\boxed{h_e = V/E} \longrightarrow (2)$$

→ Effective height is to consider the transmitting case and equate the effective height to the physical height (or length l) multiplied by the (normalized) average current or

$$\boxed{h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\,dz = \frac{I_{av}}{I_0} \cdot h_p \quad (m)} \longrightarrow (3)$$

where

$h_e \rightarrow$ Effective height, mtrs

$h_p \rightarrow$ Physical height, mtrs.

$I_{av} \rightarrow$ Average Current, Amp's.

AWP L-7 (9)

→ Let the radiation intensity of an antenna is of the form

$$U = B_0 F(\theta,\phi) \approx \frac{1}{2\eta}\left[|E_\theta^\circ(\theta,\phi)|^2 + |E_\phi^\circ(\theta,\phi)|^2\right]$$

where B_0 is a constant and E_θ° and E_ϕ° are the antennas far-zone electric field components.

The maximum value of 'U' is given by

$$U_{max} = B_0 F(\theta,\phi)\Big|_{max} = B_0 F_{max}(\theta,\phi)$$

The total radiated power using

$$P_{rad} = \oint_\Omega U(\theta,\phi)\,d\Omega = B_0\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

we now write the general expression for the directivity and maximum directivity using the following formulae respectively as

$$U(\theta,\phi) = 4\pi\,\frac{F(\theta,\phi)}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

$$D_0 = 4\pi\,\frac{F(\theta,\phi)\big|_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

The above equation can also be written as

$$D_0 = \frac{4\pi}{\left[\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi\right] \Big/ F(\theta,\phi)\big|_{max}} = \frac{4\pi}{\Omega_A}$$

where Ω_A is the beam solid angle and it is given by

$$\Omega_A = \frac{1}{F(\theta,\phi)\big|_{max}}\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

where

$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)|_{max}}$$

Thus, the electromagnetic horn may be regarded as having an aperture. The total power it extracts from a passing wave being proportional to the aperture or area of its mouth.

But the field response of the horn is not uniform across the aperture 'A' because E-field at the side walls must equal to zero. Thus, the effective aperture 'Ae' of the horn is less than the physical aperture 'Ap' as given by

$$\boxed{\text{Aperture Efficiency}(\varepsilon_{ap}) = \frac{A_e}{A_p} \text{ (dimension less)}} \longrightarrow (2)$$

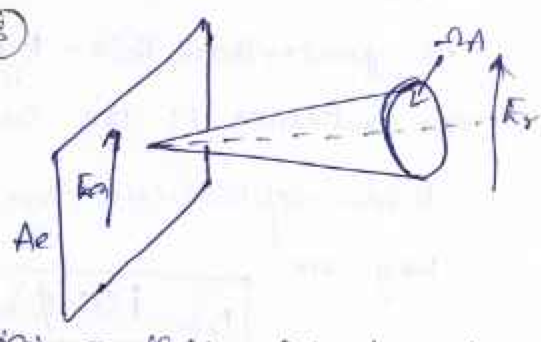
for horn and parabolic reflector antenna, ε_{ap} are commonly in the range of 50 to 80% ($0.5 \le \varepsilon_{ap} \le 0.8$).

→ Large dipole or patch arrays with uniform field to the edges of the physical aperture may attain higher aperture efficiencies approaching 100%.

However, to reduce sidelobes, fields are commonly tapered toward the edges resulting in reduced aperture efficiency.

→ Consider now an antenna with an effective aperture 'Ae' which radiates all of its power in a conical pattern of beam area Ω_A. Assuming a uniform field E_a over the aperture. The power radiated is

$$\boxed{P = \frac{E_a^2}{Z_0} A_e} \text{ (Watts)} \longrightarrow (3)$$



where Z_0 = intrinsic impedance of Medium (377 Ω for air or vacuum)

fig: Radiation over beam Area

Assuming a uniform field E_r in the far-field at a distance 'r', the power radiated is given by

$$\boxed{P = \frac{E_r^2}{Z_0} \cdot r^2 \Omega_A} \text{ (Watts)} \longrightarrow (4)$$

→ Relative Gain :— is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its reference direction".

→ The Reference antenna is usually a dipole, horn or any other antenna whose gain can be calculated or it is known.

→ However, the reference antenna is a lossless isotropic source.

thus.

$$G = \frac{4\pi U(\theta,\phi)}{P_{in}(\text{Lossless isotropic Source})} \quad (\text{dimensionless})$$

→ when the direction is not stated, the power gain is usually taken in the direction of maximum radiation.

→ We can write that the total radiated power (P_{rad}) is related to the total input power (P_{in}) by

$$\boxed{P_{rad} = e_{cd} P_{in}}$$

where e_{cd} = antenna radiation efficiency (dimensionless)

using the above expressions

$$\boxed{G(\theta,\phi) = e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{rad}}\right]}$$

which is related to the directivity by

$$G(\theta,\phi) = e_{cd} \cdot D(\theta,\phi)$$

In a similar manner, the maximum value of the gain is related to the maximum directivity by

$$G_0 = G(\theta,\phi)\Big|_{max} = e_{cd}\, D(\theta,\phi)\Big|_{max} = e_{cd} \cdot D_0 .$$

→ partial gain of an antenna for a given polarization in a given direction as that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the

Since π radian = 180

(or) 1 radian = $\left(\frac{180}{\pi}\right)$

Therefore, 4π steradians = $3282.7909 \times 4\pi$ (deg)^2

$= 13131.163 * 3.1416$ (deg)^2

$= 41252.861$ (deg)^2

$4\pi Sr \cong 41253$ (deg)^2 = solid angle in a sphere.

The beam Area (or beam Solid angle) Ω_A for an antenna is therefore given by the integral of the normalized power pattern over a sphere ($4\pi Sr$).

or $$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\, d\Omega \; Sr$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi \; Sr$$

where $P_n(\theta,\phi)$ = ~~normalized~~ normalized power pattern.

$$\boxed{P_n(\theta,\phi) = \frac{P(\theta,\phi)}{P(\theta,\phi)\big|_{max}}}$$

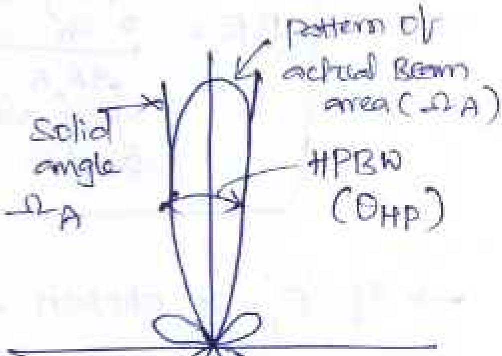


fig: A symmetrical power pattern of antenna with equival solid angle Ω_A.

→ from the figure, the beam area Ω_A of an actual pattern is equivalent to the same solid angle subtended by the spherical cap of the cone shaped pattern (triangular cross-section)

→ solid angle is also described approximately interms of the angles subtended by the half-power-points of the main lobe in principal planes.

$$\Omega_A = \theta_{HP} \cdot \phi_{HP} \; (Sr)$$

where θ_{HP} = HPBW in E-plane or θ-plane

ϕ_{HP} = HPBW in H-plane or ϕ-plane.

Since π radian $= 180°$

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Therefore, 4π steradians $= 3282.7909 \times 4\pi \ (\text{deg})^2$

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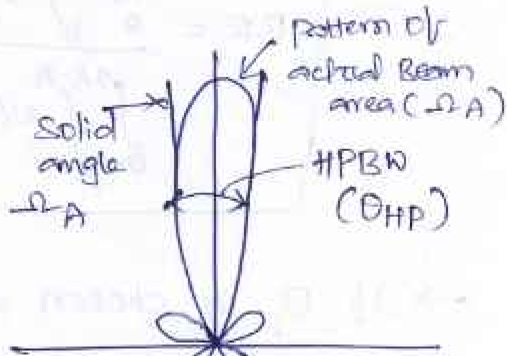

fig: A symmetrical power pattern of antenna with equivalent Solid angle Ω_A.

→ from the figure, the beam area Ω_A of an actual pattern is equivalent to the same solid angle subtended by the spherical cap of the cone shaped pattern (triangular Cross-Section)

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NI
u-I (10)

→ **Relative Gain:** — is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its reference direction".

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→ However, the reference antenna is a lossless isotropic source.

Thus.

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In a similar manner, the maximum value of the gain is related to the maximum directivity by

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AWP U-III (12)

Thus, the electromagnetic horn may be regarded as having an aperture. The total power it extracts from a passing wave being proportional to the aperture or area of its mouth.

But the field response of the horn is not uniform across the aperture 'A' because E-field at the side walls must equal to zero. Thus, the effective aperture Ae of the horn is less than the physical aperture 'Ap' as given by

$$\boxed{\text{Aperture Efficiency}(\varepsilon_{ap}) = \frac{A_e}{A_p} \quad (\text{dimension less})} \longrightarrow (2)$$

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$$\boxed{P = \frac{E_a^2}{Z_0} A_e} \quad (\text{Watts}) \longrightarrow (3)$$

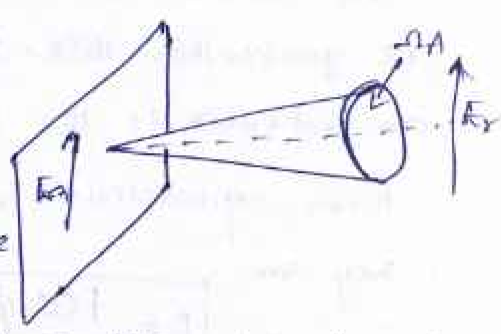


fig: Radiation over beam Area

where Z_0 = intrinsic impedance of Medium ($377\,\Omega$ for air or vacuum)

Assuming a uniform field E_r in the far-field at a distance 'r', the power radiated is given by

$$\boxed{P = \frac{E_r^2}{Z_0} \cdot r^2 \Omega_A} \quad (\text{Watts}) \longrightarrow (4)$$

ANP
L-7 9

→ Let the radiation intensity of an antenna is of the form

$$U = B_0 F(\theta,\phi) \simeq \frac{1}{2\eta}\left[|E_\theta^0(\theta,\phi)|^2 + |E_\phi^0(\theta,\phi)|^2\right]$$

where B_0 is a constant and E_θ^0 and E_ϕ^0 are the antennas far-zone electric field components.

The maximum value of 'U' is given by

$$U_{max} = B_0 F(\theta,\phi)\Big|_{max} = B_0 F_{max}(\theta,\phi)$$

The total radiated power using

$$P_{rad} = \oint\!\!\oint_\Omega U(\theta,\phi)\,d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

we now write the general expression for the directivity and maximum directivity using the following formulae respectively as

$$U(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

$$D_0 = 4\pi \frac{F(\theta,\phi)|_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

The above equation can also be written as

$$D_0 = \frac{4\pi}{\left[\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi\right] \Big/ F(\theta,\phi)|_{max}} = \frac{4\pi}{\Omega_A}$$

where Ω_A is the beam solid angle and it is given by

$$\Omega_A = \frac{1}{F(\theta,\phi)|_{max}} \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\,d\theta\,d\phi = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

where

$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)|_{max}}$$

AWP U-I (13)

When the antenna is receiving with a load resistance R_L matched to the antenna radiation resistance R_r ($R_L = R_r$), as much power is reradiated from the antenna as is delivered to the load. This is the condition of maximum power transfer (antenna assumed lossless).

Effective height (he):—

Effective height is the ratio of induced voltage at the terminal of the receiving antenna under the open circuit condition to the incident electric-field intensity or strength.

→ The effective height he (meters) of antenna is the parameter related to the aperture. Multiplying the effective height by the incident field E (V/mts) of the same polarization gives the voltage V induced.

Thus,
$$\boxed{V = h_e E} \longrightarrow (1)$$

Accordingly, the effective height may be defined as the ratio of the induced voltage to the incident field are

$$\boxed{h_e = V/E} \longrightarrow (2)$$

→ Effective height is to consider the transmitting case and equate the effective height to the physical height (or length 'l') multiplied by the (normalized) average current or

$$\boxed{h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\, dz = \frac{I_{av}}{I_0} h_p \quad (m)} \longrightarrow (3)$$

where

he → Effective height, mtrs

hp → Physical height, mtrs.

Iav → Average current, Amp's.

AWP U-I (8)

Directivity :— (D)

Directivity of an antenna is defined as 'the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged in all directions'.

The average radiation intensity is equal to the total power radiated by the antenna devided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied.

→ The direction of a non-isotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

In Mathematically, 'D' can be written as

$$\boxed{D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}}$$

→ Directivity is a measure that describes only the directional properties of the antenna.

→ If the direction is not specified, it implies the direction of maximum radiation intensity (max. directivity) expressed as

$$\boxed{D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}}$$

D = Directivity (Dimensionless)

D_0 = Maximum Directivity (Dimensionless)

U = Radiation intensity (Watts/unit solid angle)

U_{max} = Max. radiation intensity (Watts/unit solid angle)

U_0 = Radiation intensity of isotropic source [Watts/unit solid ang

P_{rad} = total radiated power (Watts)

I (14)

In oscillating dipole antenna consists of equal and opposite charges placed at two ends of the dipole (as shown in figure①). These charges oscillate up and down in harmonic motion when an electric field is applied and their separation varies with the change in period 'T'.

→ To illustrate the radiation from a dipole antenna, Let us assume a single electric field line and its variation with change in separation of charges at different time 't' as shown in figure(1)

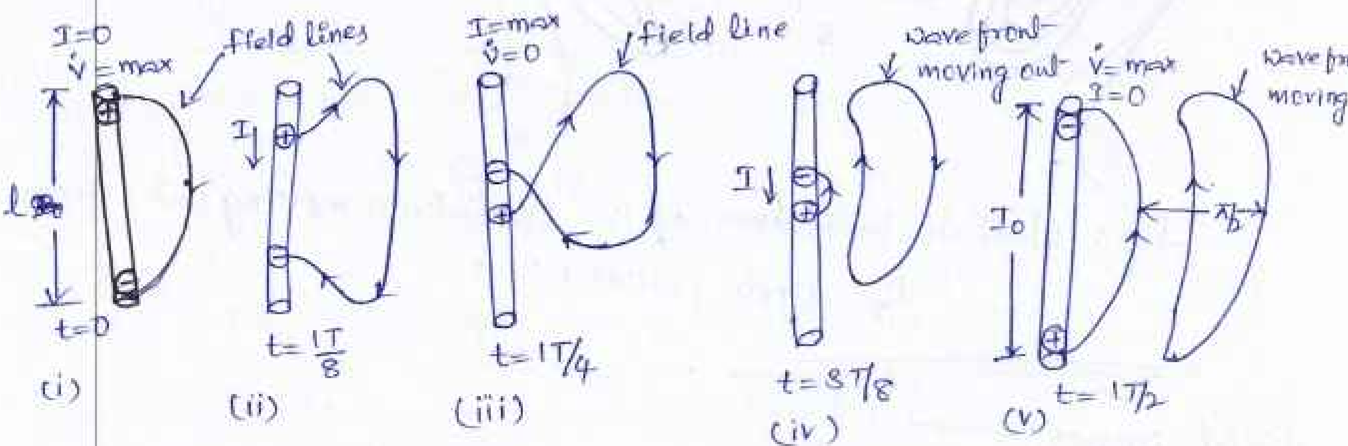


figure (1)

from the figure ①.

*→ At t=0, the separation is maximum, i.e., 'l'.

Acceleration of charges is maximum i.e., $\dot{v}$. and Current I=0.

The corresponding propagation of an electric field line is shown in figure 1(i).

*→ At t= T/8, charges move towards each other and the field line variations are as shown in figure 1(ii).

*→ At t=T/4, charges arrive at the mid point and the field lines separate to form new field lines of opposite charge as shown in figure 1(iii). In this case, the charge acceleration, $\dot{v}$=0 & Current 'I' is maximum.

Where P = instantaneous total power (W)

$\hat{n}$ = unit vector normal to the surface.

da = infinitesimal area of the closed surface (m^2).

W_{rad} = radiated power density

$W_{rad} = \frac{1}{2} Re[E \times H^*]$ W/m² → phasor notation

$W_{rad} = \hat{a}_r \cdot W_{avg}$

→ The average power radiated by an antenna (radiated power) can be written as

$$P_{rad} = P_{avg} = \oiint_S W_{rad} \cdot ds = \oiint_S W_{avg} \cdot \hat{n}\, da$$

$$= \frac{1}{2} \oiint_S Re(E \times H^*) ds$$

→ for an isotropic radiator, the total power radiated by it is given by

$$P_{rad} = \oiint_S W_0 \cdot ds = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} [\hat{a}_r W_0(r)] \cdot [\hat{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi]$$

$$\boxed{P_{rad} = 4\pi r^2 W_0(r)}$$

Since the power density is the power per unit area. but we know that Area of the sphere is $4\pi r^2$ if the radius of the sphere is 'r'. so the power density of an isotropic source given by

$$\boxed{W_0 = \hat{a}_r \cdot W_0(r) = \hat{a}_r \left[\frac{P_{rad}}{4\pi r^2}\right]} \quad W/m^2.$$

Which is uniformly distributed over the surface of sphere of radius 'r'.

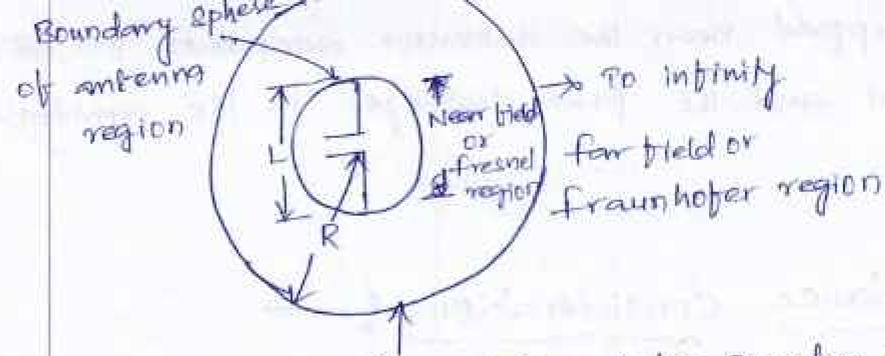


figure ①

→ This imaginary circle/sphere is called as the "fresnel-fraunhofer Boundary sphere.

field Patterns :—

→ The radiation field pattern for a dipole antenna is as shown in the figure below:

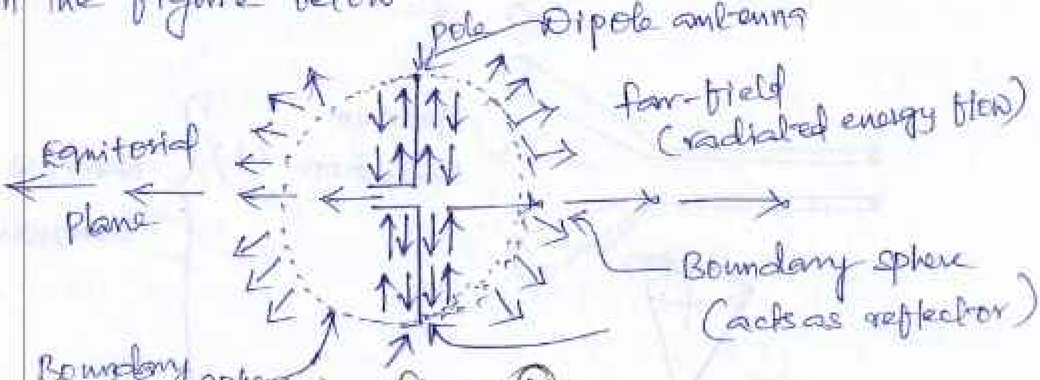


figure ②

** field components move radially out-wards from the boundary sphere in the far field zone, whereas in the near field region, the power flow is not completely radial.

** field Patterns shape depends on the distance in the near field regions whereas in far field regions it doesnot depend on the distance.

** The imaginary boundary sphere acts partially transparent and partially opaque.

** There is power leakage at the equitorial region where the sphere acts transparent and the waves escape out expanding in the perpendicular directions to the dipole as shown in figure ②.

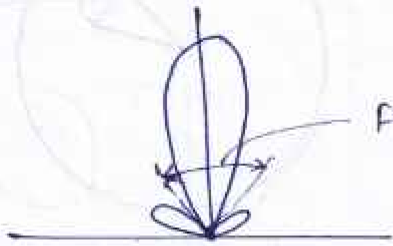
** Near the poles, the sphere acts as a reflector (opaque) resulting in reciprocating energy flow near the antenna.

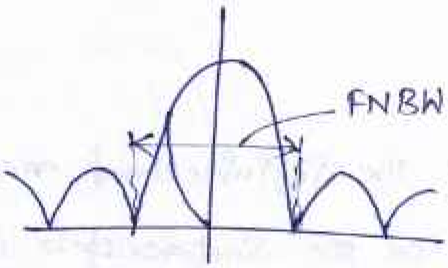
-NWT I(6)

→ Beam width Between first Nulls (FNBW):—

FNBW defined as the Beam width between first nulls.

→ Null is defined as the direction in which the radiation is zero (or) direction in which antenna doesn't radiate in that direction.





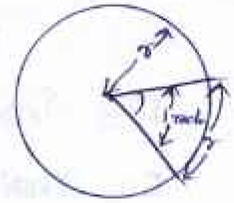
Radian & Steradian:—

Radian:—

→ The measure of a plane angle is a 'Radian'.

One Radian is defined as the plane angle with its vertex at the center of a circle of radius 'r' that is subtended by an arc whose length is 'r' since the circumference of a circle of radius 'r' is $C = 2\pi r$, there are 2π rad ($2\pi r/r$) in a full circle

i.e., 2π times of Arcs can cover the entire circle and the length of each arc is 'r' distance.



$\therefore$ 2π Arcs total distance $= 2\pi r$.

1 Arc distance $= r$.

So 1 radian $= \frac{2\pi r}{r} = 2\pi$.

Steradian:—

→ The measure of a solid angle is a steradian.

One steradian is defined as the solid angle with its vertex at the center of a sphere of radius i.e., subtended by a spherical figure 1(a) is the opened out two conductor transmission line.

In this case, the input impedance is almost constant for $d<<\lambda$ and $D\geq\lambda$ if it is extended far. Next, the curved conductors are made straight to form regular cones as shown in figure 1(b). This dipole provides maximum radiation in only one direction.

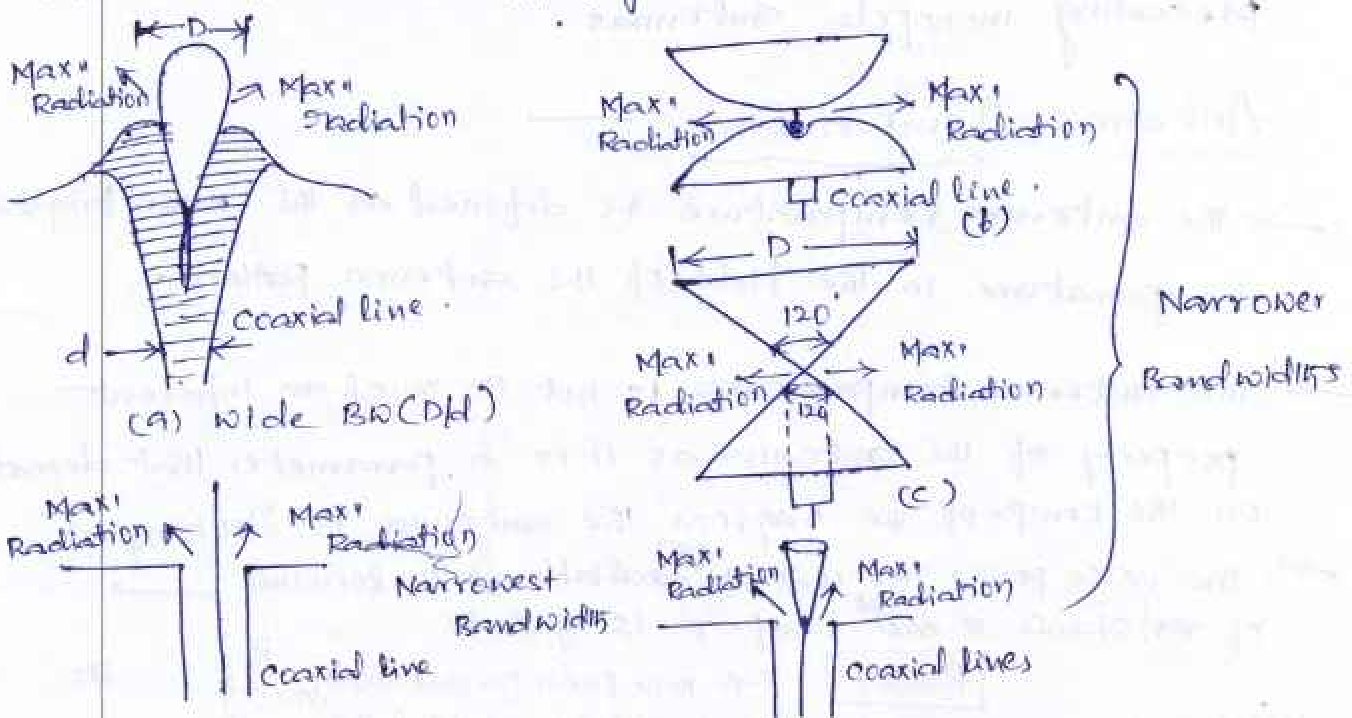
A biconical antenna is formed when the cones are arranged collinearly as shown in figure 1(c). Such a dipole antenna is omnidirectional in the horizontal plane.

Next, a thin cylindrical dipole antenna shown in figure 1(d) is formed by collapsing the cones of figure 1(c) into straight wires.

→ figure 1(e) shows a spiral antenna formed by two conductors that the sharply curved in the opposite direction. This antenna results maximum radiation in broadside case and has polarization that rotates in clockwise direction.

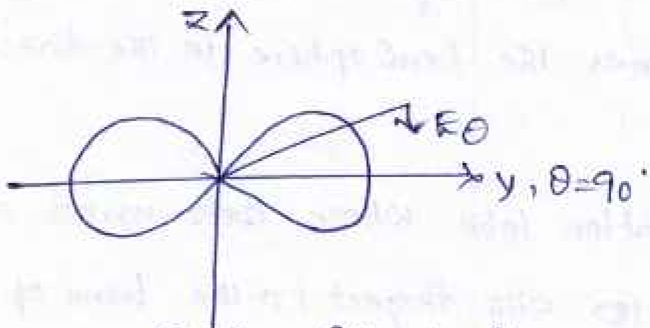
→ The dipole antenna as shown in figure (1) are all fed by two-conductor transmission lines. Hence they are balanced.

→ The monopole antennas are fed by coaxial transmission lines i.e., unbalanced transmission lines. The evolution of monopole antennas is as shown in figure(2).

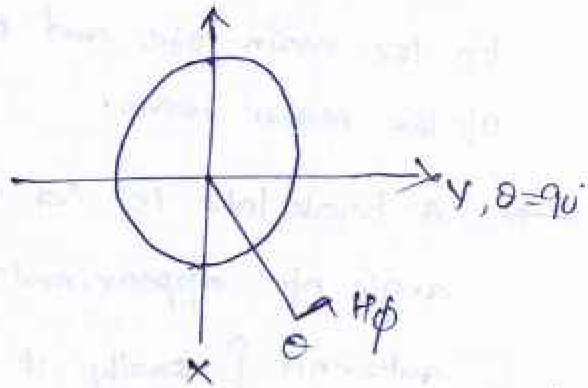


E-plane pattern: It is 'the plane containing the Electric field vector and the direction of maximum radiation'.

H-plane pattern:— It is 'the plane containing the H-field vector and the direction of maximum radiation'.



E-plane (YZ-plane)



H-plane (XY-plane).

Radiation pattern Lobes:—

→ Various parts of a radiation pattern are referred to as 'Lobes'. which may be sub-classified into major (or) main, minor, side and back lobes.

→ A radiation lobe is a 'portion of the radiation pattern bounded by regions of relatively weak radiation intensity'.

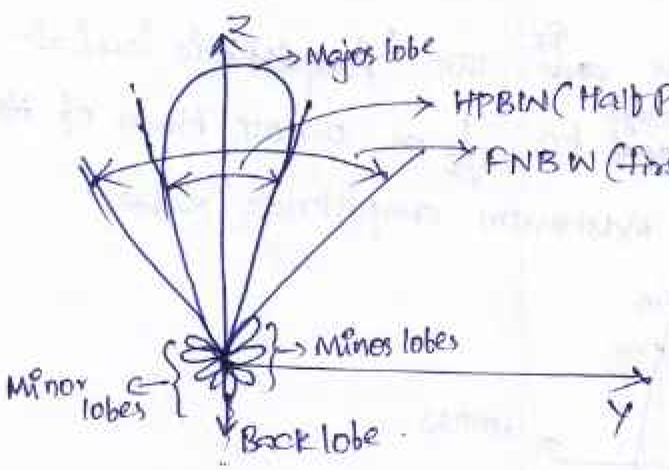


fig:- Radiation lobes & Beam widths of an antenna pattern

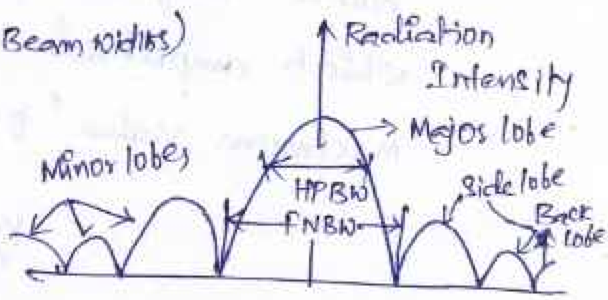
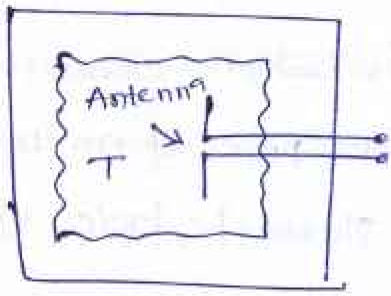


fig:- Linear plot of power pattern and its associated lobes and Beam widths.

AWP.
U-I (A)

Antenna Temperature :-

If the resistor 'R' is replaced by a lossless antenna of radiation resistance 'R' in an anechoic chamber at temp. 'T' the noise power per unit BW, available at the terminals is unchanged.



Antenna at ANECHOIC chamber at temp. 'T'.

→ If the power per unit Bandwidth $\frac{P}{B}$ is independent of freque[ncy] the total power 'P' is obtained by multiplying with the Band-width 'B' i.e.,

$$P = KTB \text{ watts}$$

$P \rightarrow$ Total power , $B \rightarrow$ Bandwidth in Hertz.

Let the antenna has an effective aperture Ae and that its beam is directed at a source of radiation which produces a power density per unit Bandwidth ~~or~~ flux density (S) at the antenna. The power received from the source is given by

$$P = S A_e B \text{ watts}$$

$S \rightarrow$ powerdensity per unit BW in Watts/m^2 Hz.

$A_e \rightarrow$ Effective aperture.

→ power density per unit Bandwidth or flux density from the source at the antenna is

$$\boxed{P = S A_e B = KTB}$$

$$\boxed{S = \frac{KT_A}{A_e}} \quad W/m^2 \cdot Hz$$

$T_A \rightarrow$ Antenna temp. due to the source, in $^\circ K$.

$$\boxed{T_A = \frac{S A_e}{K}} \quad ^\circ K.$$

AWP
U-I (4)

Let us now imagine that an isotropic radiator is situated at the center of a sphere of radius (r). Then the energy (power) radiated from it, must pass over the surface area of the sphere (assume there is no obstruction to absorb the power) is $4\pi r^2$.

→ Poynting vector (or) power density $\bar{P}$ at any point on the sphere gives the "power radiated per unit area" in any direction. Since radiated power from a isotropic source flows in radial lines, therefore for an isotropic radiator the magnitude of the poynting vector $\bar{P}$ is equal to the radial component only ($P_\theta = P_\phi = 0$).

$$|P| = P_r$$

Thus, if the poynting vector is known as all points on the sphere of radius 'r' from a point source in a loss less medium, the total power (W_t) radiated by the source is integral over the surface of the sphere of the radial component P_r of average poynting vector.

$$W_t = \iint \bar{P}\cdot ds = \iint P_r \cdot ds = P_r \iint ds$$ $\{\theta = \phi = 0$ for isotropic radiator

$$W_t = P_r \cdot 4\pi r^2$$

$$\boxed{P_r = \frac{W_t}{4\pi r^2}} \quad W/m^2$$

$|P| = P_r$

$\iint ds \cos\phi \, ds = 4\pi r^2 =$ area of surface

W_t → total power radiated, in Watts.

P_r → Radial component of average power density poynting vector, W/m^2.

ds → Infinitesimal element of area of sphere of radius $r = r^2 \sin\theta \, d\theta \, d\phi$

r → radius of sphere in meters.

AWP -1 (19)

To prove the Reciprocity theorem, space b/w two antennas are replaced by linear, passive network:

From the figure,

Z_{11} = self impedance of antenna no. 1

Z_{22} = self impedance of antenna no. 2

Z_m = Mutual impedance = Z_{12} or Z_{21}

from Superposition theorem, by making E_{21} short.

By Applying KVL to loop 1 & loop 2

from loop 1 $\Rightarrow$ $\boxed{(Z_{11}+Z_m)I_1 - Z_m I_2 = E_{12}} \longrightarrow (1)$

from loop 2 $\Rightarrow$ $\boxed{(Z_{22}+Z_m)I_2 - Z_m I_1 = 0} \longrightarrow (2)$ $[\because E_{21}=0]$

$$\boxed{I_2 = \frac{Z_m \quad I_1}{Z_{22}+Z_m}} \longrightarrow (3)$$

Substitute eqn (3) in eqn (1), then we get

$$(Z_{11}+Z_m)I_1 - \frac{Z_m^2 \; I_1}{Z_{22}+Z_m} = E_{12}$$

$$\Rightarrow I_1\left[(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2\right] = (Z_{22}+Z_m)E_{12}$$

$$I_1 = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m+\cancel{Z_m^2}-\cancel{Z_m^2}}$$

$$(4) \longleftarrow \boxed{\Rightarrow I_1 = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m} = \frac{(Z_{22}+Z_m)E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m}}$$

By Substituting eqn (4) in eqn (3), we get

$$(5) \longleftarrow \boxed{I_2 = \frac{Z_m}{\cancel{(Z_{22}+Z_m)}} * \frac{\cancel{(Z_{22}+Z_m)}E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m} = \frac{Z_m E_{12}}{Z_{11}Z_{22}+(Z_{11}+Z_{22})Z_m}}$$

Current I_1 in the meter can be obtained by symmetry:

Current I_1 in output port is

$$(6) \longleftarrow \boxed{I_1 = \frac{E_{21}Z_m}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})}}$$

AWP
U-I (3)

→ In the transmitting case, the radiated power is absorbed by objects at a distance. trees, buildings, the ground, the sky and other antennas.

→ In the receiving case, passive radiation from distant objects or active radiation from other antennas raises the apparent temparature of R_r.

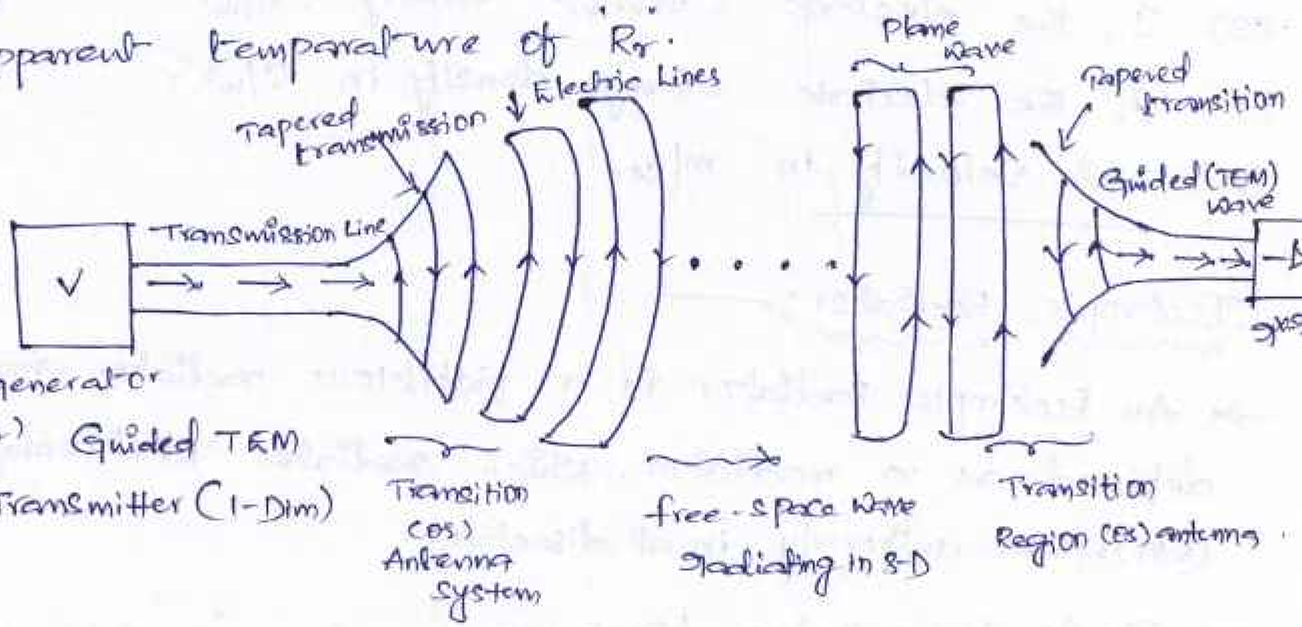


fig: wireless communication link with transmitting antenna & Receiving Antenna

→ Antenna theory is based on classification of Electromagnetic theory as described by Maxwell's equation. Therefore a review of Electromagnetic phenomena is useful in order to have a proper understanding of Antenna Theory.

→ The important vector and scalar Quantities are:

(1) E, the electric field intensity in V/m.

(2) H, the Magnetic field intensity in A/m.

(3) D, the Electric displacement density in Col/m².

(4) B, the magnetic flux density in Wb/m².

(5) A, the magnetic vector potential in A/m².

(6) F, the Electric vector potential in V/m².

(7) P, the poynting vector in W/m².

AWP U-I (20)

Power can be transferred to and from the antenna. However, almost care must be taken to ensure correct antenna matching else internal reflections will take place.

Impedance of an Isolated Antenna [used for Receiving & Transmitting] :—

Consider, the two antennas are separated with wide separation as shown in figure below.

→ The current distribution is same in case of transmitting and receiving antenna. Let antenna no.1 is the transmitting antenna and antenna no.2 is the receiving antenna.

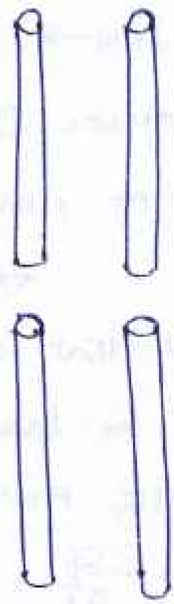


fig: Two antennas no.1 and no.2 with a wide separation.

→ The self impedance (Z_{11}) of transmitting antenna is given by

$$E_1 = Z_{11} I_1 + Z_{12} I_2 .$$

Here,

Z_{11} = self impedance of antenna (1)

Z_{12} = Mutual impedance b/w the two antennas.

Since the separation is more, mutual impedance (Z_{12}) is neglected

$$Z_{12} = 0 .$$

$$E_1 = Z_{11} I_1 + Z_{12} I_2 .$$

$$\therefore E_1 = Z_{11} I_1 \qquad [\because Z_{12} = 0] .$$

$$\boxed{Z_{11} = \frac{E_1}{I_1}} \;**$$

The receiving antenna, under open circuit and short circuit conditions are shown

AWP
U-I(2)

→ The Electric charges are the Sources of the Electromagnetic fields. When these are time varying, then the Electromagnetic waves propagate away from the Sources and the radiation takes place.

→ In general, the radiation can be considered as a process of transmitting energy. The radiation of the EM wave into the space is effectively done by using a conducting (or) dielectric structures called 'Antennas' or 'Radiators'.

→ In General, the antenna can be defined in no. of ways.

(1) A radio antenna may be defined as the structure associated with the region of transition between a guided Guided Media and a free space or viceversa.

(2) A metallic device used for radiating or receiving the radio waves is called an "Antenna".

→ The System used for launching the EM waves is either by transmission line or by a waveguide. The antenna acts as a matching device between free space and the wave launching system.

→ Antenna having directional properties. It is the important component of a wireless communication system.

Basic Radiation Equation:—

→ The basic principle of an antenna is to produce radiation by accelerating or decelerating charge. These radiations are always perpendicular (at 90°) to the direction of acceleration.

AWP Q(22)

By substitute the eqn(3) in eqn(2),

$$E = -\nabla V - \partial A/\partial t \longrightarrow (4)$$

which satisfies both the static and the time varying conditions

Since, $\nabla \cdot D = \nabla \cdot (\varepsilon E) = \varepsilon \nabla \cdot E$

$$= \varepsilon \nabla \cdot (-\nabla V - \partial A/\partial t)$$

$$= \varepsilon(-\nabla \cdot \nabla V - \partial/\partial t (\nabla \cdot A)) = \rho$$

from the above relation, $\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\rho/\varepsilon \longrightarrow (5)$

The RHS of (5) leads to the following relations:

$$\nabla^2 V = -\rho/\varepsilon \quad \text{for static conditions} \longrightarrow 6(a)$$

$$\nabla^2 V = -\rho/\varepsilon - \partial/\partial t (\nabla \cdot A) \quad \text{for time varying conditions} \longrightarrow 6(b)$$

But $\nabla \times H = J + \partial D/\partial t$

$B = \mu H$ or $H = B/\mu$

The LHS of above eqn can be written as

$$LHS = (\nabla \times B)/\mu = (\nabla \times \nabla \times A)/\mu = (\nabla(\nabla \cdot A) - \nabla^2 A)/\mu$$

This relation uses the vector identity, $\longrightarrow (7)$

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \longrightarrow (8)$$

The RHS of above eqn can also be written as

$$RHS = J + \varepsilon \frac{\partial E}{\partial t} = J + \varepsilon \frac{\partial}{\partial t}(-\nabla V - \partial A/\partial t)$$

$$= J + \varepsilon(-\nabla(\partial V/\partial t) - \partial^2 A/\partial t^2)$$

$$= J - \varepsilon\left[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2\right] \longrightarrow (9)$$

on By equaling LHS and RHS terms i.e., eqn (7) & (9), we get

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J - \mu\varepsilon\left[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2\right] \longrightarrow (10)$$

But we know that

$\nabla^2 A = -\mu J$ (in general) and $\nabla^2 A = 0$ for $J = 0$.

where as the term $\nabla \cdot A$ is yet to be defined.

→ As per the statement of Helmholtz theorem A vector field is completely defined only when both its Curl and divergence are known

UNIT-I ①

Introduction :—

Antennas are our electronic eyes and ears on the world. They are our links with space. They are essential, integral part of our civilization.

Antennas have been around for a long time, millions of years as the organ of touch or feeling of animals, birds and insects. But in the last 100 years they have acquired a new significance as the connecting link between a radio system and the out-side world.

The first radio antennas were built by Heinrich Hertz, a professor at the technical Institute in Germany. He assembled apparatus for complete radio system operating at meter wavelengths with an end-loaded dipole as the transmitting and a resonant square-loop antenna as a receiver.

Antennas are the essential communication link for aircraft and ships. Antennas for cellular phones and all types of wireless devices link us to everyone and everything. With mankind's activities expanding into space, the need for antennas will grow to an unprecedented degree. Antennas will provide the vital links to and from everything out there. The future of antennas reaches to stars also.

Antennas are 3-Dimensional and live in a world of beam area, steradians, square degrees and solid angle. Antennas have impedances (self and Mutual). They couple

A.P
U II ①

Thin Linear Wire Antennas

→ For the design of antenna systems the important requirement such as Antenna pattern, the total power radiated the input impedance of the radiator & the radiation efficiency etc.

→ The direct solution for these requirements can be obtained by solving 'Maxwell's Equations' with appropriate boundary conditions of the radiator and at infinity.

→ It is observed that most of the antenna configurations are complicated. so this direct approach is suitable for certain types of the antennas.

→ By making reasonable assumptions of the current distribution for many antennas the solution for above requirements can be obtained.

Electromagnetic Radiations:-

→ The Radiation of Electromagnetic waves from a transmitting antenna to a receiving antenna is of considerable interest. Transmitting antennas are devices used in terminating a transmission line (or) waveguide with the intent of efficiency launching Electromagnetic waves into free space and thus they may be regarded as source of such waves in space. At first the physical $\bar{E}$ & $\bar{B}$ fields are described interms of the auxilliary scalar & vector potentials V and $\bar{A}$ respectively. which in turn satisfy wave equations. A solution of the wave equation in $\bar{A}$ is then obtained in the form of an integral over the antenna current.

$$V(r',t) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho_v(r', t - R/v)}{R}\, dv'$$

$$\bar{A}(r',t) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}(r', t - R/v)}{R}\, dv'$$

potentials are delayed (or) retarded (or) propagated by the time R/v. Hence the potentials are called "Retarded potentials".

→ This unit first introduce the Concepts of electric dipoles and thin linear antennas. Later it is extended to the arrays of dipoles and apertures. The dipoles referred to here in are mostly thin linear dipoles.

Short Electric Dipoles:—

→ Any linear antenna may be considered as large no. of very short-conductors connected in series i.e, end to end and hence it is important to consider the radiation properties of such short conductors.

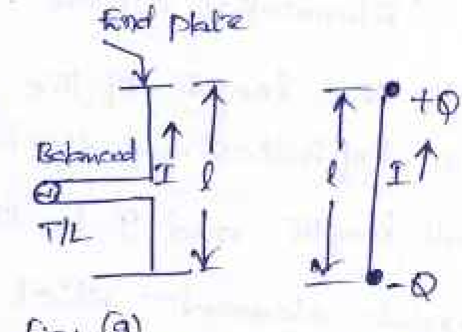


fig:- (a) short dipole fed by a balanced T/L (b) equivalent of a short dipole

→ A short linear conductor is often called "short dipole" or "a Hertzian Dipole".

→ A short linear conductor is so short that current may be assumed to be constant throughout its length as shown in figure.

→ Hertzian dipole is a hypothetical antenna, is defined as a short isolated conductor carrying uniform alternating current.

→ The importance of Hertzian dipole is more theoretical rather than practical because it can be regarded as the element from which the large antennas are constructed.

U-II
③

→ After knowing the properties of a short dipole the ideas can b extended for a long linear conductor as usually used in practise.

→ If I be the current then it is related to charge is

$$I = \frac{dq}{dt} \text{ C/sec}.$$

→ By electrically short we mean "short compared to a half wave length ($\lambda/2$)".

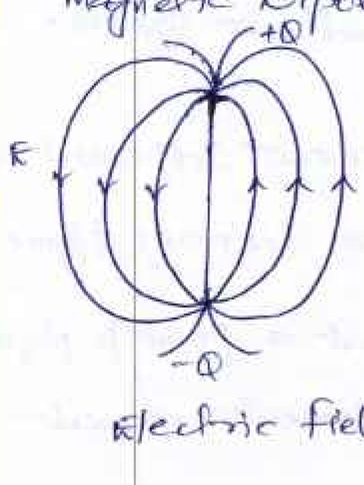
→ short dipole is usually applied to any dipole that is no longer than about one tenth of a wavelength ($\lambda/10$ or 0.1λ).

→ Isolated short dipoles donot generally have uniform current throughout their length but approximate uniformity of current may be obtained by "capacitance end loading".

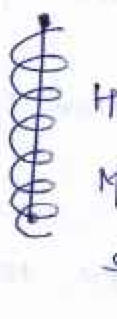
→ A short dipole that does have a uniform current will be known as "Elemental Dipole". Such a dipole will generally be considerably shorter than the $\lambda/10$ wavelength maximum for a short dipole.

→ The other terms used for elemental dipole are "Elementary Doublet" and "Hertzian Dipole".

→ In short dipole electric charge oscillates it may also be called as "oscillating Electric Dipole as against oscillating Magnetic Dipole".



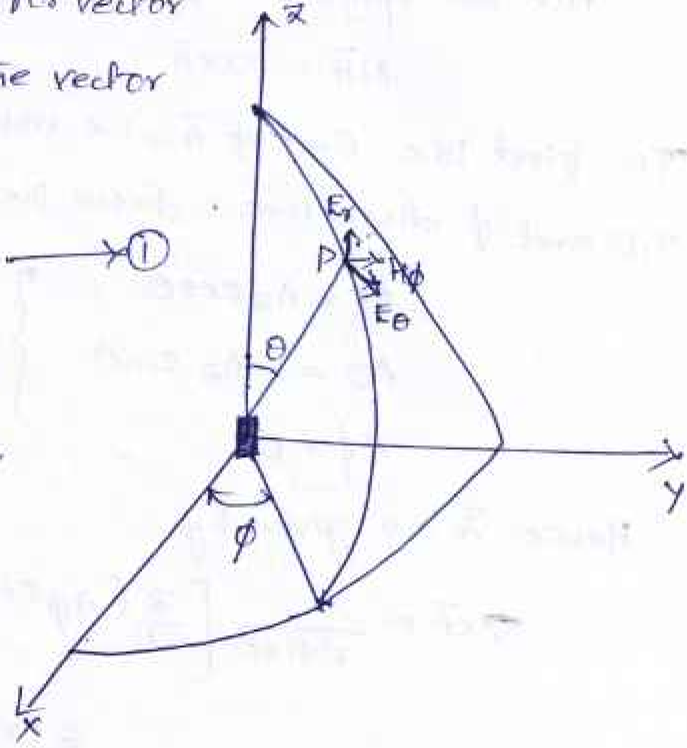
Electric field.



Magnetic field is the immediate surrounding of Elemental dipole.

Let us write the expression for vector potential $\bar{A}$ at point p. The vector potential $\bar{A}$ is given by

$$\bar{A}(r) = \frac{\mu}{4\pi}\int_V \frac{J\left(t-\frac{r}{v}\right)}{R}\,dv' \longrightarrow (1)$$



Here the vector potential is retarded in time by $\frac{r}{v}$ sec, where 'v' is the velocity of propagation.

As the current element is placed along the z-axis, the vector potential will also have only one component in positive z-direction. Hence we can write,

$$A_z = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\left(t-\frac{r}{v}\right)}{R}\,dv' \longrightarrow (2)$$

from eqn (2), it is clear that the component of vector potential 'A_z' can be obtained by integrating the current density $\bar{J}$ over the volume. This includes integration over cross section area of an element of wire and integration along its length. But the integration of current density $\bar{J}$ over a cross section area yields current I. Now this current is assumed to be constant along the length dL, the integration of $\bar{J}$ over the length dL gives value IdL. Thus mathematically we can write.

$$\int_V \bar{J}(t-r/v)\,dv' = IdL\cos w(t-r/v) \longrightarrow (3)$$

Sub. the value of integration from eqn (3) in eqn (2), the vector potential in z-direction is given by

$$A_z = \frac{\mu}{4\pi}\,\frac{IdL\cos w(t-r/v)}{R} \longrightarrow (4)$$

$$\nabla\times\bar{A} = \frac{\mu I dL\sin\theta}{4\pi r}\left[\frac{-w\sin w(t-r/v)}{v} + \frac{\cos w(t-r/v)}{r}\right]\bar{a}_\phi$$

$$\nabla\times\bar{A} = \frac{\mu I dL\sin\theta}{4\pi}\left[\frac{-w\sin w(t-r/v)}{rv} + \frac{\cos w(t-r/v)}{r^2}\right]\bar{a}_\phi \longrightarrow (9)$$

Hence the Magnetic field $\bar{H}$ is given by

$$\bar{H} = \frac{1}{\mu}\left[\nabla\times\bar{A}\right]$$

$$\therefore \bar{H} = \frac{I dL\sin\theta}{4\pi}\left[\frac{-w\sin w(t-r/v)}{rv} + \frac{\cos w(t-r/v)}{r^2}\right]\bar{a}_\phi \longrightarrow (10)$$

Eqn (10) indicates that the Magnetic field $\bar{H}$ exists only in 'ϕ' direction.

$$\therefore H_\phi = \frac{I dL\sin\theta}{4\pi}\left[\frac{-w\sin w(t-r/v)}{rv} + \frac{\cos w(t-r/v)}{r^2}\right] \longrightarrow (11)$$

Let $(t-r/v) = t'$. Sub. the value in eqn(11), we get

$$H_\phi = \frac{I dL\sin\theta}{4\pi}\left[-\frac{w\sin wt'}{rv} + \frac{\cos wt'}{r^2}\right] \longrightarrow (12)$$

After calculating the Magnetic field, now let us calculate the electric field given by

$$\nabla\times\bar{H} = \varepsilon\frac{\partial\bar{E}}{\partial t}$$

$$\therefore \partial\bar{E} = \frac{1}{\varepsilon}(\nabla\times\bar{H})\,dt$$

Integrating w.r.t. corresponding variables, we get

$$\bar{E} = \frac{1}{\varepsilon}\int(\nabla\times\bar{H})\,dt \longrightarrow (13)$$

Let us calculate each term of $\nabla\times\bar{H}$ separately.

from the definition of Curl of a vector, the component in $\bar{a}_r$ direction is given by

$$(\nabla\times\bar{H})_r = \frac{1}{r\sin\theta}\left[\frac{\partial H_\phi\sin\theta}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right]$$

AWP II (6) from equation (13), the component of $\bar{E}$ in $\bar{a}_r$ direction is given

$$E_r = \frac{1}{\varepsilon}\int (\nabla\times \bar{H})_r \, dt$$

Putting the value of $(\nabla\times\bar{H})_r$ from eqn(14),

$$E_r = \frac{1}{\varepsilon}\int \frac{2IdL\cos\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{v r^2} + \frac{\cos\omega t'}{r^3}\right]dt$$

$$= \frac{1}{\varepsilon}\int \frac{2IdL\cos\theta}{4\pi}\left[\frac{-\omega\sin\omega(t-r/v)}{v r^2} + \frac{\cos\omega(t-r/v)}{r^3}\right]dt$$

$$= \frac{2IdL\cos\theta}{4\pi\varepsilon}\left[\frac{\omega\cos\omega(t-r/v)}{v r^2}(1/\omega) + \frac{\sin\omega(t-r/v)}{r^3}(1/\omega)\right]$$

$$= \frac{2IdL\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega(t-r/v)}{v r^2} + \frac{\sin\omega(t-r/v)}{\omega r^3}\right]$$

Put $(t-r/v) = t'$

$$\therefore E_r = \frac{2I\,dL\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3}\right] \longrightarrow (16)$$

Similarly from eqn (13), the component of $\bar{E}$ in $\bar{a}_\theta$ direction is given by,

$$E_\theta = \frac{1}{\varepsilon}\int \frac{-IdL\sin\theta}{4\pi r}\left[\frac{\omega^2\cos\omega(t-r/v)}{v^2 r} + \frac{\omega\sin\omega(t-r/v)}{v r^2} - \frac{\cos\omega(t-r/v)}{r^3}\right]dt$$

$$\therefore E_\theta = \frac{-IdL\sin\theta}{4\pi\varepsilon}\int\left[\frac{\omega^2\cos\omega(t-r/v)}{v^2 r} + \frac{\omega\sin\omega(t-r/v)}{v r^2} - \frac{\cos\omega(t-r/v)}{r^3}\right]dt$$

$$\therefore E_\theta = \frac{-IdL\sin\theta}{4\pi\varepsilon}\left[\frac{\omega^2\sin\omega(t-r/v)}{v^2 r}\left(\frac{1}{\omega}\right) + \frac{-\omega\cos\omega(t-r/v)}{v r^2}\left[\frac{1}{\omega}\right] - \frac{\sin\omega(t-r/v)}{r^3}(1/\omega)\right]$$

$$E_\theta = \frac{-IdL\sin\theta}{4\pi\varepsilon}\left[\frac{\omega\sin\omega(t-r/v)}{v^2 r} - \frac{\cos\omega(t-r/v)}{v r^2} - \frac{\sin\omega(t-r/v)}{\omega r^2}\right]$$

$\because (t-r/v) = t'$

$$\therefore E_\theta = \frac{IdL\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{v^2 r} + \frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3}\right] \longrightarrow (17)$$

(3) The amplitudes of both the terms in H_ϕ have equal amplitudes. The condition at which the amplitudes are equal is given by

$$\frac{1}{r^2} = \frac{\omega}{r v}$$

$$\therefore \; r = \frac{v}{\omega} = \frac{v}{2\pi f} = \frac{(v/f)}{2\pi} = \frac{\lambda}{2\pi} \approx \lambda/6$$

(4) In the induction field term 't' is replaced by the retarded time t'. The term can be written as $\frac{I dL \sin\theta \cos\omega t'}{4\pi r^2}$. Basically this expression is similar to the expression for the magnetic field strength due to the current element derived from Biot-Savart law, extended for alternating current $I\cos\omega t$.

(5) For steady currents, the radiation field term is absent.

(6) The radiation field term indicates flow of energy away from the current element while the induction field term indicates the energy stored in the field during one quarter of the cycle which is returned back during next cycle.

Now consider the expressions of the components E_r and E_θ.

(1) The component E_θ has both the induction field and radiation terms along with a term which varies inversely with the cube of a distance 'r'.

(2) The component E_r has only induction term along with a term which varies inversely with the cube of a distance 'r'.

(3) In both the field component expressions, the term which varies inversely with cube of a distance 'r' is called "Electrostatic field or simply Electric field".

II (8)

$$P_\theta = \frac{-2IdL\cos\theta}{4\pi\epsilon}\left[\frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right] \frac{IdL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right]$$

$$= \left[\frac{2I^2dL^2\sin\theta\cos\theta}{16\pi^2\epsilon}\right]\left[\frac{-\omega\sin\omega t'\cos\omega t'}{c^2r^3} + \frac{\cos^2\omega t'}{cr^4} - \frac{\omega\sin^2\omega t'}{\omega cr^4} + \frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right]$$

since $2\sin\theta\cos\theta = \sin 2\theta$.

$$= \frac{I^2dL^2\sin 2\theta}{16\pi^2\epsilon}\left[\frac{\omega\sin 2\omega t'}{2c^2r^3} - \frac{\cos^2\omega t'}{cr^4} + \frac{\omega\sin^2\omega t'}{\omega cr^4} - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \left[\frac{I^2dL^2\sin 2\theta}{16\pi^2\epsilon}\right]\left[\frac{\omega\sin 2\omega t'}{2c^2r^3} + \frac{1}{cr^4}\left(\sin^2\omega t' - \cos^2\omega t'\right) - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

consider middle term inside the second square bracket,

$$\frac{1}{cr^4}\left[\sin^2\omega t' - \cos^2\omega t'\right] = \frac{1}{cr^4}\left[\frac{1-\cos 2\omega t'}{2} - \left(\frac{1+\cos 2\omega t'}{2}\right)\right]$$

$$= \frac{1}{cr^4}\left[\frac{-2\cos 2\omega t'}{2}\right] = \frac{-\cos 2\omega t'}{cr^4}$$

Putting value of the term considered back in the original expression.

$$P_\theta = \left[\frac{I^2dL^2\sin 2\theta}{16\pi^2\epsilon}\right]\left[\frac{\omega\sin 2\omega t'}{2c^2r^3} - \frac{\cos 2\omega t'}{cr^4} - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

The average value of $\sin 2\omega t'$ & $\cos 2\omega t'$ terms over a complete cycle is zero. This clearly indicates that for any value of r, the average of P_θ is always zero over a complete cycle. Thus there will be the power flow back and forth in θ-direction only. Hence in θ-direction, there is no net or average flow of power.

Let us calculate now radial component of the poynting vector.

$$P_r = E_\theta H_\phi$$

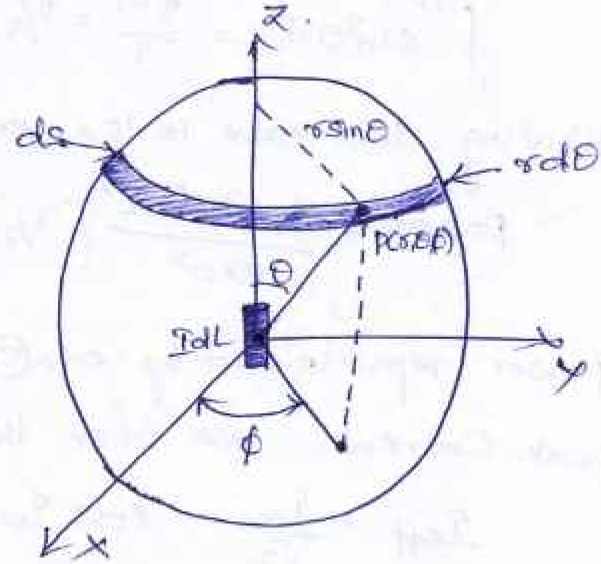
$$P_r = \frac{IdL\sin\theta}{4\pi\epsilon}\left[\frac{-\omega\sin\omega t'}{c^2r} + \frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right] * \frac{IdL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right]$$

The power component represented by eqn (3) is in radial direction. Hence it is called "radial power". Equation (5) represents the average power flow.

→ The total power radiated by the current element can be obtained by integrating the radial poynting vector over a spherical surface.

Consider a spherical shell with the current element IdL placed at the center of the spherical coordinate system as shown in the following figure.

The point p at which power radiated is to be calculated is independent of an Azimuthal angle ϕ, so the element of area ds on the spherical shell is considered as strip.



The element of area ds is given by

$$ds = 2\pi r^2 \sin\theta \, d\theta \longrightarrow (6)$$

→ The total power radiated is calculated by integrating average radial power over the spherical surface.

$$(7) \quad \text{Power} = \oint_{\text{Surface}} P_r \, ds = \oint_{\text{Surface}} \left[\frac{\eta_0}{2}\left(\frac{\omega I dL \sin\theta}{4\pi r c}\right)^2 (2\pi r^2 \sin\theta \, d\theta)\right]$$

$$= \oint \left[\frac{\eta_0}{2}\right]\left[\frac{\omega^2 I^2 dL^2 \sin^2\theta}{16\pi^2 r^2 c^2}\right]\left[2\pi r^2 \sin\theta\right] d\theta$$

$$= \oint \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \sin^3\theta \, d\theta = \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \oint \sin^3\theta \, d\theta$$

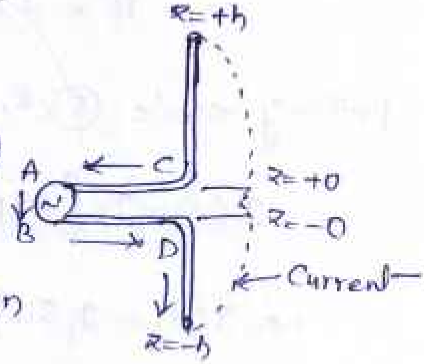
In spherical co-ordinate system, θ varies from 0 to π. Hence

II (10) Equation (10) is another form of the power radiated in terms of the effective current. As we know that the power is in the form of I^2R. Thus the coefficient of power in the eqn (10) is nothing but the resistance. This resistance is called 'Radiation Resistance' of the current element, represented by R_{rad}.

$$\therefore \boxed{R_{rad} = 80\pi^2 \frac{dL^2}{\lambda^2}.} \longrightarrow (11)$$

Quarter Wave Monopole and Half Wave Dipole :—

Asymptotic Current Distributions in Dipole :—



Let the dipole antenna be fed by a two-wire transmission line and the generator is connected at the terminals AB.

→ figure shows asymptotic current distribution in a dipole fed symmetrically.

Since in each antenna arm the current is sinusoidal and hence,

$$\left.\begin{aligned} I(z) &= A'\cos\beta z + B'\sin\beta z \ \ldots\ldots\ z > +0 \\ I(z) &= A''\cos\beta z + B''\sin\beta z \ \ldots\ldots\ z < -0 \end{aligned}\right\} \longrightarrow (A)$$

At the ends of dipole current must vanish as there is no conductor where current can flow. Thus

$$z = \pm h$$

$$I(+h) = I(-h) = 0. \longrightarrow (1)$$

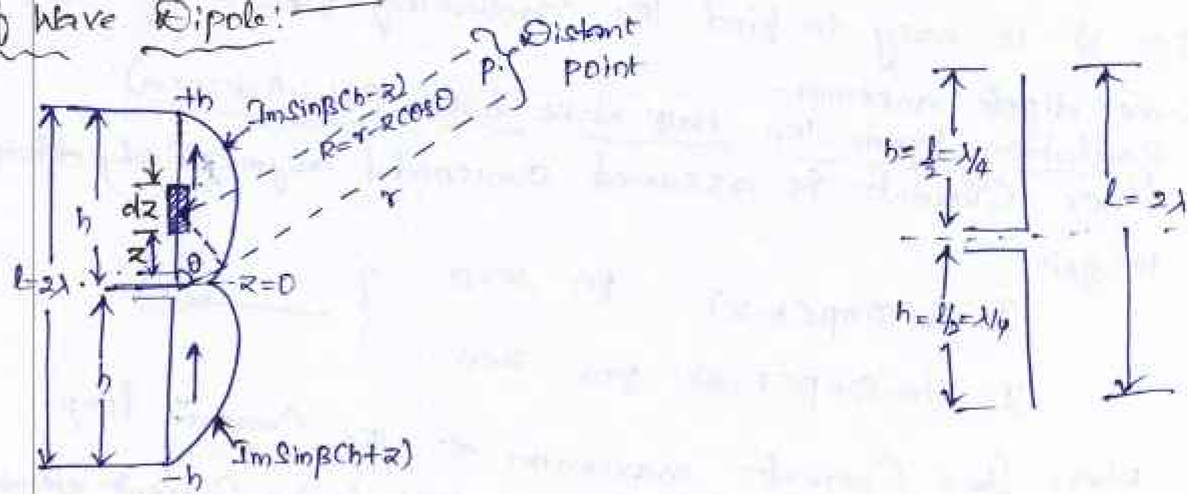
Since the figure is symmetrical and if the generator is balanced w.r.t. the feeder line at A, B. The currents in AC and BD are equal and opposite. Thus antenna current entering at 'C' must be equal to that leaving D, i.e.,

$$I(-0) = I(+0). \longrightarrow (2)$$

particular fixed frequency of operation.

→ If the frequency of operation changes then we have to change the length of Quarter wave Monopole antenna according to the operating frequency. Otherwise, Quarter wave Monopole antenna cannot work other frequencies.

Half Wave Dipole:—



→ Half wave length dipole or simply Half wave dipole ($\lambda/2$ antenna) is one of the simplest antenna.

→ A $\lambda/2$ antenna is the fundamental radio antenna of metal rod or tubing or thin wire which has a physical length of half wavelength in free space at the frequency of operation.

→ A $\lambda/2$ antenna is also known as "Hertz Antenna" or "Half wave Doublet".

→ A dipole Antenna may be defined as a symmetrical antenna in which the two ends are at equal potential relative to Mid-point.

→ The dipole is usually fed at the centre having maximum current at the centre i.e., Maximum Radiation in the plane normal to the axis.

→ By deviding ~~this~~ the length of half wave dipole into small dipoles (or) current elements. We can find out the total radiation of the half wave dipole antenna by integrating the fields of individual (either short dipole or current element) elements over

II (12)

Thus in denominator R may be replaced by 'r' but not in numerator because 'R' is involved in the phase factor and hence the difference between R & r is important.

Eqn (3) reduces to.

$$A_z = \frac{\mu}{4\pi}\int_{-h}^{0} \frac{I_m \sin\beta(h+z)}{r} \cdot e^{-j\beta(r-z\cos\theta)} dz + \frac{\mu}{4\pi}\int_{0}^{h} \frac{I_m \sin\beta(h-z)}{r} \cdot e^{-j\beta(r-z\cos\theta)} dz$$

$$\because e^{-j\beta(r-z\cos\theta)} = e^{-j\beta r} \cdot e^{+j\beta z\cos\theta}$$

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-h}^{0} \sin\beta(h+z)\, e^{+j\beta z\cos\theta} \cdot dz + \int_{0}^{+h} \sin\beta(h-z)\, e^{+j\beta z\cos\theta} \cdot dz\right]}$$

for a $\lambda/2$ Antenna :$\longrightarrow$

$$l = 2h = \lambda/2$$

$$\Rightarrow \boxed{h = \lambda/4} \longrightarrow (5)$$

→ By observing the figure, the current is maximum at the centre because feed is connected at the center and it is minimum at the ends.

for the length of $\lambda/2$, the variation of current along the length completes or takes only half of the cycle.

so it takes π radians (or) 180° degrees.

$$\therefore\ l = 2h = \pi$$

$$\therefore\ h = \pi/2 \qquad \text{since } h = \lambda/4$$

$$\therefore \boxed{h = \lambda/4 = \pi/2} \longrightarrow (6)$$

so for $h = \pi/2$, $\sin\beta(h+z) = \sin\beta(\pi/2 + z) = \cos\beta z$

$\sin\beta(h-z) = \sin\beta(\pi/2 - z) = \cos\beta z$

$$\therefore \boxed{\sin\beta(h+z) = \sin\beta(h-z) = \cos\beta z} \longrightarrow (7)$$

U-II (13)

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos[\pi/_2\cos\theta] - \cancel{\cos\theta\cos[\pi/_2\cos\theta]} + \cos(\pi/_2\cos\theta) + \cancel{\cos\theta\cos[\pi/_2\cos\theta]}}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{2\cos[\pi/_2\cos\theta]}{\sin^2\theta}\right]$$

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos[\pi/_2\cos\theta]}{\sin^2\theta}\right]} \longrightarrow (9)$$

→ But for a current element along z-axis, from Maxwell eq

$\nabla \times A = \mu H$ [$\because$ A is a vector] for a distant field from the current element.

$$\therefore \text{Since } (\nabla\times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\theta \cdot r) - \frac{\partial A_r}{\partial\theta}\right]$$

where $A_\theta = -A_z\sin\theta$ & $A_r = A_z\cos\theta$

Due to symmetry, $\partial/\partial\phi = 0$, $A_\phi = 0$.

$$\therefore (\nabla\times A)_\phi = \frac{1}{r}\left[r\frac{\partial A_\theta}{\partial r} + A_\theta\cdot\frac{\partial r}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$$

$$= \frac{1}{r}\left[r\frac{\partial A_\theta}{\partial r} + A_\theta - \frac{\partial}{\partial\theta}[A_z\cos\theta]\right] = \frac{1}{r}\left[r\cdot\frac{\partial A_\theta}{\partial r} - \cancel{A_z\sin\theta} + \cancel{A_z\sin\theta}\right]$$

$$\mu H_\phi = (\nabla\times A)_\phi = \frac{\partial A_\theta}{\partial r} = \frac{\partial}{\partial r}[-A_z\sin\theta] = -\sin\theta\cdot\frac{\partial A_z}{\partial r}$$

$$\therefore \boxed{\mu H_\phi = -\sin\theta\frac{\partial A_z}{\partial r}} \longrightarrow (10)$$

Putting eqn (9) in eqn (10), we get

$$\mu H_\phi = -\sin\theta\left[\frac{\partial}{\partial r}\left\{\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left(\frac{\cos(\pi/_2\cos\theta)}{\sin^2\theta}\right)\right\}\right]$$

$$= -\sin\theta\cdot\frac{\mu I_m}{2\pi\beta}\cdot\frac{\cos(\pi/_2\cos\theta)}{\sin^2\theta}\cdot\frac{\partial}{\partial r}\left[\frac{e^{-j\beta r}}{r}\right]$$

$$= -\frac{\mu I_m}{2\pi\beta\sin\theta}\cos(\pi/_2\cos\theta)\cdot\left[\frac{r\cdot e^{-j\beta r}(-j\beta) - e^{-j\beta r}}{r^2}\right]$$

II (4)

Since E_θ and H_ϕ are in time phase, therefore, the maximum value in time of the poynting vector is just the product of the peak values of E_θ and H_ϕ i.e.,

$$\boxed{W_{max} = |E_\theta||H_\phi|} \longrightarrow (15)$$

Therefore, the average value in time of the poynting vector is given

$$W_{avg} = \frac{E_\theta}{\sqrt{2}} \cdot \frac{H_\phi}{\sqrt{2}} = \frac{1}{2} E_\theta H_\phi = \frac{P_{max}}{2}$$

$$W_{avg} = \frac{1}{2} E_\theta H_\phi = \frac{1}{2} \cdot \frac{I_m}{2\pi r} \cdot \frac{60 I_m}{r} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2 \longrightarrow (16)$$

$$\therefore \boxed{W_{avg} = \frac{15 I_m^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2} \longrightarrow (17)$$

In practice, the rms current is of importance for measurement purposes etc. so

$$\left[\because \frac{I_m}{\sqrt{2}} = I_{rms} \text{ (or) } I_m = \sqrt{2} I_{rms} \right]$$

$$W_{avg} = \frac{15 (\sqrt{2} I_{rms})^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2$$

$$\therefore \boxed{W_{avg} = \frac{30 I_{rms}^2}{\pi r^2} \left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \right]} \; W/m^2 \longrightarrow (18)$$

This is the expression for average power in terms of rms current.

Power Radiation by a Half Wave Dipole (i.e. $\lambda/2$ antenna) and its Radiation Resistance:—

For the calculation of total power radiation by a $\lambda/2$ antenna and its radiation resistance can be found using the poynting vector.

→ From the figure, the elemental area of the spherical shell is given by eqn.

$$ds = r^2 \sin\theta \, d\theta \longrightarrow (1)$$

$$\left[\because ds = (r\sin\theta d\phi)(r d\theta) \right]$$

Hence the total power radiated from a $\lambda/2$ antenna is given by the surface integral of the poynting vector over any

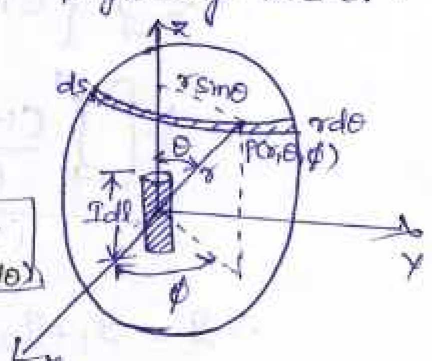


Fig: An Elemental Area on a spherical

II (15)

Let us make another substitution

Say $t=-x$ when $t=+1,\ x=-1$

or $dt=-dx$ and $t=-1,\ x=+1$.

Putting these substitutions in I_2 expression, we get—

$$I_2=\frac{1}{4}\int_{+1}^{-1}\frac{1+\cos(-\pi x)}{1+x}(-dx) \qquad \left[\because \cos(-\theta)=\cos\theta\right]$$

$$I_2=\frac{1}{4}\int_{-1}^{1}\frac{1+\cos\pi x}{1+x}\,dx \longrightarrow (6)$$

Then from eqn (4), we have

$$I=I_1+I_2=2I_2=2\cdot\frac{1}{4}\int_{-1}^{+1}\frac{1+\cos\pi x}{1+x}\,dx$$

$$I=\frac{1}{2}\int_{-1}^{+1}\frac{(1+\cos\pi x)}{1+x}\,dx \longrightarrow (7)$$

Again put—

$$(1+x)=y/\pi \quad \text{or} \quad \pi+\pi x=y \quad \text{(or)} \quad \pi x=y-\pi$$

(or) $dx=\dfrac{dy}{\pi}$ when $x=-1,\ y=0,\quad x=+1,\ y=2\pi$

Thus from eqn (7), we get—

$$I=\frac{1}{2}\int_{0}^{2\pi}\frac{(1+\cos(y-\pi))}{y/\pi}\cdot\frac{dy}{\pi}=\frac{1}{2}\int_{0}^{2\pi}\frac{1+\cos(y-\pi)}{y}\cdot dy$$

$$I=\frac{1}{2}\int_{0}^{2\pi}\frac{1-\cos y}{y}\,dy \qquad \because \cos(y-\pi)=-\cos y.$$

(or) $$I=\frac{1}{2}\int_{0}^{2\pi}\left\{\frac{\cancel{1}-\left[1-\frac{y^2}{2!}+\frac{y^4}{4!}-\frac{y^6}{6!}+\frac{y^8}{8!}+\cdots\right]}{y}\right\}dy$$

$$=\frac{1}{2}\int_{0}^{2\pi}\left\{\frac{\frac{y^2}{2!}-\frac{y^4}{4!}+\frac{y^6}{6!}-\frac{y^8}{8!}+\cdots}{y}\right\}dy$$

$$=\frac{1}{2}\int_{0}^{2\pi}\left\{\frac{y}{2!}-\frac{y^3}{4!}+\frac{y^5}{6!}-\frac{y^7}{8!}+\cdots\right\}dy \longrightarrow (8)$$

$$\therefore \boxed{I=\frac{1}{2}\left[\frac{y^2}{2!\cdot 2}-\frac{y^4}{4!\cdot 4}+\frac{y^6}{6!\cdot 6}-\frac{y^8}{8!\cdot 8}+\cdots\right]_0^{2\pi}} \longrightarrow (9)$$

Eqn (9) is a series and does not converge rapidly. Therefore, a number

HWT
U.II (16)

This is the expression of total power radiated by a Half wave Dipole in free space remote from the ground. Eqn (12) gives that radiation resistance of a Centre fed half dipole or simply dipole antenna is 73.14 Ω or approximately 73 Ω.

→ for a Quarter wave Monopole antenna, the radiation resistance is half of the dipoles' radiation resistance, i.e. 73.14/2 (or) 36.57 Ω

→ only Difference in A $\lambda/2$ antenna & $\lambda/4$ Antenna (Marconi Antenna) is that dipole radiate power more or less in all directions whereas Monopole radiates power in a hemisphere Surface and this is why its radiation resistance is half of the dipole.

Directivity of Half Wave Dipole :—

We know that, the relation between poynting vector (or) power density (W) and field in $\lambda/2$ antenna is,

$$W = \frac{E^2}{\eta} \longrightarrow (1)$$

$$\text{Since } W_{avg} = \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos\left[\pi/2 \cos\theta\right]}{\sin\theta}\right]^2 \; W/m^2 \longrightarrow (2)$$

Since this average radiated power is considered as the radiation field at the distance r from the source (or center of Half Wave Dipole antenna].

$$\therefore W = \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta}{\sin\theta}\right]^2 W/m^2 . \longrightarrow (3)$$

where [I] → Retarded Current.

∴ Radiation Intensity, $U = r^2 \cdot W_{rad}$

$$U = r^2 W_{avg} \qquad \left[\because W_{rad} = E^2\right]$$

(or)

$$U = r^2 \cdot E \longrightarrow (4)$$

from eqn ⑥ & ⑦,

$$P_{rad} = 60 I_{rms}^2 [1.218]$$

$$P_{rad} = 73.08 I_{rms}^2 \longrightarrow ⑧$$

$$\therefore U_0 = \frac{P_{rad}}{4\pi} = \frac{73.08 I_{rms}^2}{4\pi} \text{ W/Sr} \longrightarrow ⑨$$

from ⑤, ⑥ & ⑨ we get

$$D = \frac{30 I_{rms}^2/\pi}{73.08 I_{rms}^2/4\pi} = \frac{30 * 4\pi}{73.08 * \pi}$$

$$D = \frac{120}{73.08} = 1.64$$

$$D(dB) = 2.15 dB.$$

This is the Directivity of $\lambda/2$ Dipole Antenna.

Effective Aperture [or Effective Area]:—

→ Effective Area is defined as the ratio of power received at the antenna load terminal to the poynting vector (P), of the incident wave.

→ Effective aperture also called as 'Capture Area'.

→ Power radiated or power received by the antenna depends on the effective aperture.

Relation between Directivity and Effective Area:—

→ The directivity of receiving antenna is directly proportional to the maximum effective aperture (Effective Area).

Let us consider, two antennas 1 and 2 whose Directivities and Maximum effective apertures are D_1, D_2 and $(Ae_1)_{max}$, $(Ae_2)_{max}$ respectively.

$$D_1 \propto (Ae_1)_{max} \quad , \quad D_2 \propto (Ae_2)_{max}$$

In general, we have,

$$\boxed{D = \frac{4\pi}{\lambda^2} (A_e)_{max}}$$

$$\boxed{(A_e)_{max} = \frac{\lambda^2}{4\pi} (D)}$$

Effective Aperture of HWD ($\lambda/2$ Antenna):-

Since the directivity of $\lambda/2$ antenna is

$$D = 1.65$$

$$\therefore A_e = \frac{\lambda^2}{4\pi} (1.65)$$

$$A_e = \frac{1.65}{4*3.14} \lambda^2$$

$$A_e = \frac{1.65}{12.56} \lambda^2$$

$$\therefore \boxed{A_e = 0.131 \lambda^2} \; m^2$$

Effective length of a HWD:-

→ The Effective length of an antenna is defined as the ratio of induced voltage at the terminal of the receiving antenna under open circuited condition to the incident electric field intensity. i.e.,

$$\text{Effective Length, } l_e = \frac{\text{open circuited voltage}}{\text{Incident field strength}}$$

$$l_e = \frac{V}{E}$$

However the induced voltage 'V' depends on the effective aperture as

$$A_e = \frac{V^2 R_L}{[(R_A+R_L)^2 + (X_A+X_L)^2] P}$$