Control System

System:- An arrangement (or) combination of different physical components that are connected (or) related together to form an entire unit to achieve a certain objective is called a system

Ex:- Kite, classroom

Control:- The meaning of control means to regulate, direct (or) command a system so that a desired object is obtained.

Control system:- It is a system where the output quantity is controlled by varying the i/p quantity.

Basic Components of control system:-

Plant :- It is defined as the portion of system which is to be controlled (or) regulated. it is also called as Process.

Controller :- It is the element of the system itself (or) external to the System It controls the plant (or) the process.

Input :- Input is an applied signal (or) excitation signal from an external source
command signal

Output :- It is a signal of actual response i.e, obtained from a control system due to application of input.
controlled variable

Disturbance :- The signal that has some adverse effect ~~under~~ on the value of o/p of a system is disturbance.

Classification of control system :-

1. Natural control system
2. Man-made control system
3. Combinational control system
4. Time varying and Time ~~invarying~~ invariant control system
5. Linear and non-linear control system
6. Continuous and discrete-time control system
7. Deterministic and stochastic control system
8. lumped - parameters and distributor parameters control system.
9. single i/p - single o/p and multiple i/p - multiple o/p (SISO + MIMO)

1. Natural control systems :- The system inside a human being (or) a biological system are known as natural control system

   Ex:- Animals.

2. Man-made control systems:- The various control systems that are designed and developed by man are known as man-made control system.

   Ex:- Automobiles.

3. Combinational control system:- The Combination of a natural control system and man-made control system is known as Combinational system.

   Ex:- Driver driving a vehicle.

4. Time varying and time invarying control systems:- If the parameters of control system vary with the time, the control system is termed as time varying control system.

Ex:- A space vehicle leaving the earth.

If the parameters of a control systems are not varying with the time, the control system is termed as time invarying control system

Ex:- capacitors, resistors & Inductors

5. Linear & non-linear Control System

A control system is known as linear if it satisfies the additive polarity as well as the homogeneous property.

Ex - resistors, iron core coil.

a) Additive property:- If x and y belong to the domain of the function F, we can write,

$$f(x+y) = f(x) + f(y)$$

b) Homogeneous property:- For x and y belonging to the domain of the function f and for any scalar constant β, we can write

$$f(\beta, x) = \beta f(x)$$

The principle of super position is a combination of the above two properties

If a function satisfies the above two properties, it is said to be linear in nature.

If $f(x) = x^3$, it is obvious that

$$f(x+y) = (x+y)^3 \neq x^3 + y^3 \text{ and}$$

$$f(\alpha x) = (\alpha x)^3 = \alpha^3 x^3 \neq \alpha(x^3)$$

the function $f(x) = x^3$ is non linear

Ex:- diodes, transistors, thyristors

6. Continous and discrete time control System:- If all the system variables of a control system are functions of time, it is termed as continous time control system. (or) A system in which all the variables are continous w.r.t time.

Ex:- The speed control of a dc motor with tachogenerator feed back.

⇒ If one or more system variables of a control system are known at a certain discrete time control systems

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If few variables in a system are discrete w.r.t time.
Ex:- the microprocessor (or) computer based system

To control temp of electric furnace with

7. Deterministic and stochastic control system:-

If the response to input, and to external disturbances, of a control system is known / Predictable & repetitive as deterministic system. Any control system is called stochastic if such response is unpredictable.
Ex:- a ship mounted mini gun

8. Lumped parameters and distributor parameter control system:-

If a control system can be represented by ordinary differential equations a control system is called lumped parameters.

Ex:- Resistance, Inductance, capacitance

If a control system can be described by partial differential eq^ns such a control system is known as distributed control system. In a transmission line, its parameters such as resistances and inductances are totally distributed along it. Transmission line characteristics are always described by partial differential equations. [the feed back signal is proportional to o/p signal and is fed to the error detector. The error detector] X

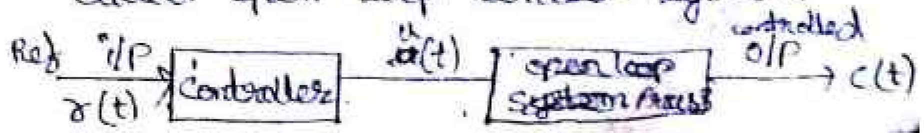
9. Single i/p - single o/p & multiple i/p - multiple o/p (SISO & MIMO) control system:-

If a control system has one i/p and one o/p it is termed as SISO system. If a control system has multiple i/p & multiple o/p it is known as MIMO system.

Open loop control system:-

An open loop system (or) any physical system which does not automatically correct the variations in o/p is called an O.L.S. (or)

The control system in which the o/p quantity has no effect upon the i/p quantity is called open loop control system.



⇒ If any disturbance occurs in this system the i/p can be charged manually in order to control the o/p

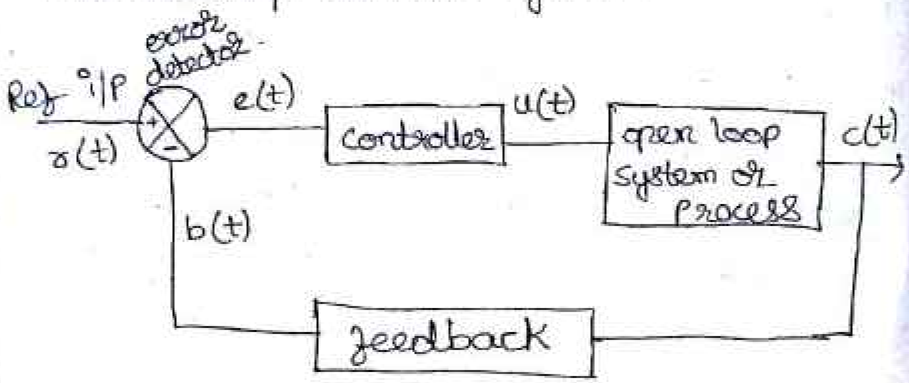
Advantages:-

1. The open loop system has simple & economical
2. The systems are easier to construct
3. Generally the open loop systems are stable

Disadvantages:-

1. Inaccurate and unreliable
2. The change in o/p due to external disturbances ~~can't be accurate~~ are not corrected automatically

Closed loop control system:-



⇒ control system in which the o/p has an effect upon their i/p quantity in order to maintain the desired o/p value is called as closed loop control system

The provision of feedback makes open loop system as closed loop, the f b signal covers the i/p signal and modifies with reference to the o/p signal. The feedback signal is proportional to o/p signal & is fed to the error detector.

The error detector detects the signal which is proportional to reference signal & control signal the controller controls the error and gives modified o/p which is given to plant in order to get desired o/p

Advantages:-

1. Accuracy will be more even, in the presence of non-linearities
2. Sensitivity of the system may be made ~~will be~~ small to make ~~& hence~~ the system ~~will~~ be more stable.
3. The systems ~~is~~ are less affected by noise.

Disadvantages:-

1. This system ~~will be~~ are complex & costly ~~more cost~~
2. f.b reduces overall gain of the system
3. Stability is the a major problem in c.l system & more care is needed to design a stable closed loop system ~~It needs careful~~ design
4. f.b in closed loop system leads to oscillatory response.

Diff b/w open loop & closed loop

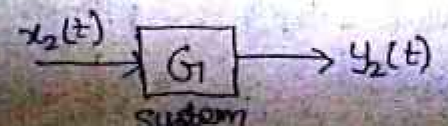
open loop	closed loop
⇒ measurement of o/p is not required for operation of system	⇒ measurement of o/p is neccessary for operation of system
⇒ Feedback element is absent	⇒ Feedback element is present
⇒ Highly sensitive to the disturbances	⇒ less sensitive to the disturbances
⇒ It is inacurate and open loop unrealiable	⇒ highly acurate & reliability will be more closed loop
⇒ change in o/p has no effect on the i/p	⇒ change in o/p effects input
⇒ Error detector is absent	⇒ error detector is present
⇒ It is simple to construct and it is cheap	⇒ complicated to design & hence cost will be more
⇒ open loop system will be stable in nature.	⇒ stability is the major consideration while desiging
⇒ highly effected by non linearities	⇒ Reduced effect of non linearities

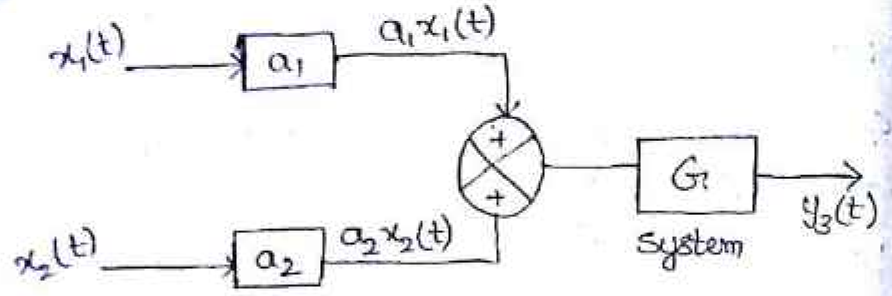
Mathematical method of control systems:-

control system is a collection of physical objects (or) components connected together to perform an object. The i/p & o/p relations & various physical compoments of a system are governed by differential eq^n's. The mathematical model of a control system constitutes a set of differential eq^n's. The response or o/p of the system can be obtained by soluing the differential eq^n's for various i/p conditions. The mathematical model of a system is linear. If it satisfies the principle of superposition & homogenity.

If a system model as responses $y_1(t)$ & $y_2(t)$ to any i/p's $x_1(t)$ & $x_2(t)$ then the system response to the linear combination of these i/p's $[a_1x_1(t)+a_2x_2(t)]$ is given by linear combination of the individual o/p's $[a_1y_1(t)+a_2y_2(t)]$.

Let G be a linear system





$$y_3(t) = a_1y_1(t) + a_2y_2(t)$$

The system 'G' is linear

Mathematical model will be linear, if the diff eq^n describing the system as constant coefficients.

⇒ If the coefficients of the differential eq^n describing the system are constant then the system is <u>linear time invarient</u>.

⇒ If the coefficient of the differential eq^{n}s describing the system are functions of time then the system is <u>linear time varient.</u>

⇒ The differential eq^{n}s of a linear time in varient system can be solved by SISO system in the help of transfer function of system

⇒ Transfer function system is defined as the ratio of laplace transfer of o/p to the laplace transfer of i/p with '0' initial condition.

$$\text{Transfer function (TF)} = \left.\frac{\text{Laplace transfer of o/p}}{\text{Laplace transfer of i/p}}\right| \text{with '0' initial con}^n$$

Transfer function can be obtained by taking Laplace transform of the differential eq^n describing the system with zero initial conditions and arranging the resultant algebric eq^{n}s in the ratio of o/p to i/p.

Mechanical system:- Mechanical systems are classified into 2 types.

1. Translation motion 2) Rotational motion

1. Translation mechanical system.

symbols used

x = displacement = meters (m)

v = velocity = $\frac{dx}{dt}$ = m/sec

a = acce = $\frac{dv}{dt}$ = m/sec^2 = $\frac{d^2x}{dt^2}$

f = applied force

f_m = opposing force offered by mass of the body "

f_b = opposing force offered by friction of the body (dash pot) "

f_k = opposing force offered by elasticity of the body (spring) "

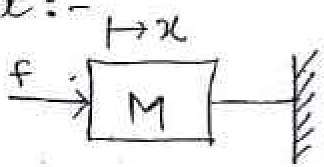
M = mass

K = stiffness of the spring

B = balance, viscous friction coefficient

Force balance eq^{n}s of idealize elements:-

1. Ideal mass element :-



$f_m \propto a$

$f_m \propto \frac{d^2x}{dt^2}$

$f_m = \frac{md^2x}{dt^2}$

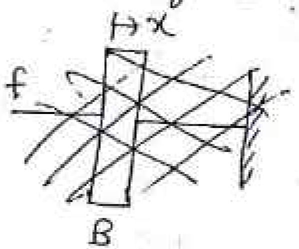
Force balance eq^n

By newton's second law

$f = f_m$

$f = \frac{md^2x}{dt^2}$

2. Ideal dashpot with one end fixed :-

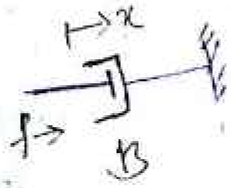


$f_b \propto V$

$f_b \propto \frac{dx}{dt}$

$f_b = B\frac{dx}{dt}$

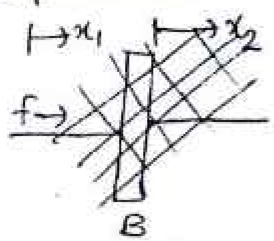
By newton's second law



$f = f_b$

$f = B\frac{dx}{dt}$

3. Ideal dashpot with displacement at both ends:-



$f_b \propto V$

$f_b \propto \frac{d}{dt}(x_1 - x_2)$

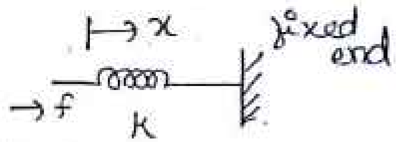
$f_b = B\frac{d}{dt}(x_1 - x_2)$

By newton's second law

$f = f_b$

$f = B\frac{d}{dt}(x_1 - x_2)$

4. Ideal spring with one end fixed:-



$f_K \propto x$

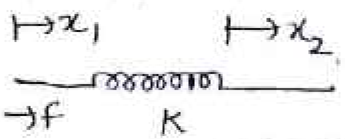
$f_K = Kx$

by newton's second law.

$f_K = f$

$f = Kx$

5. Ideal spring with displacement at both ends:-



$f_K \propto (x_1 - x_2)$

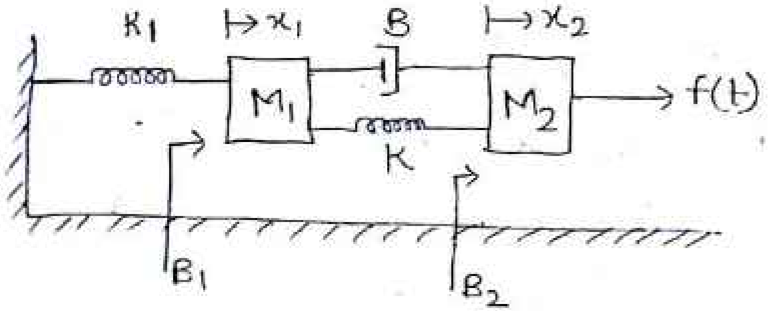
$f_K = K(x_1 - x_2)$

by newton's second law

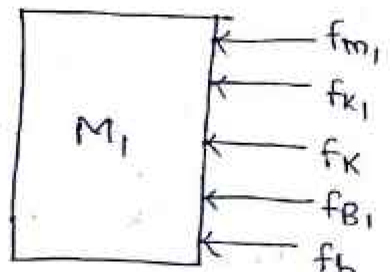
$f = f_K$

$f = K(x_1 - x_2)$

1. Write the differential eq^ns describing the mechanical system shown in fig & determine transfer function



Sol:-



$f_{m_1} = M_1 \frac{d^2x_1}{dt^2}$

$f_{K_1} = K_1 x_1$

$f_K = K(x_1 - x_2)$

$f_{b_1} = B_1 \frac{dx_1}{dt}$

$f_b = B \frac{d}{dt}(x_1 - x_2)$

Free body diagram for M_1

From Newton's second law,

$$f_{m_1} + f_{K_1} + f_K + f_{b_1} + f_b = 0$$

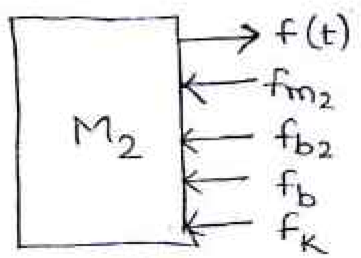
$$m_1 \frac{d^2x_1}{dt^2} + k_1 x_1 + k(x_1 - x_2) + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x_2) = 0 \quad \text{--- (1)}$$

Applying laplace transform,

$$m_1 s^2 x_1(s) + K_1 x_1(s) + K x_1(s) - K x_2(s) + B_1 s x_1(s) + BS x_1(s) - BS x_2(s) = 0$$

$$x_1(s)\left[m_1 s^2 + K_1 + K + B_1 s + BS\right] = x_2(s)\left[BS + K\right]$$

$$x_1(s) = \frac{BS + K}{m_1 s^2 + K_1 + K + B_1 s + BS} x_2(s) \quad \text{--- (2)}$$



$f_{m_2} = m_2 \frac{d^2x_2}{dt^2}$

$f_{b_2} = B_2 \frac{dx_2}{dt}$

$f_b = B \cdot \frac{d}{dt}(x_2 - x_1)$

$f_K = K(x_2 - x_1)$

By Newton's second law,

$$f_{m_2} + f_{b_2} + f_b + f_K = f(t)$$

$$m_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B\frac{d}{dt}(x_2 - x_1) + K(x_2 - x_1) = f(t) \quad \text{--- (3)}$$

Applying laplace transform

$$M_2 s^2 x_2(s) + B_2 s x_2(s) + BS x_2(s) - BS x_1(s) + K x_2(s) - K x_1(s) = F(s)$$

$$x_2(s)\left[M_2 s^2 + B_2 s + BS + K\right] - x_1(s)\left[BS + K\right] = F(s) \quad \text{--- (4)}$$

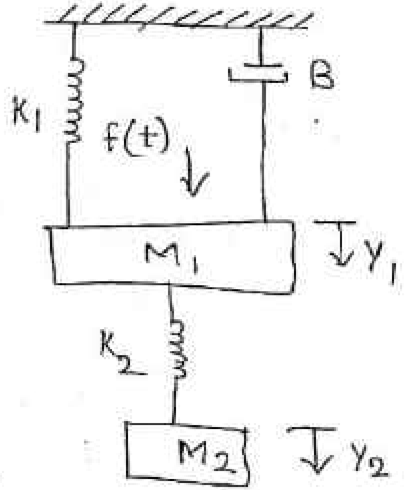
Sub (2) in (4)

$$x_2(s)\left[M_2 s^2 + B_2 s + BS + K\right] - \left[\frac{(BS + K)^2}{M_1 s^2 + K_1 + K + B_1 s + BS}\right] x_2(s) = F(s)$$

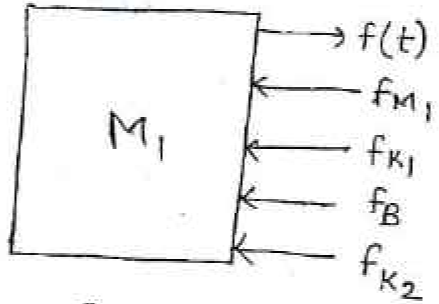
$$X_2(s)\left[\frac{(m_1s^2+B_1s+Bs+K+K_1)(m_2s^2+B_2s+Bs+K)-(Bs+K)^2}{m_1s^2+B_1s+Bs+K_1+K}\right] = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{M_1s^2+(B_1+B)s+K_1+K}{(M_2s^2+s(B_2+B)+K)(M_1s^2+(B_1+B)s+K+K_1)-(Bs+K)^2}$$

2. obtain the transfer function $\frac{Y_2(s)}{F(s)}$ of the system.



Sol:-



$$f_{M_1} = M_1\frac{d^2y_1}{dt^2}$$

$$f_{K_1} = K_1Y_1$$

$$f_{K_2} = K_2(Y_1-Y_2)$$

$$f_B = B\frac{dY_1}{dt}$$

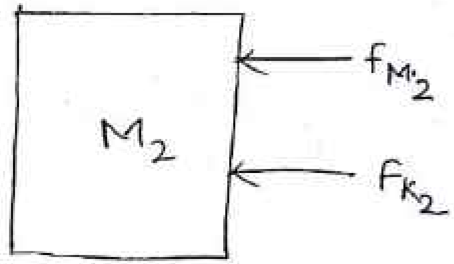
By newton's second law,

$$M_1\frac{d^2Y_1}{dt^2} + K_1Y_1 + K_2(Y_1-Y_2) + B\frac{dY_1}{dt} = f(t) \quad \text{— (1)}$$

Applying laplace transform

$$M_1s^2Y_1(s) + K_1Y_1(s) + K_2Y_1(s) - K_2Y_2(s) + BsY_1(s) = F(s)$$

$$Y_1(s)\left[M_1s^2+K_1+K_2+Bs\right] - K_2Y_2(s) = F(s) \quad \text{— (2)}$$



$$f_{m_2} = M_2\frac{d^2y_2}{dt^2}$$

$$f_{K_2} = K_2(Y_2-Y_1)$$

By Newton's second law,

$$M_2\frac{d^2y_2}{dt^2} + K_2(y_2-y_1)=0 \quad \text{—— (3)}$$

$M_2 s^2 y_2$ Laplace transform

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s) = 0$$

$$Y_2(s)\left[M_2 s^2 + K_2\right] = K_2 Y_1(s)$$

$$Y_1(s) = \left[\frac{M_2 s^2 + K_2}{K_2}\right] Y_2(s) \quad \text{—— (4)}$$

Sub (4) in (2)

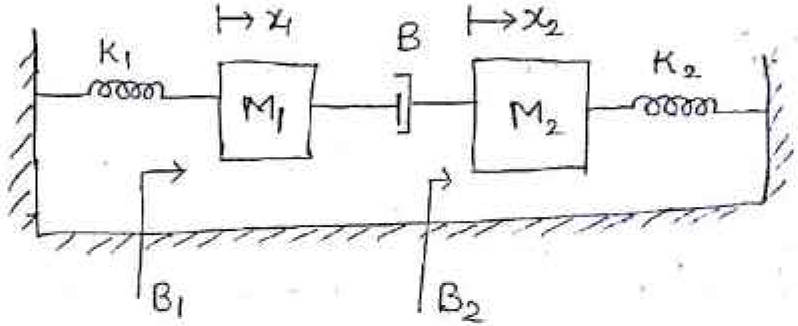
$$\left[\frac{M_2 s^2 + K_2}{K_2}\right] Y_2(s)\left[M_1 s^2 + K_1 + K_2 + Bs\right] - K_2 Y_2(s) = F(s)$$

$$Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + K_1 + K_2 - Bs)}{K_2} - K_2\right] = F(s)$$

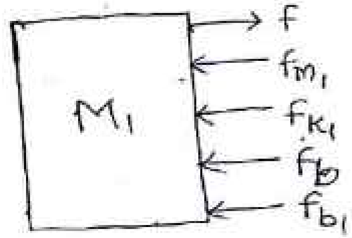
$$Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + K_1 + K_2 - Bs) - K_2^2}{K_2}\right] = F(s)$$

$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{(M_2 s^2 + K_2)(M_1 s^2 + K_1 + K_2 - Bs) - K_2^2}$$

Determine the transfer function $\frac{X_1(s)}{F(s)}$ and $\frac{X_2(s)}{F(s)}$ of the following system



Sol:-



Free body diagram

$$f_{m_1} = M_1\frac{d^2x_1}{dt^2}$$

$$f_{K_1} = K_1 x_1$$

$$f_b = B\frac{d}{dt}(x_1 - x_2)$$

$$f_{b_1} = B_1\frac{d}{dt}x_1$$

From Newton's second of law

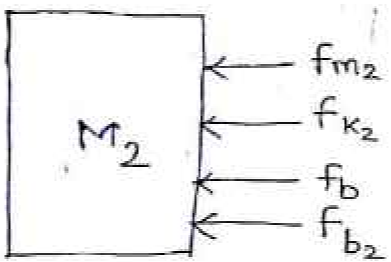
$$f_{m_1} + f_{K_1} + f_{b_1} + f_b = f$$

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}x_1 + B\frac{d}{dt}(x_1-x_2) + K_1x_1 = f(t) \quad \text{—— (1)}$$

Apply laplace transformation

$$M_1s^2x_1(s) + B_1sx_1(s) + BS\left[x_1(s) - x_2(s)\right] + K_1x_1(s) = F(s)$$

$$x_1(s)\left[M_1s^2 + B_1S + BS + K_1\right] - BSx_2(s) = F(s) \quad \text{—— (2)}$$



$$f_{m_2} = M_2\frac{d^2x_2}{dt^2}$$

$$f_{k_2} = K_2x_2$$

$$f_b = B\frac{d}{dt}(x_2-x_1)$$

$$f_{b_2} = B_2\frac{d}{dt}x_2$$

From Newton's second law,

$$f_{m_2} + f_{k_2} + f_b + f_{b_2} = 0$$

$$M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B\frac{d}{dt}(x_2-x_1) + B_2\frac{d}{dt}x_2 = 0 \quad \text{—— (3)}$$

Apply Laplace transformation

$$M_2s^2x_2(s) + K_2x_2(s) + BS\left[x_2(s) - x_1(s)\right] + B_2Sx_2(s) = 0$$

$$x_2(s)\left[M_2s^2 + K_2 + BS + B_2S\right] = BSx_1(s) \quad \text{—— (4)}$$

Sub (4) in (2)

$$x_1(s)\left[M_1s^2 + B_1S + BS + K_1\right] - \frac{B^2S^2}{\left[M_2s^2 + B_2S + BS + K_2\right]}x_1(s) = F(s)$$

$$x_1(s)\left[\left[M_1S^2 + B_1S + BS + K_1\right] - \frac{B^2s^2}{\left[M_2s^2 + B_2S + BS + K_2\right]}\right] = F(s)$$

$$\frac{x_1(s)}{F(s)} = \frac{M_2s^2 + BS(B_2+B) + K_2}{\left(M_1s^2 + s(B_1+B) + K_1\right)\left(M_2s^2 + s(B_2+B) + K_2\right) - B^2s^2}$$

Sub (4) in (2).

$$\frac{\left(M_2s^2 + s(B_2+B) + K_2\right)\left(M_1s^2 + s(B_1+B) + K\right)}{BS}x_2(s) - BSx_2(s) = F(s)$$

$$x_2(s)\left[\frac{\left(M_2s^2 + s(B_2+B) + K_2\right)\left(Ms^2 + s(B_1+B) + K\right)}{BS} - BS\right] = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{BS}{\left(M_1s^2+s(B_1+B)+K\right)\left(M_2s^2+s(B_2+B)+K_2\right)-(BS)^2}$$

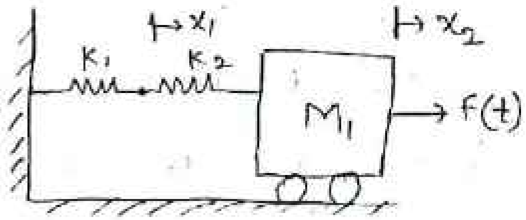
Output:-

Differential eqⁿ.

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}x_1 + B\frac{d}{dt}(x_1-x_2) + K_1x_1 = f(t)$$

$$M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + B\frac{d}{dt}(x_2-x_1) + K_2x_2 = 0$$

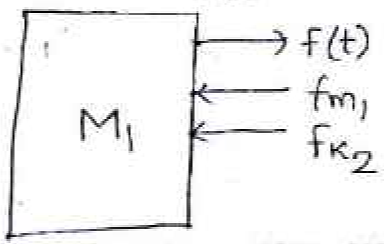
4. Find the transfer function of a given system



Sol:- $i/p = f(t) = L\{f(t)\} = F(s)$

$o/p = x_2(t) = L\{x_2(t)\} = X_2(s)$

$$T.F = \frac{X_2(s)}{F(t)}$$



$$f_{m_1} = M_1\frac{d^2x_2}{dt^2}$$

$$f_{K_2} = K_2(x_2-x_1)$$

From Newton's II law

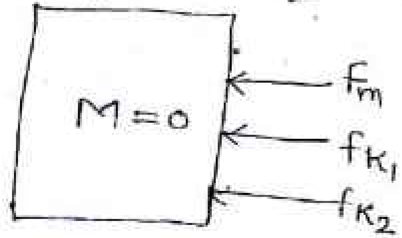
$$f_{m_1} + f_{K_2} = f$$

$$M_1\frac{d^2x_2}{dt^2} + K_2(x_2-x_1) = F(t) \quad \text{—— (1)}$$

Laplace transform

$$M_1s^2 X_2(s) + K_2\left[X_2(s) - X_1(s)\right] = F(s)$$

$$X_2(s)\left[M_1s^2 + K_2\right] - X_1(s)K_2 = F(s) \quad \text{—— (2)}$$



$f_m = 0$; $f_{K_1} = K_1x_1$; $f_{K_2} = K_2[x_1-x_2]$

Newton's II law

$$f_m + f_{K_1} + f_{K_2} = 0$$

$$K_1x_1 + K_2[x_1-x_2] = 0 \quad \text{—— (3)}$$

Laplace transform

$$K_1X_1(s) + K_2\left[X_1(s) - X_2(s)\right] = 0$$

$$X_1(s)\left[K_1+K_2\right] = K_2X_2(s)$$

$$X_1(s) = \frac{K_2}{K_1+K_2} X_2(s) \quad \text{——— (4)}$$

Sub (4) in (2).

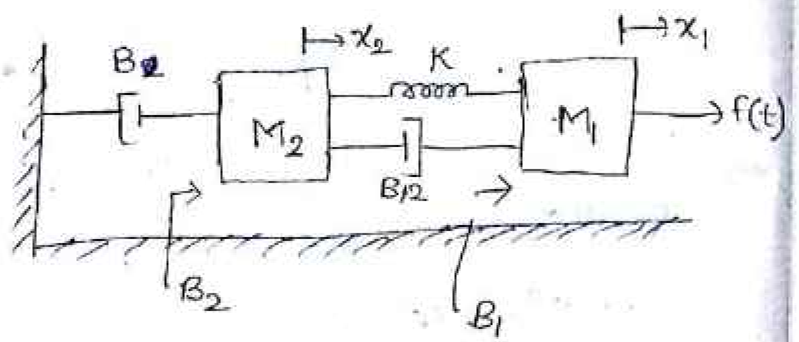
$$X_2(s)\left[M_1 s^2 + K_2\right] - \frac{K_2^2}{K_1+K_2} X_2(s) = F(s)$$

$$X_2(s)\left[M_1 s^2 + K_2 - \frac{K_2^2}{K_1+K_2}\right] = F(s)$$

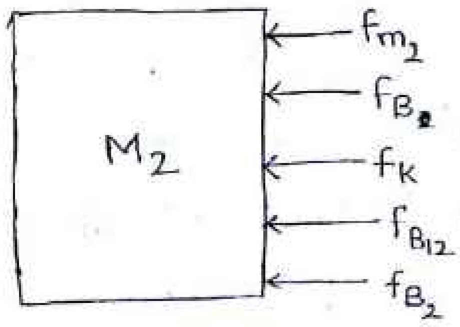
$$\frac{X_2(s)}{F(s)} \left[\frac{(M_1 s^2+K_2)(K_1+K_2) - K_2^2}{K_1+K_2}\right] = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{K_1+K_2}{(M_1 s^2+K_2)(K_1+K_2) - K_2^2}$$

5.



sol:-



$$f_{m_2} = M_2 \frac{d^2 x_2}{dt^2}$$

$$f_{B} = B_2 \frac{d}{dt} x_2$$

$$f_K = K(x_2 - x_1)$$

$$f_{B_{12}} = B_{12} \frac{d}{dt}(x_2 - x_1)$$

$$f_{B_2} = B_2 \frac{d}{dt} x_2$$

By newton's second law,

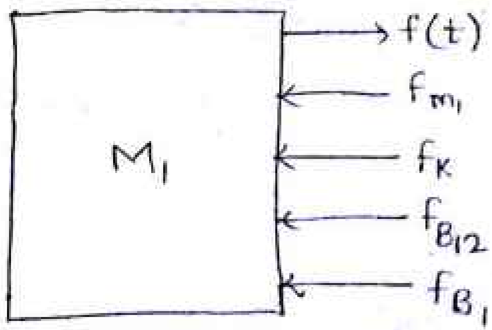
$$f_{m_2} + f_{B} + f_K + f_{B_{12}} + f_{B_2} = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d}{dt} x_2 + K(x_2 - x_1) + B_{12} \frac{d}{dt}(x_2 - x_1) + B \frac{d}{dt} x_2 = 0 \quad \text{——— (1)}$$

Apply laplace transform,

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + K X_2(s) - K X_1(s) + B_{12} s X_2(s) - B_{12} s X_1(s) + B s X_2(s) = 0$$

$$X_2(s)\left[M_2 s^2 + B_2 s + K + B_{12} s + B s\right] = X_1(s)\left[K + B_{12} s\right] \quad \text{——— (2)}$$



$f_{m_1} = M_1 \frac{d^2x_1}{dt^2}$

$f_K = K(x_1 - x_2)$

$f_{B_{12}} = B_{12} \frac{d}{dt}(x_1 - x_2)$

$f_{B_1} = B_1 \frac{d}{dt} x_1$

By newton's second law,

$f_{m_1} + f_K + f_{B_{12}} + f_{B_1} = f(t)$

$$M_1 \frac{d^2x_1}{dt^2} + K(x_1 - x_2) + B_{12}\frac{d}{dt}(x_1 - x_2) + B_1 \frac{d}{dt}x_1 = f(t) \quad \text{——} (3)$$

Apply laplace transform

$$M_1 s^2 X_1(s) + K X_1(s) - K X_2(s) + B_{12} s X_1(s) - B_{12} s X_2(s) + B_1 s X_1(s) = F(s)$$

$$X_1(s)\left[M_1 s^2 + K + B_{12}s + B_1 s\right] - X_2(s)\left[K + B_{12}s\right] = F(s) \quad \text{——} (4)$$

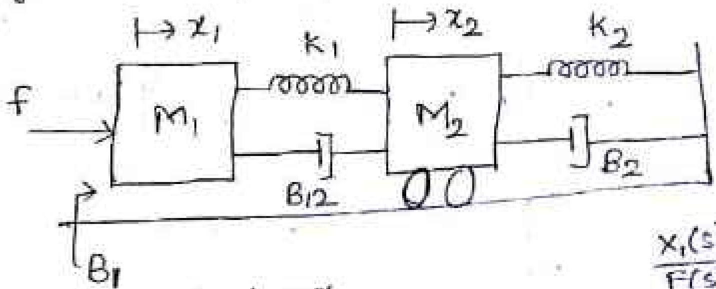
Sub (2) in (4).

$$X_1(s)\left[M_1 s^2 + (B_{12}+B_1)s + K\right] - \frac{(K+B_{12}s)(K+B_{12}s)}{\left[M_2 s^2 + B_2 s + K + B_{12}s + B_2 s\right]} X_1(s) = F(s)$$

$$X_1(s)\left[\frac{(M_1 s^2 + s(B_{12}+B_1)+K)(m_2 s^2 + B_2 s + K + B_{12}s + B_2 s) - (K+B_{12}s)(K+B_{12}s)}{(m_2 s^2 + B_2 s + K + B_{12}s + B_2 s)}\right] = F(s)$$

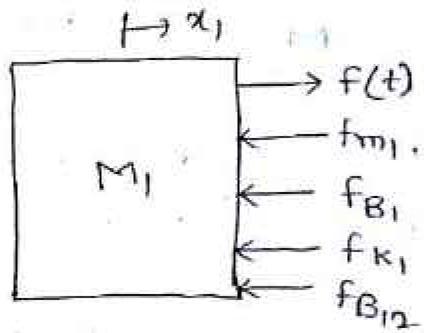
$$\frac{X_1(s)}{F(s)} = \frac{m_2 s^2 + B_2 s + K + B_{12}s + B_2 s}{(M_1 s^2 + s(B_{12}+B_1)+K)(m_2 s^2 + B_2 s + K + B_{12}s + B_2 s) - (K+B_{12}s)(K+B_{12}s)}$$

6. Find the transfer function of the system.



$\frac{X_1(s)}{F(s)}$

Sol:-



$f_{m_1} = M_1 \frac{d^2x_1}{dt^2}$

$f_{B_1} = B_1 \frac{d}{dt} x_1$

$f_{K_1} = K_1(x_1 - x_2)$

$f_{B_{12}} = B_{12}\frac{d}{dt}(x_1 - x_2)$

By Newton's second law,

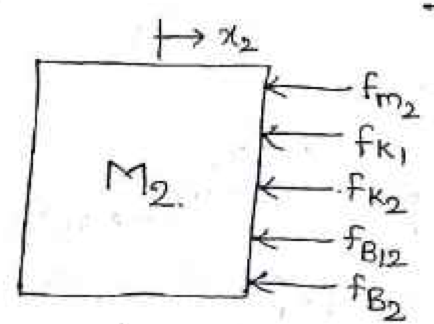
$f_{m_1} + f_{B_1} + f_{K_1} + f_{B_{12}} = f(t)$

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}x_1 + K_1(x_1 - x_2) + B_{12}\frac{d}{dt}(x_1 - x_2) = f(t) \quad \text{—— (1)}$$

Apply Laplace transform.

$$M_1S^2X_1(s) + B_1SX_1(s) + K_1X_1(s) - K_1X_2(s) + B_{12}SX_1(s) - B_{12}SX_2(s) = F(s)$$

$$X_1(s)\left[M_1S^2 + B_1S + K_1 + B_{12}S\right] - X_2(s)\left[K_1 + B_{12}S\right] = F(s) \quad \text{—— (2)}$$



$f_{m_2} = M_2\frac{d^2x_2}{dt^2}$

$f_{K_1} = K_1(x_2 - x_1)$ $\qquad f_{B_2} = B_2\frac{d}{dt}x_2$

$f_{K_2} = K_2x_2$

$f_{B_{12}} = B_{12}\frac{d}{dt}(x_2 - x_1)$

By Newton's second law,

$f_{m_2} + f_{K_1} + f_{K_2} + f_{B_{12}} + f_{B_2} = 0$

$$M_2\frac{d^2x_2}{dt^2} + K_1(x_2 - x_1) + K_2x_2 + B_{12}\frac{d}{dt}(x_2 - x_1) + B_2\frac{d}{dt}x_2 = 0$$

Apply Laplace transform

$$M_2S^2X_2(s) + K_1X_2(s) - K_1X_1(s) + K_2X_2(s) + B_{12}SX_2(s) - B_{12}SX_1(s) + B_2SX_2(s) = 0$$

$$X_2(s)\left[M_2S^2 + K_1 + K_2 + B_{12}S + B_2S\right] = X_1(s)\left[K_1 + B_{12}S\right]$$

$$X_2(s) = \frac{K_1 + B_{12}S}{M_2S^2 + K_1 + K_2 + (B_{12} + B_2)S}X_1(s) \quad \text{—— (4)}$$

sub (4) in (2).

$$X_1(s)\left[M_1S^2 + (B_1 + B_{12})S + K_1\right] - \frac{(K_1 + B_{12}S)^2}{M_2S^2 + (B_2 + B_{12})S + K_1 + K_2}X_1(s) = F(s)$$

$$X_1(s)\left[\frac{(M_1S^2 + (B_1 + B_{12})S + K_1)(M_2S^2 + (B_2 + B_{12})S + K_1 + K_2) - (K_1 + B_{12}S)^2}{M_2S^2 + (B_2 + B_{12})S + K_1 + K_2}\right] = F(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{M_2s^2+(B_2+B_{12})s+K_1+K_2}{(M_1s^2+(B_1+B_{12})s+K_1)(M_2s^2+(B_2+B_{12})s+K_1+K_2) - (K_1+B_{12}s)^2}$$

Mechanical Rotational System:-

Symbols ~~used~~

To obtain mechanical translational system we require 3 basic elements Inertia of mass or Moment of inertia of mass(J) dash pot with rotational friction coeff(B) Torsional spring (K)

Symbols used

θ = angular displacement in radians

$\frac{d\theta}{dt}$ = angular velocity radians/sec

$\frac{d^2\theta}{dt^2}$ = angular acceleration rad/sec^2

T = applied torque - ~~N/~~ N-m

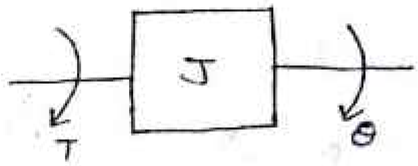
J = moment of inertia Kg-m^2/rad

B = rotational frictional coeff N-m

K = stiffness of the spring N-m

Torque balance eq^ns of idealized elements moment of inertia of mass:-

$$T_j \propto \frac{d^2\theta}{dt^2}$$

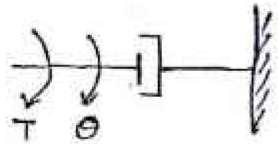


$$T_j = J\frac{d^2\theta}{dt^2}$$

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Ideal rotational dash pot with one end fixed to the reference:-

$$T_b \propto \frac{d\theta}{dt}$$

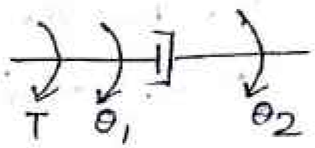


$$T_b = B\frac{d\theta}{dt}$$

$$T = T_b$$

Ideal rotational dash pot with angular displacement at both ends:-

$$T_b \propto \frac{d}{dt}(\theta_1-\theta_2)$$

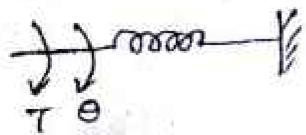


$$T_b = B\frac{d}{dt}(\theta_1-\theta_2)$$

$$T = T_b$$

Ideal torsional spring fixed to one end:-

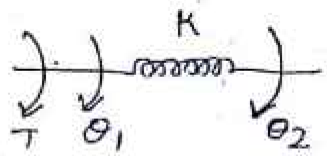
$$T_K \propto \theta$$



$T_K = K\theta$

$T = T_K$

Ideal torsional spring with displacement at both ends:-

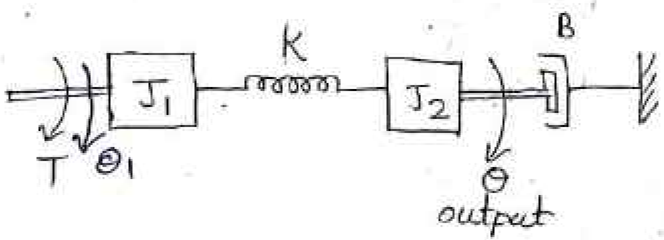


$T_K \propto (\theta_1 - \theta_2)$

$T_K = K(\theta_1 - \theta_2)$

$T = T_K$

1. write the differential eq's governing the mechanical rotational system shown in fig & obtain the transfer function



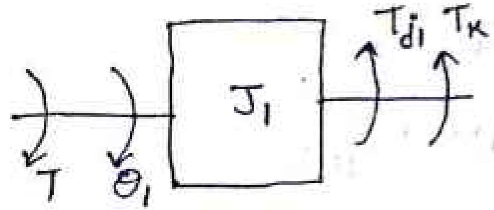
Sol:- $T = L\{T\} = T(s)$

$\theta = L\{\theta\} = \theta(s)$

Consider θ_1

$\theta_1 = L\{\theta\} = \theta_1(s)$

$T.F = \dfrac{\theta(s)}{T(s)}$



$T_{j_1} = J_1 \dfrac{d^2\theta_1}{dt^2}$

$T_K = K(\theta_1 - \theta)$

By newton's second law,

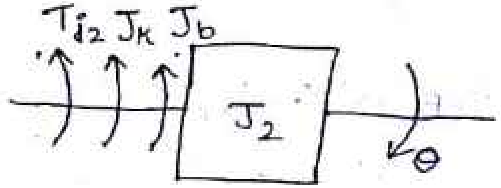
$T_{j_1} + T_K = T$

$J_1 \dfrac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$ ——— (1)

Apply Laplace transform

$J_1 s^2\theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$

$\theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s) = T(s)$ ——— (2)



$T_{j_2} = J_2 \dfrac{d^2\theta}{dt^2}$

$J_K = K(\theta - \theta_1)$

$J_b = B\dfrac{d}{dt}\theta$

By newton's second law

$$T_{j_2} + T_K + T_B = 0$$

$$J_2\frac{d^2\theta}{dt^2} + K(\theta-\theta_1) + B\frac{d}{dt}\theta = 0 \quad \text{——} (3)$$

Apply laplace transform

$$J_2 s^2\theta(s) + K\theta(s) - K\theta_1(s) + BS\,\theta(s) = 0$$

$$\theta(s)\left[J_2 s^2 + K + BS\right] = K\theta_1(s)$$

$$\theta_1(s) = \frac{J_2 s^2 + BS + K}{K}\,\theta(s) \quad \text{——} (4)$$

Put eqn (4) in eqn (2)

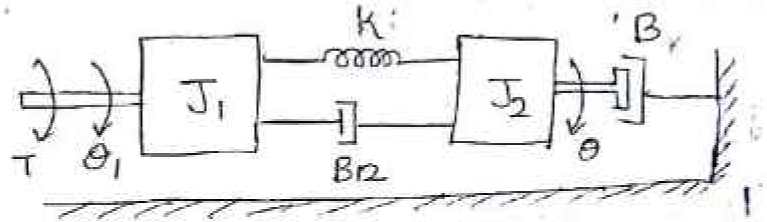
$$\left[\frac{J_2 s^2 + BS + K}{K}\right](J_1 s^2 + K)\,\theta s - K\theta(s) = T(s)$$

$$\theta(s)\left[\frac{(J_2 s^2 + BS + K)(J_1 s^2 + K)}{K} - K\right] = T(s)$$

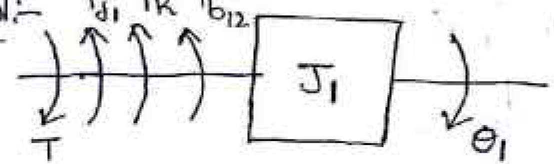
$$\theta(s)\left[\frac{(J_2 s^2 + BS + K)(J_1 s^2 + K) - K^2}{K}\right] = T(s)$$

$$\frac{\theta(s)}{T(s)} = \frac{K}{(J_2 s^2 + BS + K)(J_1 s^2 + K) - K^2}$$

2. Write the differential eqn of the mechanical system shown in fig & obtain $\frac{\theta_1(s)}{T(s)}$ & $\frac{\theta(s)}{T(s)}$



Sol:-



$$T_{j_1} = J_1\frac{d^2\theta}{dt^2}$$

$$T_K = K(\theta_1 - \theta)$$

$$T_{b_{12}} = B_{12}\frac{d}{dt}(\theta_1 - \theta)$$

By Newton's second law,

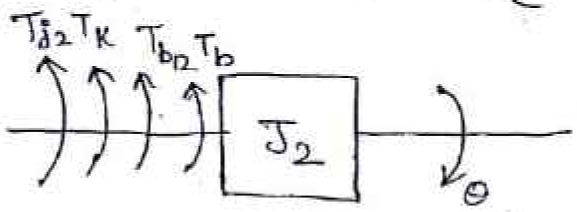
$$T_{j_1} + T_K + T_{b_{12}} = T$$

$$J_1\frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) + B_{12}\frac{d}{dt}(\theta_1 - \theta) = T \quad \text{——} (1)$$

Apply laplace transform

$$J_1 s^2\theta_1(s) + K\theta_1(s) - K\theta(s) + B_{12}S\theta_1(s) - B_{12}S\theta(s) = T(s)$$

$$\theta_1(s)\left[J_1s^2+K+B_{12}s\right]-\theta(s)\left[K+B_{12}s\right]=T(s) \quad \text{--- (2)}$$



$$T_{j2} = J_2\frac{d^2\theta}{dt^2}$$

$$T_K = K(\theta-\theta_1)$$

$$T_{b12} = B_{12}\frac{d}{dt}(\theta-\theta_1)$$

$$T_b = B\frac{d\theta}{dt}$$

By newton's second law,

$$T_{j2}+T_K+T_{b12}+T_b=0$$

$$J_2\frac{d^2\theta}{dt^2}+K(\theta-\theta_1)+B_{12}\frac{d}{dt}(\theta-\theta_1)+B\frac{d\theta}{dt}=0 \quad \text{--- (3)}$$

Apply laplace transform

$$J_2s^2\theta(s)+K\theta(s)-K\theta_1(s)+B_{12}s\theta(s)-B_{12}s\theta_1(s)+Bs\theta(s)=0$$

$$\theta(s)\left[J_2s^2+K+B_{12}s+Bs\right]=\theta_1(s)\left[K+B_{12}s\right] \quad \text{--- (4)}$$

Sub eqⁿ (4) in eqⁿ (2)

$$\theta_1(s)\left[J_1s^2+K+B_{12}s\right]-\theta_1(s)\frac{(K+B_{12}s)^2}{J_2s^2+K+s(B_{12}+B)}=T(s)$$

$$\theta_1(s)\left[\frac{(J_1s^2+B_{12}s+K)(J_2s^2+(B_{12}+B)s+K)-(K+B_{12}s)^2}{J_2s^2+(B_{12}+B)s+K}\right]=T(s)$$

$$\frac{\theta_1(s)}{T(s)}=\frac{J_2s^2+(B_{12}+B)s+K}{(J_1s^2+B_{12}s+K)(J_2s^2+(B_{12}+B)s+K)-(K+B_{12}s)^2}$$

Again sub eqⁿ (4) in eqⁿ (2).

$$\cancel{\frac{(K+B_{12}s)(J_1s^2+K+B_{12}s)}{J_2s^2+(B_{12}+B)s+K}\theta_1}$$

$$\left[\frac{(J_2s^2+(B_{12}+B)s+K)(J_1s^2+K+B_{12}s)}{K+B_{12}s}-(K+B_{12}s)\right]\theta(s)=T(s)$$

$$\frac{\theta(s)}{T(s)}=\frac{(K+B_{12}s)}{(J_2s^2+(B_{12}+B)s+K)(J_1s^2+K+B_{12}s)-(K+B_{12}s)^2}$$

$$\theta(s)\left[\frac{(J_2s^2+B_{12}s+K_1+K)(J_1s^2+B_{12}s+K)-(K+B_{12}s)^2}{(K+B_{12}s)}\right] = T(s)$$

$$\frac{T(s)}{\theta(s)} = \frac{(J_2s^2+B_{12}s+K_1+K)(J_1s^2+B_{12}s+K)-(K+B_{12}s)^2}{(K+B_{12}s)}$$

Again sub eqⁿ ④ in eqⁿ ①

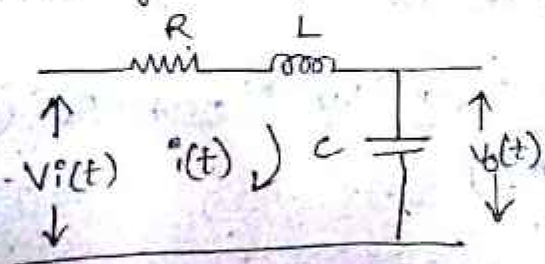
$$\theta_1(s)\left[J_1s^2+B_{12}s+K\right]-\theta_1(s)\left[\frac{(K+B_{12}s)^2}{J_2s^2+K+B_{12}s+K_1}\right]=T(s)$$

$$\theta_1(s)\left[\frac{(J_1s^2+B_{12}s+K)(J_2s^2+B_{12}s+K+K_1)-(K+B_{12}s)^2}{J_2s^2+B_{12}s+K+K_1}\right]=T(s)$$

$$\frac{\theta_1(s)}{T(s)} = \frac{J_2s^2+B_{12}s+K+K_1}{(J_1s^2+B_{12}s+K)(J_2s^2+B_{12}s+K+K_1)-(K+B_{12}s)^2}$$

Electrical systems:-

1. Find the transfer function of the circuit given below.



i/P $V_i(t) = L\{V_i(t)\} = V_i(s)$

o/P. $V_o(t) = L\{V_o(t)\} = V_o(s)$

$$T.F = \frac{V_o(s)}{V_i(s)}$$

Apply KVL to ckt.

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V_i(t) \quad \text{—— ①}$$

$$\frac{1}{C}\int i(t)dt = V_o(t) \quad \text{—— ②}$$

Apply Laplace to eqⁿ ① & ②

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = V_i(s)$$

$$R + Ls + \frac{1}{Cs} = \frac{V_i(s)}{I(s)}$$

$$\left[\frac{RCs+CLs^2+1}{Cs}\right]I(s) = V_i(s) \quad \text{—— ③}$$

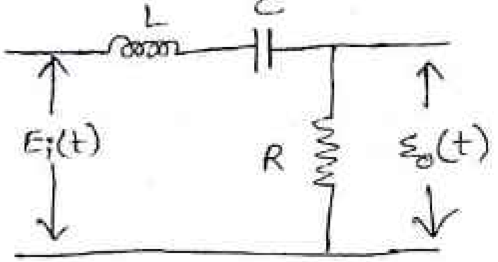
$$② \Rightarrow \frac{I(s)}{C(s)} = V_o(s) \quad \text{—— ④}$$

$$\frac{④}{③}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{I(s)}{Cs}}{\left[\frac{RCs+CLs^2+1}{Cs}\right]I(s)}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+CLs^2+1}$$

2.



$i/p = E_i(t) = L\{E_i(t)\} = E_i(s)$

$o/p = E_o(t) = L\{E_o(t)\} = E_o(s)$

Apply KVL

$$E_i(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt + Ri(t) \text{ --- } 1$$

$$E_o(t) = Ri(t) \text{ --- } 2$$

Apply Laplace to 1 & 2

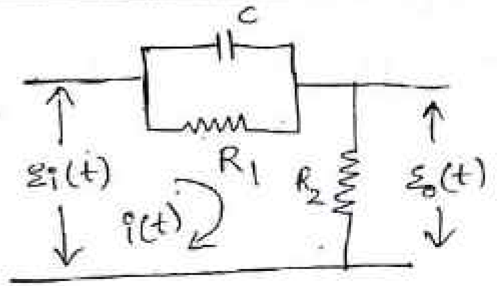
$$E_i(s) = LsI(s) + \frac{1}{Cs}I(s) + RI(s) \text{ --- } 3$$

$$E_o(s) = RI(s) \text{ --- } 4$$

$$\frac{4}{3} = \frac{E_o(s)}{E_i(s)} = \frac{I(s)R}{I(s)\left[\frac{1}{Cs} + Ls + R\right]}$$

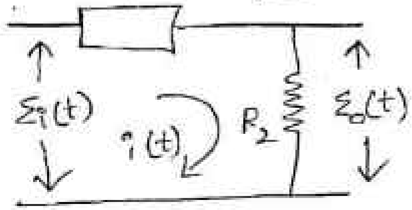
$$T(s) = \frac{RCs}{1 + LCs^2 + RCs}$$

3.



Sol:- Let Z_1 is the impedance of parallel combination of R_1 & C.

$Z_1 = R_1 // \frac{1}{j\omega C}$



$$Z_1 = R_1 // \frac{1}{j\omega C}$$

$$= \frac{R_1 \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}}$$

$$= \frac{R_1}{R_1 j\omega C + 1}$$

$$E_i(t) = \frac{R_1}{R_1 j\omega C + 1} i(t) + R_2 i(t) \text{ --- } 1$$

$$E_o(t) = R_2 i(t) \text{ --- } 2$$

Apply Laplace

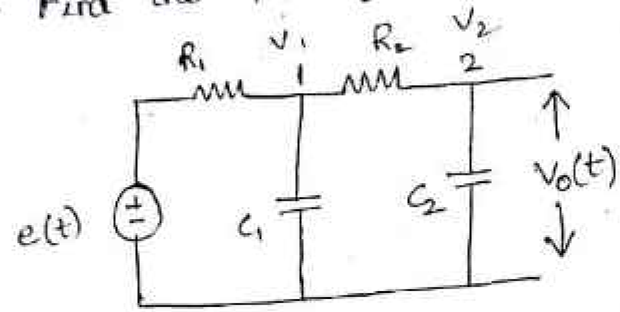
$$E_i(s) = \frac{R_1}{R_1 Cs + 1} I(s) + R_2 I(s) \text{ --- } 3$$

$$E_o(s) = R_2 I(s) \text{ --- } 4$$

$$\frac{E_o(s)}{E_i(s)} = \frac{I(s)R_2}{I(s)\left[R_2 + \frac{R_1}{R_1 Cs + 1}\right]}$$

$$T(s) = \frac{R_2[R_1 Cs + 1]}{R_2 R_1 Cs + R_2 + R_1}$$

4. Find the T.F of the given ckt.



$$T(s) = \frac{V_0(s)}{E(s)}$$

Apply KCL at node ①

$$\frac{V_1(t)}{R_1} + C_1\frac{dV_1(t)}{ds} + \frac{V_1 - V_2}{R_2} = \frac{e}{R_1} \text{ --- ①}$$

Apply KCL at node ②

$$C_2\frac{dV_2}{dt} + \frac{V_2 - V_1}{R_2} = 0 \text{ --- ②}$$

Laplace.

$$\frac{V_1(s)}{R_1} + C_1 S V_1(s) + \frac{V_1(s) - V_2(s)}{R_2} = \frac{e(s)}{R_1}$$

$$V_1(s)\left[\frac{1}{R_1} + C_1 S + \frac{1}{R_2}\right] - \frac{e(s)}{R_1} = \frac{V_2(s)}{R_2} \text{ --- ③}$$

② → $$C_2 S V_2(s) + \frac{V_2(s) - V_1(s)}{R_2} = 0$$

$$V_2(s)\left[C_2 S + \frac{1}{R_2}\right] = \frac{V_1(s)}{R_2}$$

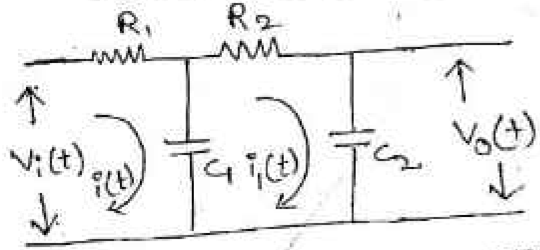
$$V_1(s) = V_2(s)\left[C_2 S R_2 + 1\right]$$

$$V_2(s)\left[C_2R_2S+1\right]\left[\frac{R_2 + C_1SR_1R_2 + R_1}{R_1R_2}\right] - \frac{e(s)}{R_1} = \frac{V_2(s)}{R_2}$$

$$V_2(s)\left[(C_2R_2S+1)\left(\frac{R_2+R_1+C_1SR_1R_2}{R_1R_2}\right) - \frac{1}{R_2}\right] = \frac{e(s)}{R_1}$$

$$V_2(s)\left[\frac{(C_2R_2S+1)(R_1+R_2+C_1SR_1R_2) - R_1}{R_2}\right] = e(s)$$

$$\frac{V_2(s)}{e(s)} = \frac{R_2}{(C_2R_2S+1)(R_1+R_2+C_1SR_1R_2) - R_1}$$



$$T(s) = \frac{V_0(t)}{V_i(t)}$$

Apply KVL ~~at~~ loop ①

$$V_i(t) = R_1 i(t) + \frac{1}{C_1}\int (i(t) - i_1(t))\,dt \text{ --- ①}$$

Apply KVL at loop ②

$$\frac{1}{C_1}\int (i_1(t) - i(t)) + R_2 i_1(t) + \frac{1}{C_2}\int i_1(t) = 0 \text{ --- ②}$$

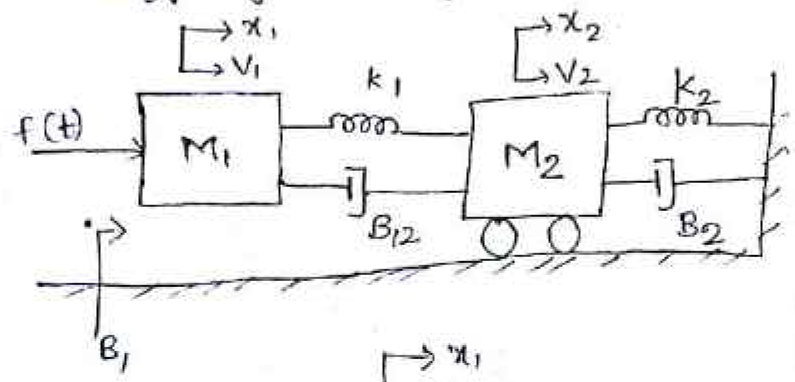
$$\frac{1}{C_2}\int i_1(t) = V_0(t) \text{ --- ③}$$

Mechanical System — Electrical System

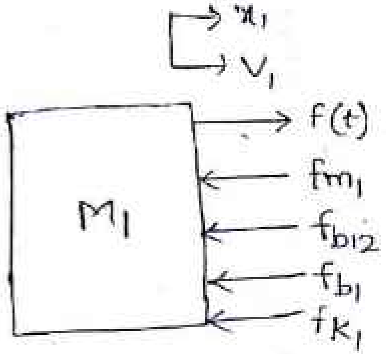
Analogous ckts (or) Analogy

Translational	Rotational	Force-voltage (F-V)/(T-V)	Force-current (F-I)/(T-I)
Force (F)	Torque (T)	voltage source (V)	current source (I)
displacement (x)	Angular displacement (θ)	Charge (q)	flux (ϕ)
Mass (M)	Inertia of Mass (J)	Inductance (L)	capacitance (C)
Dash pot (B)	Dash pot (B)	Resistance (R)	Conductance $G = \frac{1}{R}$
Spring (K)	Spring (K)	Inverse of capacitance ($\frac{1}{C}$)	Inverse of Inductance ($\frac{1}{L}$)
Velocity (v)	Angular velocity (ω)	current (i)	Voltage (V)
$\Sigma F = 0$	$\Sigma T = 0$	$\Sigma V = 0$	$\Sigma I = 0$
Lever $\frac{l_1}{l_2} = \frac{l_1}{l_2}$	Gear $\frac{T_1}{T_2} = \frac{N_1}{N_2}$	$\frac{e_1}{e_2} = \frac{N_1}{N_2}$ T/F	$\frac{i_1}{i_2} = \frac{N_2}{N_1}$

1. Write Differential eq^ns of the mechanical system shown in fig. Draw F-V & F-I electrical analogous ckts & verify by writing Mesh & Node eq^ns.



Sol:-



Free body diagram of M_1

$$f_{m_1} = M_1\frac{d^2x_1}{dt^2}$$

$$f_{b_{12}} = B_{12}\frac{d}{dt}(x_1-x_2)$$

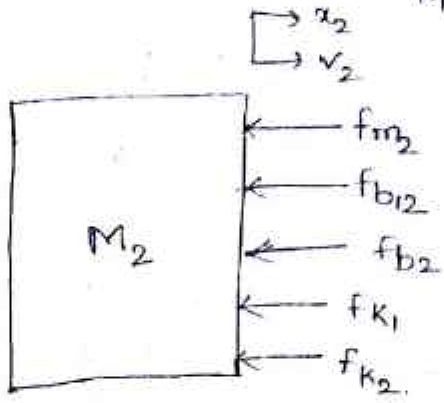
$$f_{b_1} = B_1\frac{d}{dt}x_1 .$$

$$f_{K_1} = K_1[x_1-x_2]$$

Newton's II law :-

$$f_{m_1} + f_{b_{12}} + f_{b_1} + f_{K_1} = f(t)$$

$$M_1\frac{d^2x_1}{dt^2} + B_{12}\frac{d}{dt}(x_1-x_2) + B_1\frac{d}{dt}x_1 + K_1(x_1-x_2) = f(t) \quad \text{—— (1)}$$



$$f_{m_2} = M_2\frac{d^2x_2}{dt^2}$$

$$f_{b_{12}} = B_{12}\frac{d}{dt}(x_2-x_1)$$

$$f_{b_2} = B_2\frac{d}{dt}x_2$$

$$f_{K_1} = K_1[x_2-x_1]$$

$$f_{K_2} = K_2x_2$$

From Newton's II law

$$f_{m_2} + f_{b_{12}} + f_{b_2} + f_{K_1} + f_{K_2} = 0$$

$$M_2\frac{d^2x_2}{dt^2} + B_{12}\frac{d}{dt}(x_2-x_1) + B_2\frac{d}{dt}x_2 + K_1[x_2-x_1] + K_2x_2 = 0 \quad \text{—— (2)}$$

Now, replacing displacement with corresponding velocity

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} ; \quad \frac{dx}{dt} = v \quad ; \quad x = \int v\,dt$$

eq^n ①

$$M_1 \frac{dv_1}{dt} + B_{12}(v_1 - v_2) + B_1 v_1 + K_1 \int (v_1 - v_2)\,dt = f(t) \quad \text{—— ③}$$

eq^n ②

$$M_2 \frac{dv_2}{dt} + B_{12}(v_2 - v_1) + B_2 v_2 + K_1 \int [v_2 - v_1]\,dt + K_2 \int v_2\,dt \quad \text{—— ④}$$

Force-voltage Analogy:-

The electrical analogous elements to the elements of mechanical system are given below

$f(t) \rightarrow V$; $M_1 \longrightarrow L_1$; $B_1 \longrightarrow R_1$

$M_2 \longrightarrow L_2$ $\quad B_2 \longrightarrow R_{12}$

$K_1 \rightarrow \frac{1}{C_1}$ $\quad v_1 \rightarrow i_1$

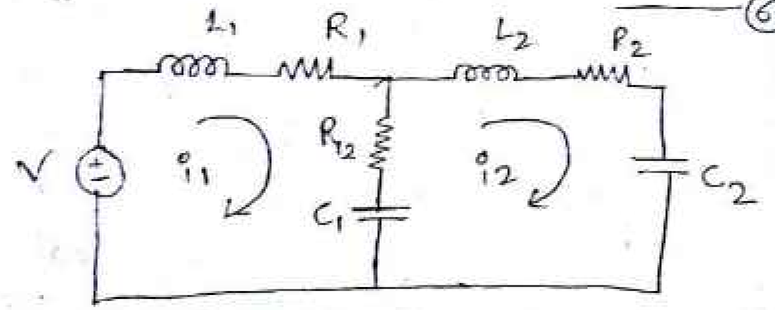
$K_2 \rightarrow \frac{1}{C_2}$ $\quad v_2 \rightarrow i_2$

eq^n ③

$$L_1 \frac{di_1}{dt} + R_{12}(i_1 - i_2) + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2)\,dt = V \quad \text{—— ⑤}$$

eq^n ④

$$L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + R_2 i_2 + \frac{1}{C_1} \int (i_1 - i_2)\,dt + \frac{1}{C} \int i_2\,dt = 0 \quad \text{—— ⑥}$$



Force current Analogy:-

$f(t) \longrightarrow I$; $M_1 \longrightarrow C_1$; $B_1 \longrightarrow \frac{1}{R_1}$;

$M_2 \longrightarrow C_2$ $\quad B_2 \longrightarrow \frac{1}{R_{12}}$

$K_1 \rightarrow \frac{1}{L_1}$ $\quad v_1 \longrightarrow V_1$

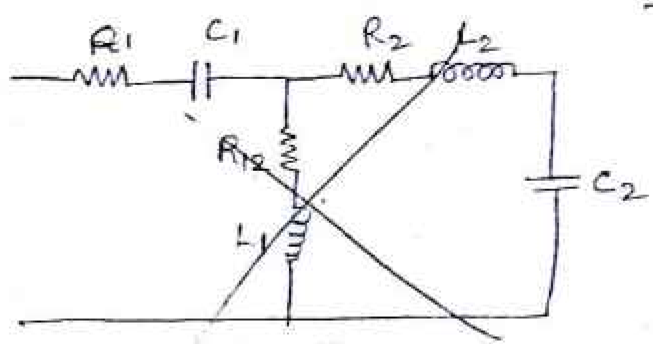
$K_2 \longrightarrow \frac{1}{L_2}$ $\quad v_2 \rightarrow V_2$

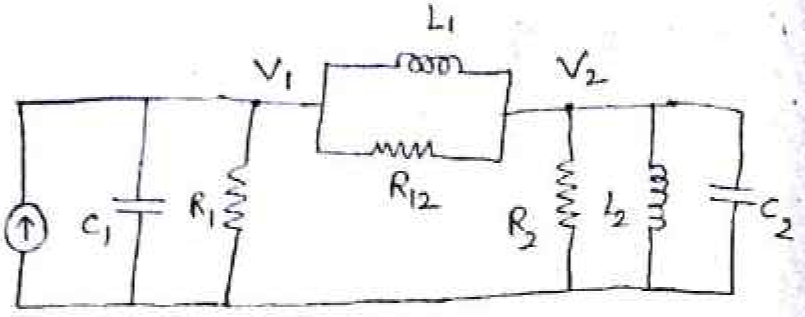
eq^n ③,

$$C_1 \frac{dV_1}{dt} + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{R_1} V_1 + \frac{1}{L_1} \int (V_1 - V_2)\,dt = I \quad \text{—— ⑦}$$

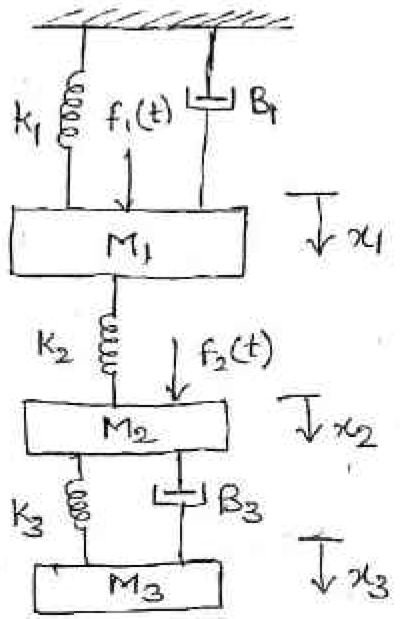
eq^n ④

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{R_2} V_2 + \frac{1}{L_1} \int (V_2 - V_1)\,dt + \frac{1}{L_2} \int V_2\,dt = 0 \quad \text{—— ⑧}$$

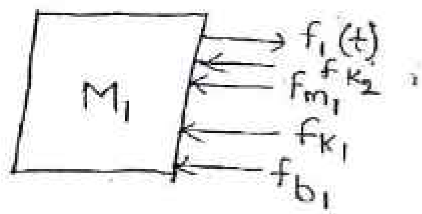




2. write differential eqⁿs of the mechanical system shown in fig. Draw F-V & F-I electrical analogous ckts.



Sol:-



$f_{m_1} = M_1\frac{d^2x_1}{dt^2}$ $\qquad f_{K_2} = K_2(x_1-x_2)$

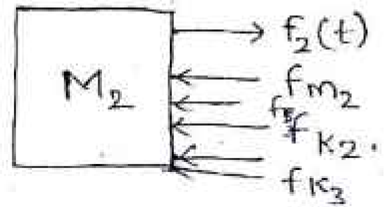
$f_{K_1} = K_1x_1$

$f_{b_1} = B_1\frac{d}{dt}x_1$

By newton's II law,

$f_{K_2} + f_{m_1} + f_{K_1} + f_{b_1} = f_1(t)$

$$M_1\frac{d^2x_1}{dt^2} + K_1x_1 + B_1\frac{dx_1}{dt} + K_2(x_1-x_2) = f_1(t) \quad \text{—— (1)}$$



$f_{m_2} = M_2\frac{d^2x_2}{dt^2}$ $\qquad f_{K_3} = K_3(x_2-x_3)$

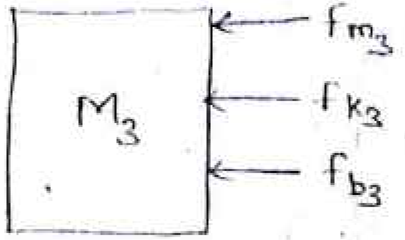
$f_{K_2} = K_2(x_2-x_1)$ $\qquad f_{b_3} = B_3\frac{d}{dt}(x_2-x_3)$

By newton's II law,

$f_{K_3} + f_{b_3} + f_{m_2} + f_{K_2} = f_2(t)$

$$M_2\frac{d^2x_2}{dt^2} + K_2(x_2-x_1) = \cancel{f_2(t)} + K_3(x_2-x_3) + B_3\frac{d}{dt}(x_2-x_3) = f_2(t)$$

—— (2)



$$f_{m_3} = M_3 \frac{d^2 x_3}{dt^2}$$

$$f_{k_3} = K_3 (x_3 - x_2)$$

$$f_{b_3} = B_3 \frac{d}{dt}(x_3 - x_2)$$

By newton's II law,

$$f_{m_3} + f_{k_3} + f_{b_3} = 0$$

$$M_3 \frac{d^2 x_3}{dt^2} + K_3 (x_3 - x_2) + B_3 \frac{d}{dt}(x_3 - x_2) = 0 \quad \text{——— (3)}$$

Replacing displacement with corresponding veloc

$$x = \int v\,dt \; ; \quad v = \frac{dx}{dt} \; ; \quad \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

eq^n (1)

$$M_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_1 v_1 + K_2 \int (v_1 - v_2) dt = f_1(t) \quad \text{——— (4)}$$

eq^n (2)

$$M_2 \frac{dv_2}{dt} + K_2 \int (v_2 - v_1) dt + K_3 \int (v_2 - v_3) dt + B_3 (v_2 - v_3) = f_2(t) \quad \text{——— (5)}$$

eq^n (3)

$$M_3 \frac{dv_3}{dt} + K_3 \int (v_3 - v_2) dt + B_3 (v_3 - v_2) = 0 \quad \text{——— (6)}$$

Force voltage analogy :-

$f(t) \rightarrow V$ $\quad M_1 = L_1 \quad B_1 = R_1 \quad K_1 = \frac{1}{C_1}$

$M_2 = L_2 \quad B_3 = R_3 \quad K_2 = \frac{1}{C_2}$

$M_3 = L_3 \quad K_3 = \frac{1}{C_3}$

$v_1 = i_1$

$v_2 = i_2$

$v_3 = i_3$

eq^n (4).

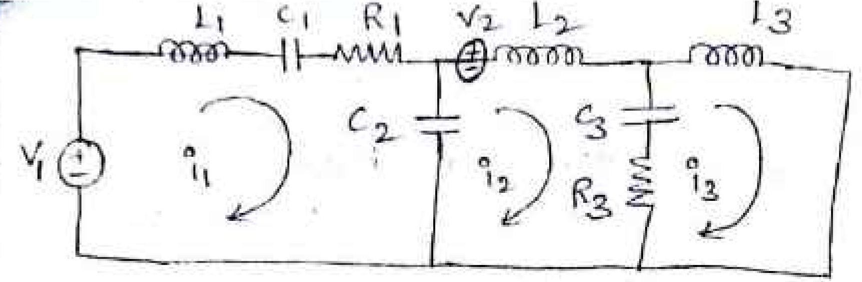
$$L_1 \frac{di_1}{dt} + \frac{1}{C_1} \int i_1 dt + R_1 i_1 + \frac{1}{C_2} \int (i_1 - i_2) dt = V_1 \quad \text{——— (7)}$$

eq^n (5)

$$L_2 \frac{di_2}{dt} + \frac{1}{C_2} \int (i_2 - i_1) dt + \frac{1}{C_3} \int (i_2 - i_3) dt + R_3 (i_2 - i_3) = V_2 \quad \text{——— (8)}$$

eq^n (6)

$$L_3 \frac{di_3}{dt} + \frac{1}{C_3} \int (i_3 - i_2) dt + R_3 (i_3 - i_2) = 0 \quad \text{——— (9)}$$



Force current Analogy

$f(t) = I$; $M_1 = C_1$; $B_1 = \frac{1}{R_1}$

$M_2 = C_2$ $\quad B_3 = \frac{1}{R_3}$

$M_3 = C_3$

$K_1 - \frac{1}{L_1}$ $\quad \vartheta_1 = V_1$

$K_2 - \frac{1}{L_2}$ $\quad \vartheta_2 = V_2$

$K_3 - \frac{1}{L_3}$ $\quad \vartheta_3 = V_3$

eq^n 4

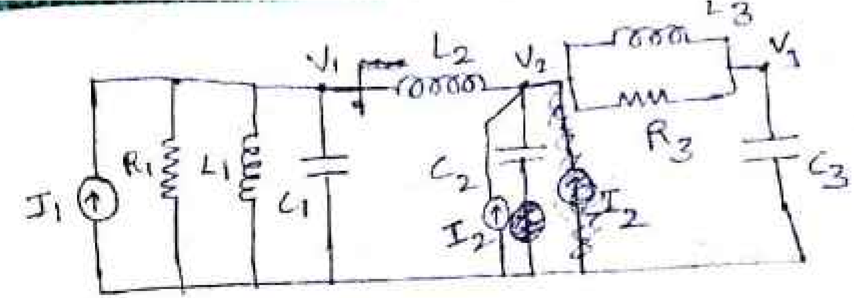
$$C_1 \frac{dv_1}{dt} + \frac{1}{L_1}\int v_1 dt + \frac{1}{R_1}v_1 + \frac{1}{L_2}\int (v_1 - v_2)dt = I_1$$

eq^n 5

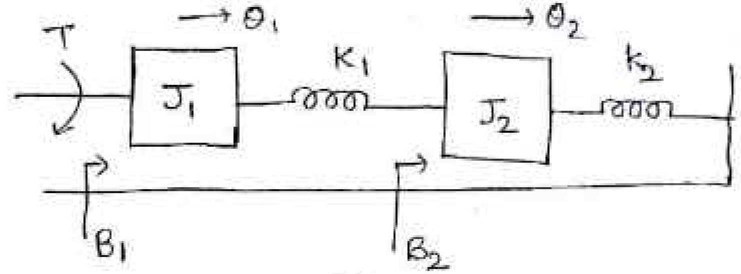
$$C_2 \frac{dv_2}{dt} + \frac{1}{L_2}\int (v_2 - v_1)dt + \frac{1}{L_3}\int (v_2 - v_3)dt + \frac{1}{R_3}(v_2 - v_3) = I_2$$

eq^n 6

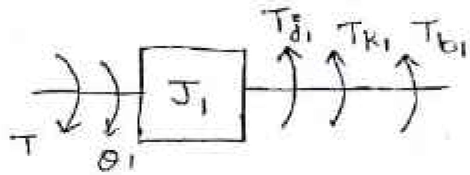
$$C_3 \frac{dv_3}{dt} + \frac{1}{L_3}\int (v_3 - v_2)dt + \frac{1}{R_3}(v_3 - v_2) = 0$$



1. Write the differential eq^n's of the rotational system shown in fig. Draw T-V & T-I analogous ckts.



Sol:-



$$T_{J_1} = J_1 \frac{d^2\theta_1}{dt^2}$$

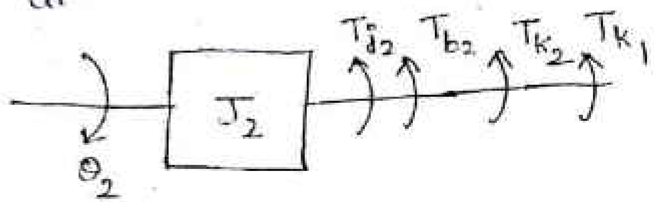
$$T_{K_1} = K_1(\theta_1 - \theta_2)$$

$$T_{b_1} = B_1 \frac{d\theta_1}{dt}$$

By Newton's II^nd law,

$$T_{J_1} + T_{K_1} + T_{b_1} = T$$

$$J_1\frac{d^2\theta_1}{dt^2} + K_1(\theta_1-\theta_2) + B_1\frac{d\theta_1}{dt} = T \quad \text{——} (1)$$



$$T_{j_2} = J_2\frac{d^2\theta_2}{dt^2}$$

$$T_{b_2} = B_2\frac{d\theta_2}{dt}$$

$$T_{k_2} = K_2\theta_2$$

$$T_{k_1} = K_1(\theta_2-\theta_1)$$

By newtons II law,

$$T_{j_2} + T_{b_2} + T_{k_2} + T_{k_1} = 0$$

$$J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2-\theta_1) = 0 \quad \text{——} (2)$$

Replacing angular displacements with angular velocity.

$$\frac{d^2\theta}{dt} = \frac{d\omega}{dt} \; ; \; \frac{d\theta}{dt} = \omega \; ; \; \theta = \int \omega dt.$$

eq^n (1)

$$J_1\frac{d\omega_1}{dt} + B_1\omega_1 + K_1\int(\omega_1-\omega_2)dt = T \quad \text{——} (3)$$

eq^n (2)

$$J_2\frac{d\omega_2}{dt} + B_2\omega_2 + K_1\int(\omega_2-\omega_1)dt + K_2\int\omega_2 = 0 \quad \text{——} (4)$$

T-V Analogy:-

$T \to V$ $J_1 \to L_1$ $B_1 \to R_1$ $K_1 \to \frac{1}{C_1}$

$J_2 \to L_2$ $B_2 \to R_2$ $K_2 \to \frac{1}{C_2}$

$\omega_1 \to i_1$

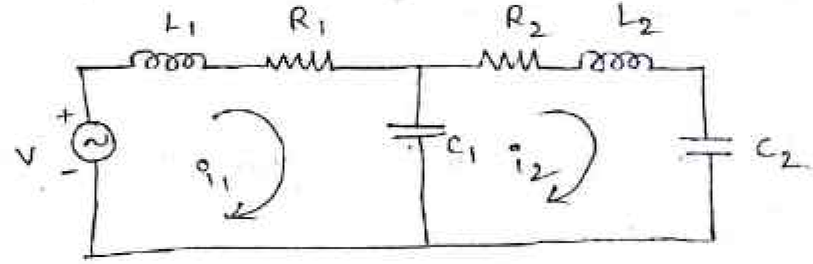
$\omega_2 \to i_2$.

~~$J_1\frac{di_1}{dt} + B_1\theta_1 dt$~~ · eq^n (3)

$$L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int(i_1-i_2)dt = V$$

eq^n (4)

$$L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_1}\int(i_2-i_1)dt + \frac{1}{C_2}\int i_2 = 0$$



T-I Analogy:-

$T \to I$ $\omega_1 \to V_1$ $J_1 \to C_1$ $B_1 \to \frac{1}{R_1}$

$\omega_2 \to V_2$ $J_2 \to C_2$ $B_2 \to \frac{1}{R_2}$

$K_1 \to \frac{1}{L_1}$

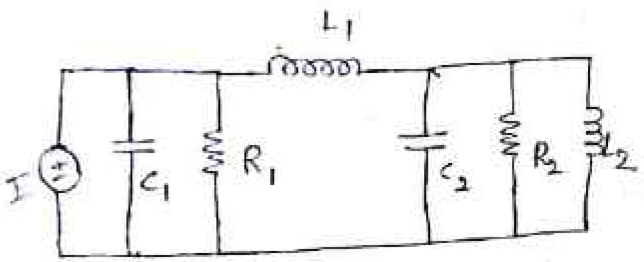
$K_2 \to \frac{1}{L_2}$

eq^n (3)

$$C_1\frac{dV_1}{dt} + \frac{1}{R_1}V_1 + \frac{1}{L_1}\int(V_1-V_2)dt = I$$

eq^n (4)

$$C_2\frac{dv_2}{dt} + \frac{1}{R_2}v_2 + \frac{1}{L_1}\int(v_2 - v_1)dt + \frac{1}{L_2}\int v_2 dt = 0$$



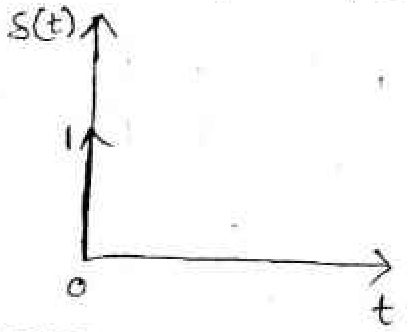
Impulse Response :-

Mathematically impulse function is defined as $F(t) = A$ for $t = 0$

$= 0$ for $t \neq 0$.

Mathematically a unit impulse is defined as $\delta(t) = 1$ for $t = 0$

$= 0$ for $t \neq 0$



$$\boxed{L\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt}$$

$$L\{\delta(t)\} = \int_0^\infty \delta(t)e^{-st}dt$$

$$= \int_0^\infty e^{-st}dt$$

$$= e^{-st}/_{t=0} \text{ by sampling property}$$

$$L\{\delta(t)\} = \delta(s) = 1$$

Now consider the i/p as unit impulse

$r(t) = \delta(t)$

$R(s) = L\{\delta(t)\}$.

$R(s) = 1$

wkt, transfer function, T.F.

$$T(s) = \frac{C(s)}{R(s)}$$

$$T(s) = \frac{C(s)}{1}$$

$$T(s) = C(s)$$

$$c(t) = L^{-1}\{T(s)\}$$

$\therefore$ Laplace transform of an impulse response of a linear time invariant system is its transfer function with all the initial cond^n are assumed to be zero.

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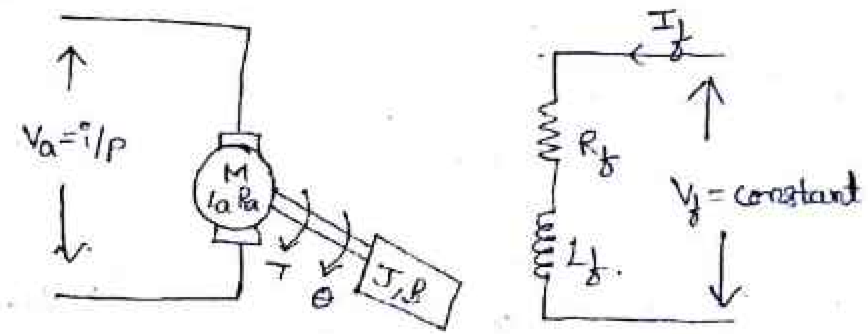
1. Gain is reduced by a factor $\frac{1}{1+G(s)H(s)}$
2. There is a reduction of parameter variation by a factor $1+G(s)H(s)$
3. There is improvement in sensitivity
4. There may be reduction of stability

Ramesh sir
9491471702

Unit - II

Servo motors:- Motor which is used in automatically control system is called servo motor. It has linear charn. It converts electric sig to angular displacement
A.C
D.C
special

TF of armature controlled DC servo motor :-



Let,

R_a = armature resistance, Ω

L_a = armature inductance, H

i_a = armature current, A

V_a = armature voltage, V

e_b = back emf, V

K_t = Torque constant, N-m/A

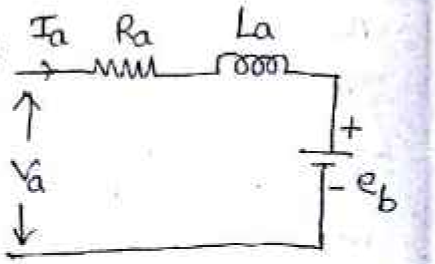
T = Torque developed by the motor, N-m

θ = angular displacement of shaft, rad

J = moment of inertia of motor & load
B = frictional coeff of motor & load N-m/(rad/s)
K_b = back emf constant. V/(rad/sec)

$$N \propto \frac{V}{\phi}$$

The equivalent ckt of armature winding



eq. ~~electrical~~ ckt of armature

Apply KVL,

$$R_a I_a + L_a \frac{dI_a}{dt} + e_b = V_a \quad \text{——— (1)}$$

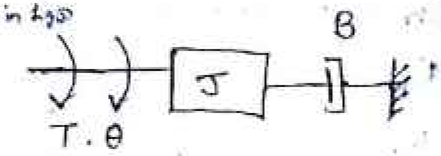
Torque developed in dc motor is directly proportional to flux & armature current.

$$T \propto \phi I_a$$

Here field voltage is kept constant. Hence flux are maintained constant. so Torque is directly proportional to Armature current.

$$T = K_t I_a \quad \text{——— (2)}$$

The mechanical system of the motor is shown in fig.



Mechanical eq. ckt.

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{——— (3)}$$

Back emf of the motor is directly proportional to angular velocity.

$$e_b \propto \frac{d\theta}{dt}$$

$$e_b = K_b \frac{d\theta}{dt} \quad \text{——— (4)}$$

Now apply laplace transform to eq^n (1), (2), (3), (4).

$$R_a I_a(s) + L_a s I_a(s) + E_b(s) = V_a(s)$$

$$I_a(s)\left[R_a + L_a s\right] + E_b(s) = V_a(s) \quad \text{——— (5)}$$

$$T(s) = K_t I_a(s) \quad \text{——— (6)}$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \quad \text{——— (7)}$$

$$E_b(s) = K_b s\theta(s) \quad \text{——— (8)}$$

Sub eq^n (6) in eq^n (7)

$$Js^2\theta(s) + Bs\theta(s) = K_t I_a(s)$$

$$\theta(s)\left[Js^2 + Bs\right] = K_t I_a(s)$$

$$I_a(s) = \frac{(Js^2 + Bs)}{K_t}\theta(s) \quad \text{——— (9)}$$

Sub eq^n (8), (9) in eq^n (5)

$$\frac{(Js^2+Bs)}{K_t}\theta(s)(R_a + L_a s) + K_b s\theta(s) = V_a(s)$$

$$\theta(s)\left[\frac{(Js^2+Bs)(R_a+L_a s) + K_b K_t s}{K_t}\right] = V_a(s)$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(Js^2+Bs)(R_a+L_as) + K_bK_t s.}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{Bs\left[1+\frac{Js}{B}\right]R_a\left[1+\frac{L_as}{R_a}\right] + K_bK_t s}$$

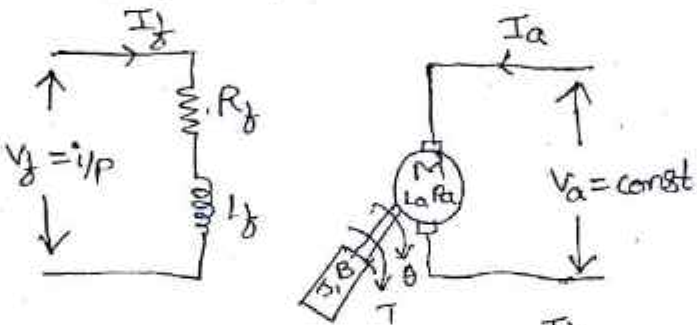
$$= \frac{K_t / BR_a}{s\left[(1+\tau_m s)(1+\tau_e s)+\frac{K_bK_t}{BR_a}\right]}$$

where

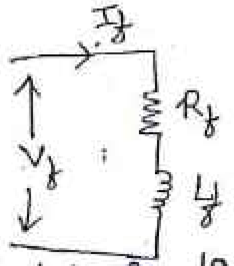
$\tau_m = \frac{J}{B}$ = mech const

$\tau_e = \frac{L_a}{R_a}$ = Elect const.

T.F of field controlled DC servo motors:-



$$I_fR_f + L_f\frac{dI_f}{dt} = V_f \quad \text{—— (1)}$$



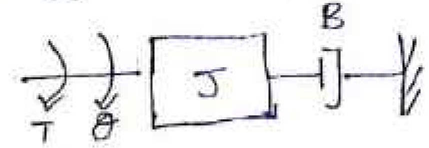
The torque developed in motor is directly proportional to flux & armature current.

$T \propto \phi$

$T \propto I_f$

$T = K_{ft} I_f$ —— (2)

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{—— (3)}$$



Apply laplace transformation to eq^n (1), (2), (3)

$R_f I_f(s) + L_f s I_f(s) = V_f(s)$ —— (4)

$T(s) = K_{ft} I_f(s)$ —— (5)

$Js^2\theta(s) + Bs\theta(s) = T(s)$ —— (6)

Sub in eq^n (5) in eq^n (6).

$\theta(s)\left[Js^2+Bs\right] = K_{ft} I_f(s)$.

$$I_f(s) = \frac{\left[Js^2+Bs\right]}{K_{ft}}\theta(s). \quad \text{—— (7)}$$

Sub eq^n (7) in eq^n (4).

$$R_f\theta(s)\frac{\left[Js^2+Bs\right]}{K_{ft}} + L_f s\frac{\left[Js^2+Bs\right]}{K_{ft}}\theta(s) = V_f(s)$$

$$\theta(s)\left[\frac{(Js^2+Bs)}{K_{ft}}R_f + \frac{(Js^2+Bs)}{K_{ft}}L_f s\right] = V_f(s)$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_f t}{(Js^2+Bs)(R_f+L_f s)}$$

$$= \frac{K_f t}{Bs\left(1+\frac{Js}{B}\right)R_f\left(1+\frac{L_f}{R_f}s\right)}$$

$$= \frac{K_f t / BR_f}{s\left(1+\frac{J}{B}s\right)\left(1+\frac{L_f}{R_f}s\right)}$$

$$= \frac{K_f t / BR_f}{s(1+\tau_m s)(1+\tau_e s)}$$ $K_f t / BR_f = k_m$

k_m = [illegible]

where

$\tau_m = \frac{J}{B}$ = mech time const

$\tau_e = \frac{L_f}{R_f}$ = (elec time const) or field time const

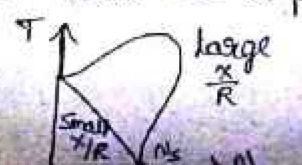
A.C Servo Motor:-

1. 2-~~ϕ IM is~~ rotor of the I.M is build with high resistance so that

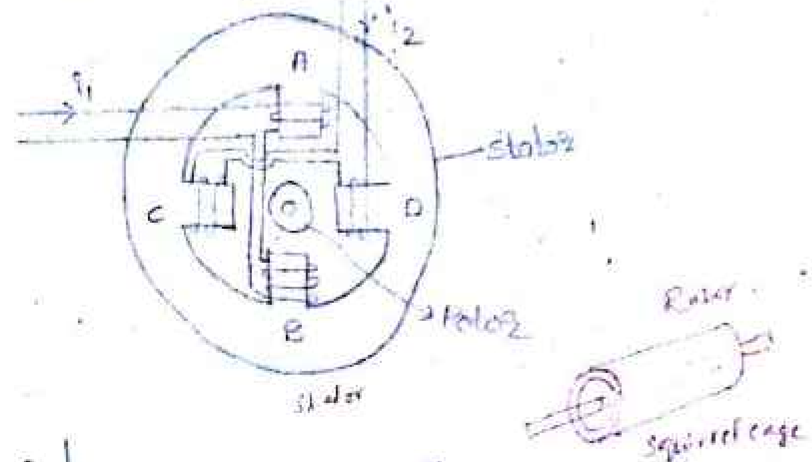
$\frac{X}{R}$ ratio $\rightarrow$ small $\rightarrow$ which results in linear speed - Torque relation

2. Excitation voltage applied to ~~the voltage~~ of
~~2 wdgs of rotor~~ must be a ~~phase of 90°~~

Two stator windings should have a phase diff of 90°.

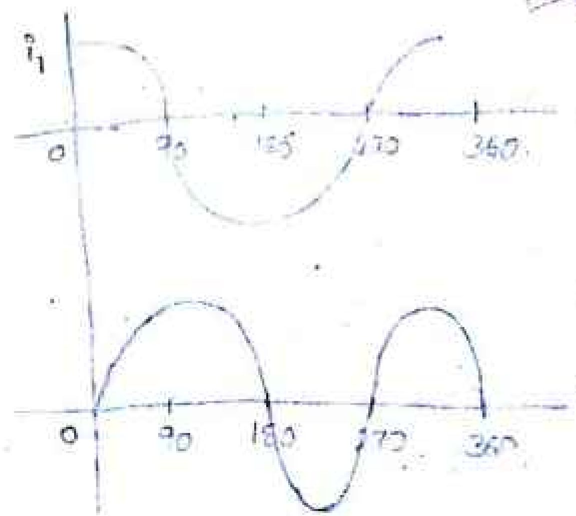


Construction:-



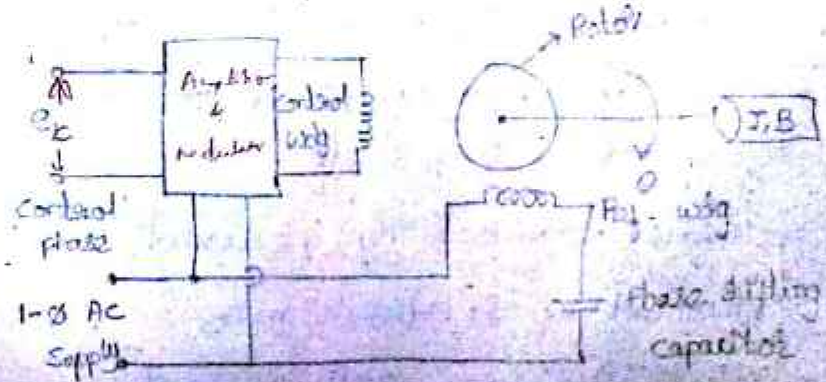
Rotor

squirrel cage (or) drag-cup [illegible]



Exciting Currents

[illegible]



Symbolic Rep of AC servomotor

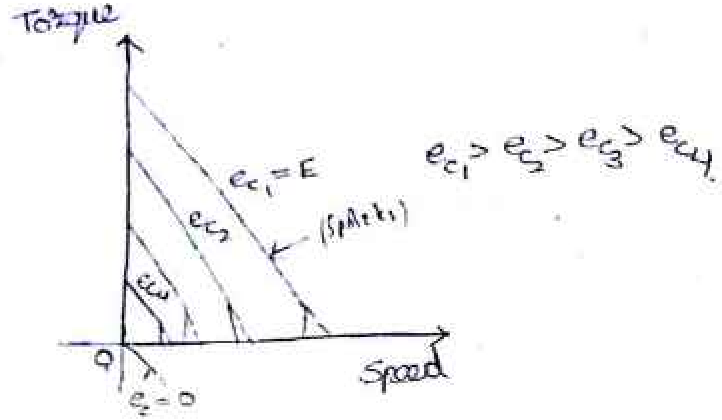


fig (a) Speed – torque curve

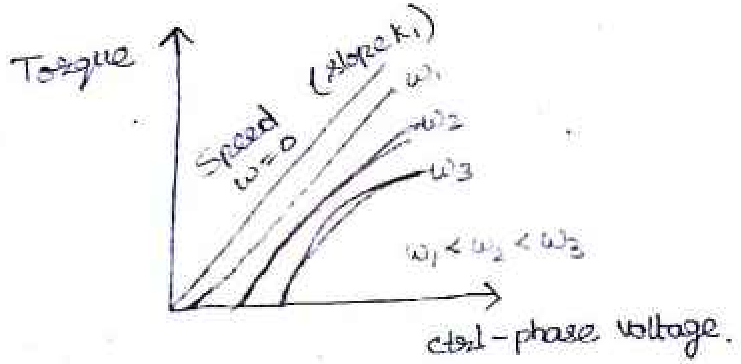


fig (b) Control Voltage vs Torque.

T.F of AC servomotor:-

let

T_m = Torque developed by servomotor

θ = angular displacement of rotor

ω = angular velocity $= \frac{d\theta}{dt}$

T_l = torque required by the load

J = moment of inertia of mass

B = Viscous frictional coeff of load

K_1 = slope of controlled phase voltage. vs Torque char^s

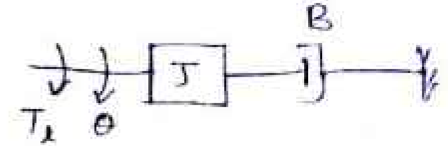
K_2 = slope of speed Torque char^s

with reference to fig (a) & (b) we can say that for speeds near zero, all the curves are st. lines parallel to the char^s at rated i/p voltage. ($e_c = E$) & are equally spaced for equal increments of the i/p voltage under this assumption.

The torque developed by the motor is represented by the eq^n given below. ~~Torque developed by motor is.~~

$$T_m = K_1 e_c - K_2 \frac{d\theta}{dt} \quad \text{——— (1)}$$

The rotating part of motor and the load can be modeled by the eq^n



$$T_l = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad \text{——— (2)}$$

Under equilibrium cond^n, $T_m = T_l$

Now,

$$K_1 e_c - K_2 \frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

$$J\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt}(B+K_2) - K_1 e_c = 0$$

Apply Laplace.

$$Js^2\theta(s) + s\theta(s)(B+K_2) = K_1 E_c(s)$$

$$\theta(s)\left[Js^2 + (B+K_2)s\right] = K_1 E_c(s)$$

$$\frac{\theta(s)}{E_c(s)} = \frac{K_1}{Js^2+(B+K_2)s}$$

$$= \frac{K_1}{(B+K_2)s\left[1+\left(\frac{J}{B+K_2}\right)s\right]}$$

$$= \frac{K_1/B+K_2}{s\left[1+\frac{Js}{B+K_2}\right]}$$

$$= \frac{K_1/B+K_2}{s\left[1+\tau_m s\right]}$$

$$= \frac{K_m}{s(1+\tau_m s)}.$$

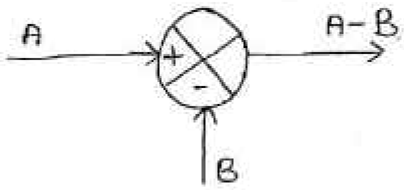
Where

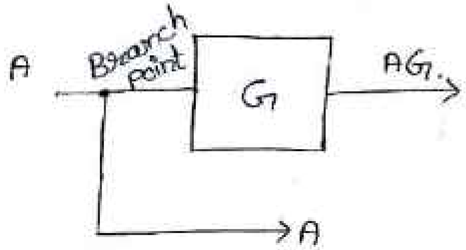
$K_m = \frac{K_1}{B+K_2}$ = motor gain constant

$\tau_m = \frac{J}{B+K_2}$ = motor time constant

Block diagram:-

1. Block
2. Summing point
3. Branch point





1. Construct block diagram of field control DC servomotor.

Sol:- differential eq^n of field controlled DC servomotor.

$$i_f R_f + L_f \frac{di_f}{dt} = V_f \quad \text{——— (1)}$$

$$T = K_{ft}\, i_f \quad \text{---} \; (2)$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{---} \; (3)$$

Apply L.P

$$I_f(s)R_f + L_f s I_f(s) = V_f(s) \quad \text{---} \; (4)$$

$$T(s) = K_{ft}\, I_f(s) \quad \text{---} \; (5)$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \quad \text{---} \; (6)$$

eq^n (4)

$$I_f(s)(R_f + L_f s) = V_f(s)$$

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s}$$

$$V_f(s) \longrightarrow \boxed{\frac{1}{R_f + L_f s}} \longrightarrow I_f(s)$$

eq^n (5)

$$T(s) = K_{ft}\, I_f(s).$$

$$I_f(s) \longrightarrow \boxed{K_{ft}} \longrightarrow T(s)$$

eq^n (6)

$$\theta(s)\left[Js^2 + Bs\right] = T(s)$$

$$T(s) = \left[Js^2 + Bs\right]\theta(s)$$

$$T(s) \longrightarrow \boxed{\frac{1}{Js^2 + Bs}} \longrightarrow \theta(s)$$

$$V_f(s) \longrightarrow \boxed{\frac{1}{R_f + L_f s}} \xrightarrow{I_f(s)} \boxed{K_{ft}} \xrightarrow{T(s)} \boxed{\frac{1}{Js^2 + Bs}} \longrightarrow \theta(s)$$

∴ Block diagram of field control of DC servomotor

2. Construct block diagram of armature controlled DC servomotor.

Sol:- write differential eq^n are

$$R_a I_a + L_a\frac{dI_a}{dt} + e_b = V_a \quad \text{---} \; (1)$$

$$T = K_t I_a \quad \text{---} \; (2)$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{---} \; (3)$$

$$e_b = K_b\frac{d\theta}{dt} \quad \text{---} \; (4)$$

$$\omega = \frac{d\theta}{dt} \quad \text{---} \; (5)$$

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Now,

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad \text{—— (1)}$$

$$T = K_t i_a \quad \text{—— (2)}$$

$$J\frac{d\omega}{dt} + B\omega = T \quad \text{—— (3)}$$

$$e_b = K_b \omega \quad \text{—— (4)}$$

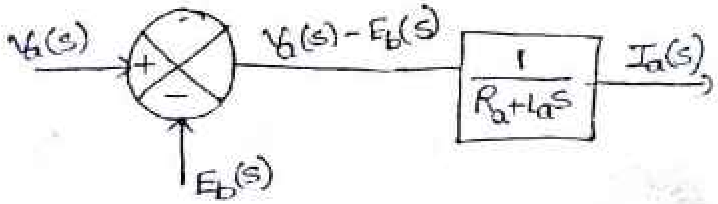
Apply L.P

eqⁿ (1).

$$I_a(s)R_a + L_a s I_a(s) + E_b(s) = V_a(s)$$

$$V_a(s) - E_b(s) = I_a(s)\left[R_a + L_a s\right]$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + L_a s}$$



eqⁿ (2),

$$T = K_t i_a$$

$$T(s) = K_t I_a(s)$$

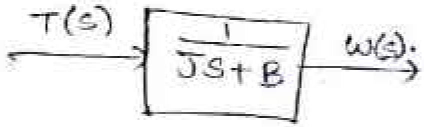
$$\cancel{I_a(s) = \frac{T(s)}{K_t}}$$

eqⁿ (3)

$$J\frac{d\omega}{dt} + B\omega = T$$

$$Js\,\omega(s) + B\,\omega(s) = T(s)$$

$$T(s) = \omega(s)\left[Js + B\right]$$



eqⁿ (4)

$$e_b = K_b \omega$$

$$e_b(s) = K_b \omega(s)$$

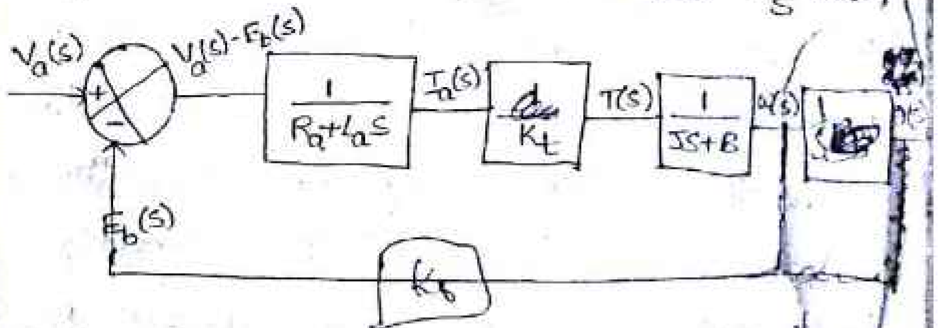
$$\cancel{\omega(s) =} \frac{e_b(s)}{K_b}$$

eqⁿ (5)

$$\omega = \frac{d\theta}{dt}$$

$$\omega(s) = s\,\theta(s)$$

$$\theta(s) = \frac{1}{s}\,\omega(s)$$



Synchro receiver:-

In synchro transmitter the terminal to terminal voltage is governed by angular position of the rotor. In the receiver it is in reverse i.e, voltage to position transducer. The construction is the same except that a heavy fly wheel is attached to the shaft of rotor to reduce oscillations. A single phase voltage is applied to each of stator wdgs and A.C excitation to the rotor wdgs and mutual repulsion gives the angle of rotor shaft.

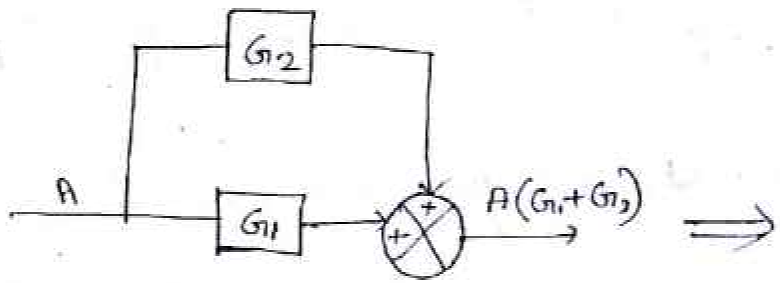
Initially the magnetic fields set by the stator and rotor wdgs cause the rotor to be aligned in the line with the stator wdg S_2. when the magnitude of stator voltages changes the stator current also changes. The resultant stator field changes its direction causing the rotor field to follow it. As the rotor field follows the resultant stator field the rotor shaft starts to turn. Hence the rotor motion is damped by the fly wheel.

Block diagram reduction:-

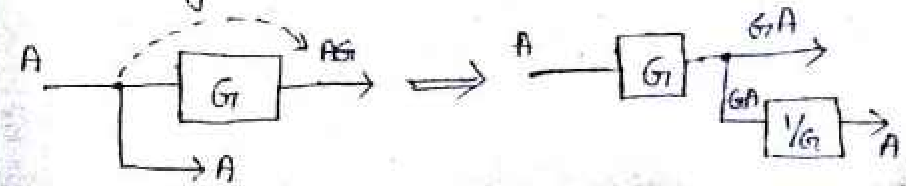
Rules of Block diagram algebra:-

⇒ Blocks are in cascade

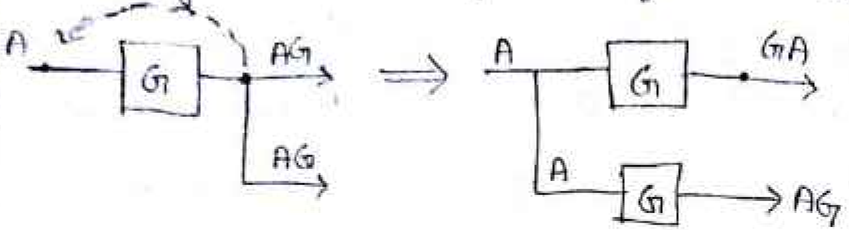
⇒ Combining parallel blocks



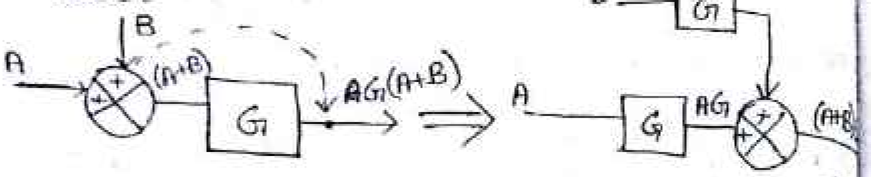
⇒ Moving the branch point ahead



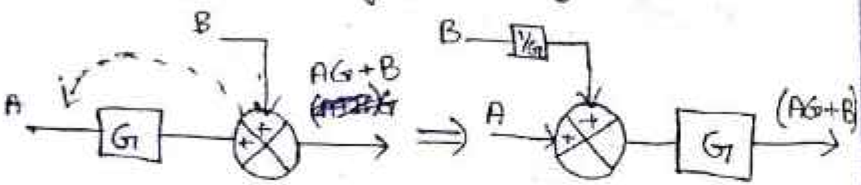
⇒ Moving the branch point before the block



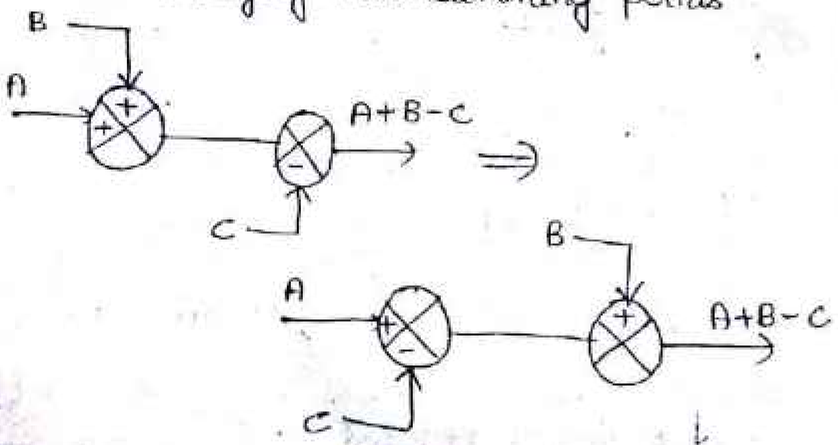
⇒ Moving summing point ahead of the block



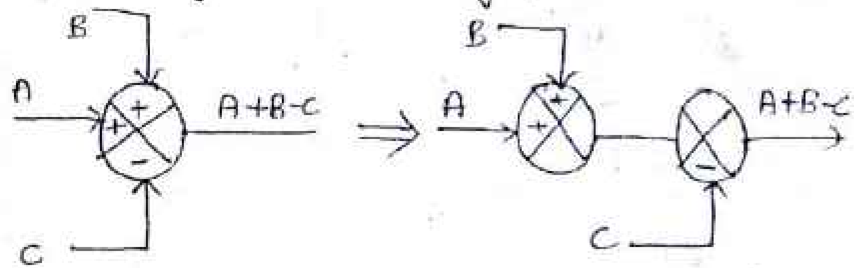
⇒ Moving summing point before the block



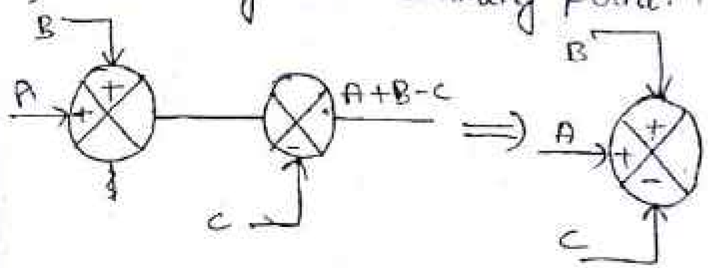
⇒ Interchanging the summing points



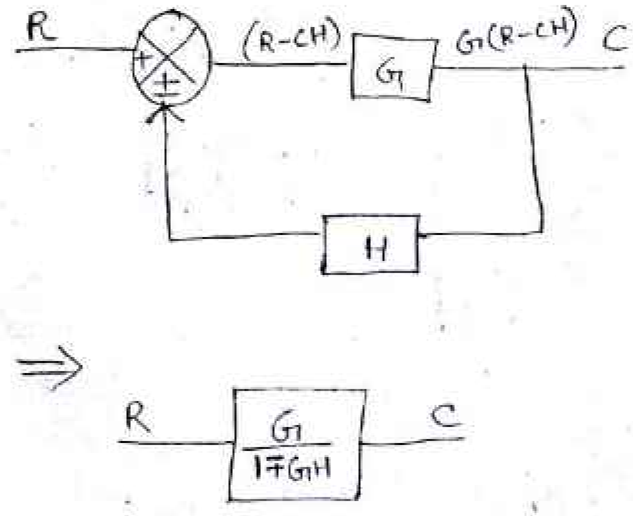
⇒ Splitting the summing point



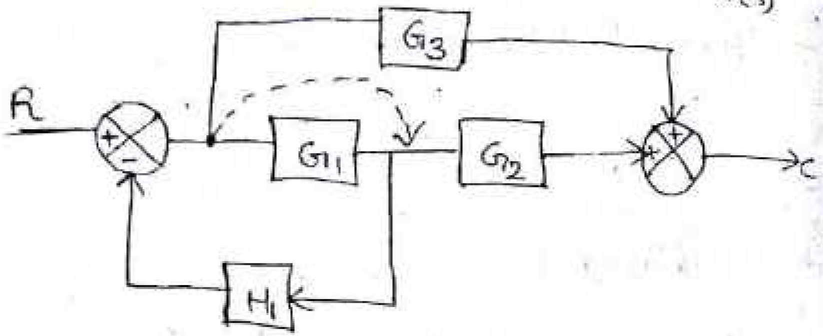
⇒ Combining the summing point.



⇒ Eliminating the feedback ~~scale~~ loop

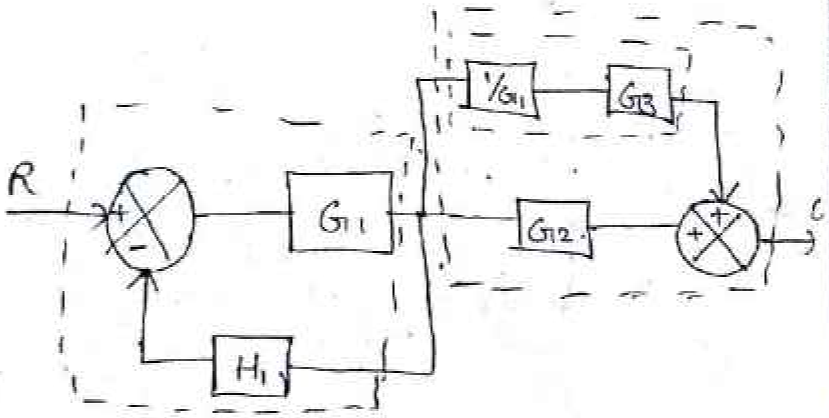


1. Reduce the block diagram shown in fig using reduction rules & find. $\frac{C(s)}{R(s)}$

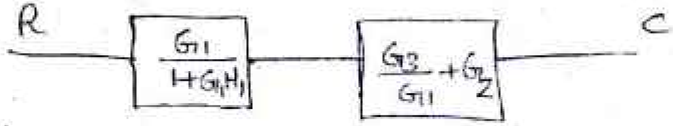


Sol:- Given
Block diagram is

Step -1
Moving branch point ahead of block G_1



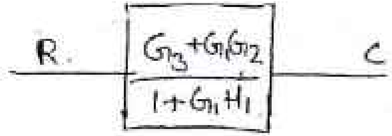
Step 2:-
Eliminating the closed loop and combining the parallel parts.



Step ③. combining cascade blocks.

$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H_1}\right)\left(\frac{G_3}{G_1}+G_2\right)$$

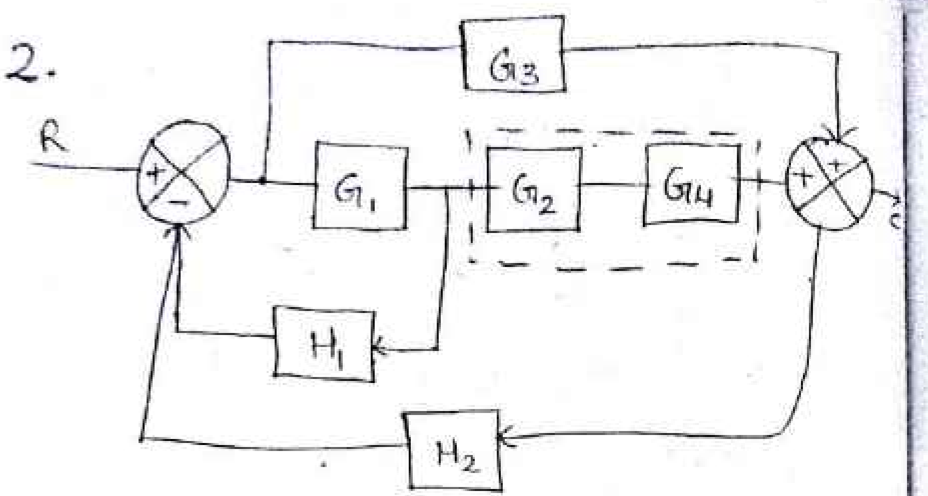
$$= \left(\frac{G_1}{1+G_1H_1}\right)\left(\frac{G_3+G_1G_2}{G_1}\right) = \frac{G_3+G_1G_2}{1+G_1H_1}$$



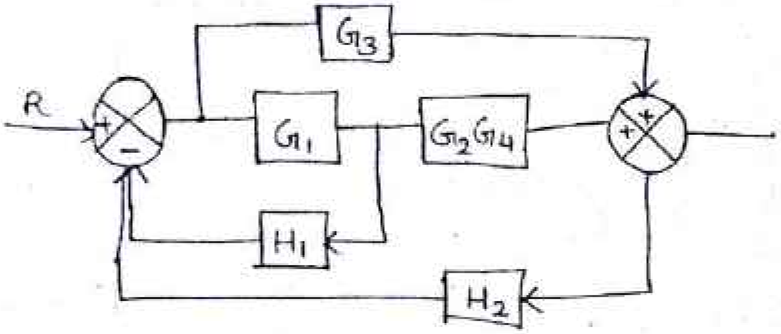
Procedure for reduction of block diagram.

⇒ Reduce the cascaded blocks
⇒ Reduce the parallel blocks
⇒ Reduce the internal feedback loops
⇒ It is advisable to shift branch points towards right & summing points towards left.
⇒ Repeat step1 to Step4 until the simple form is obtained
⇒ Find T.F of the overall system using the form $C(s)/R(s)$.

2.



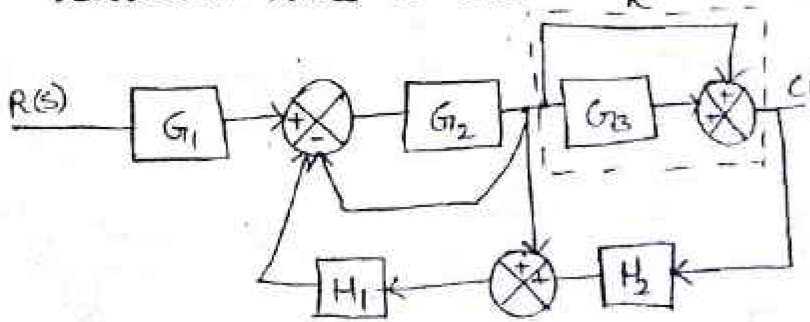
Sol:- Step-1 cascade the blocks.



Step 2:-

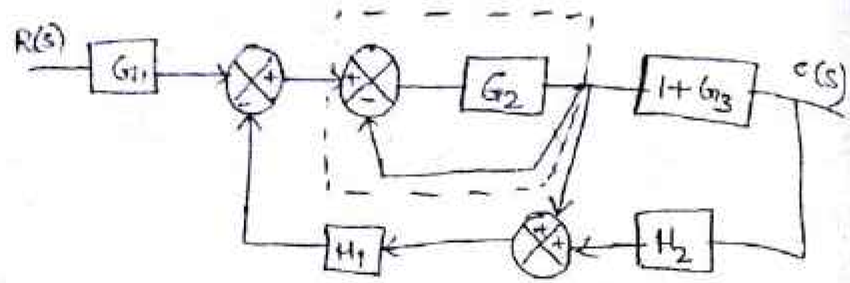
$$\frac{C}{R} = \frac{G_1G_2G_4 + G_3}{1 + G_1H_1 + G_1G_2G_4H_2 + G_3H_2}$$

3. Reduce the block diagram by using reduction rules & obtain $\frac{C}{R}$

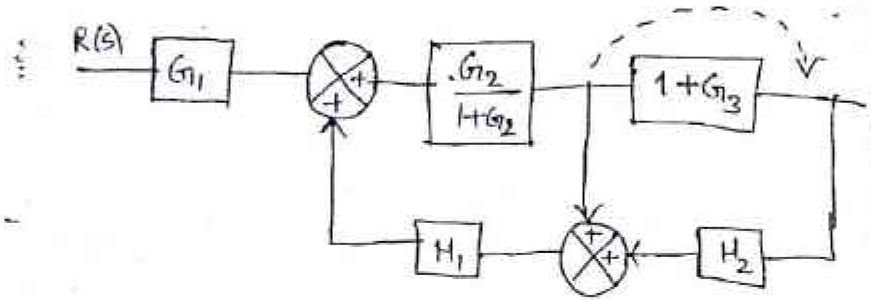


Sol:- Step 1:-

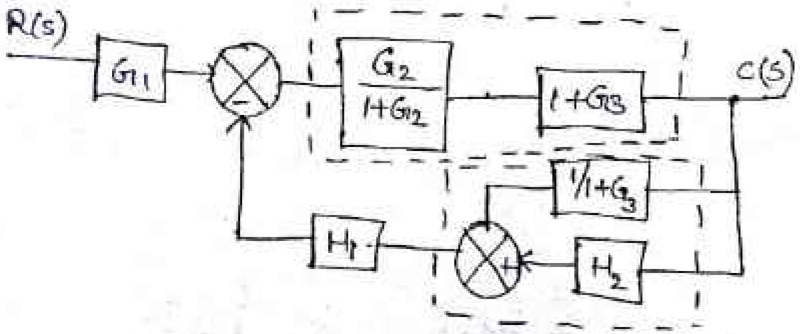
Combining parallel paths & split summing point.



Step 2:- Eliminate feedback loop.

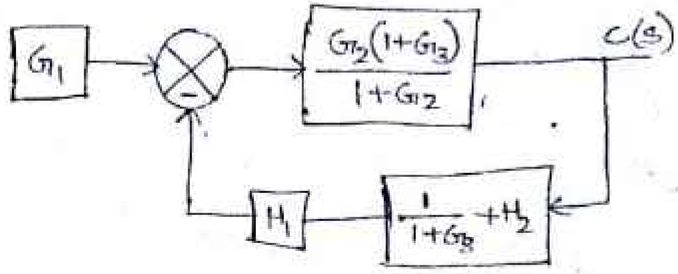


Step 3:- Move branch point ahead of block $(1+G_3)$



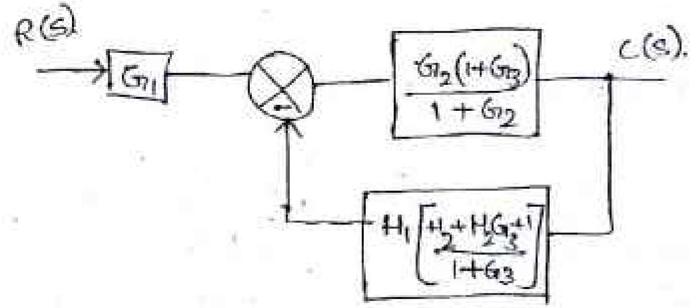
Step 4:-

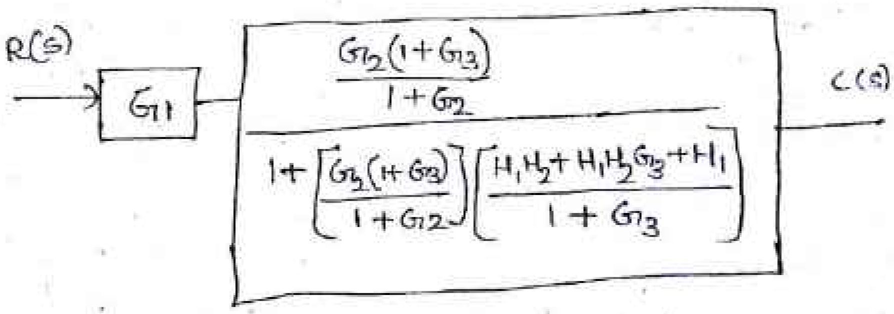
Combine cascades blocks & parallel blocks.

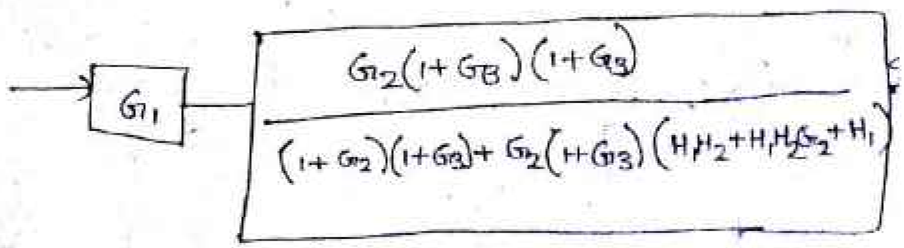


Step 5:-

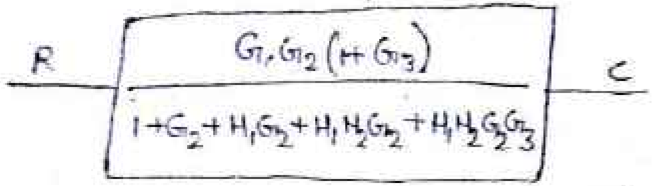
combine blocks in cascade $\frac{H_2+H_2G_3+1}{1+G_3}$





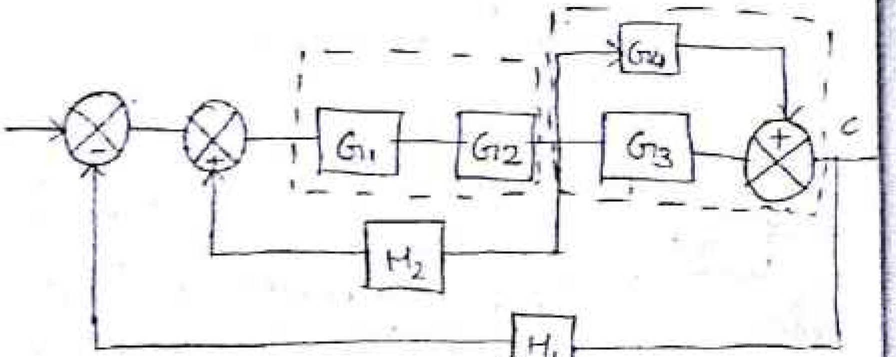


Step 6 :-
cascade blocks

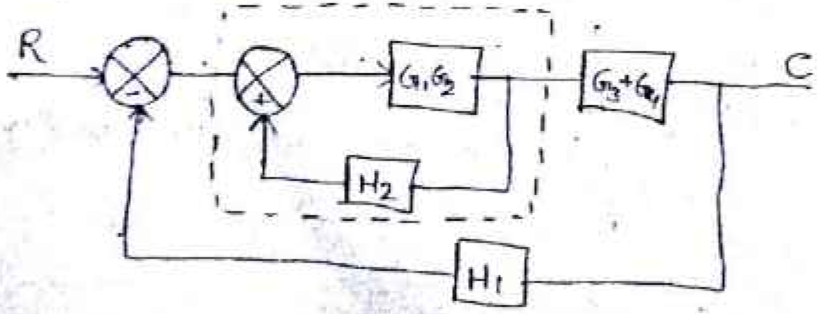


$$\frac{C}{R} = \frac{G_1G_2 + G_1G_2G_3}{1+G_2+H_1G_2+H_1H_2G_2+H_1H_2G_2G_3}$$

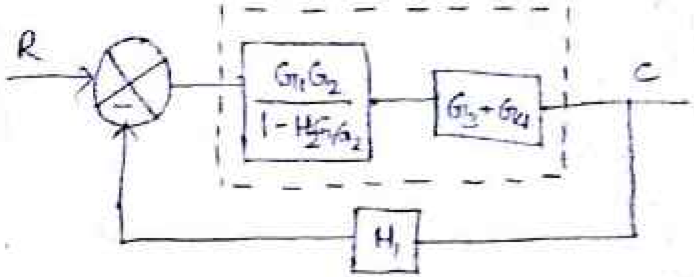
4. Reduce the block diagram using reduction rule



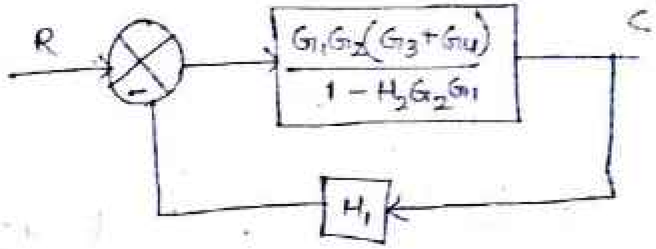
Step 1 :- cascade & parallel.



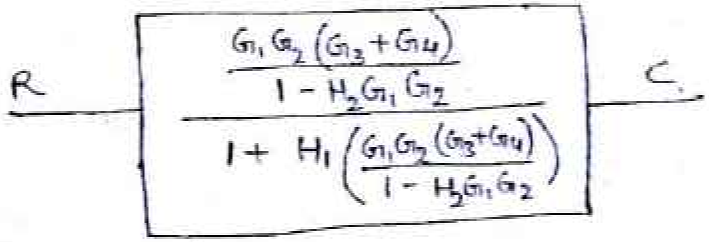
Step 2 :- eliminate feedback loop.

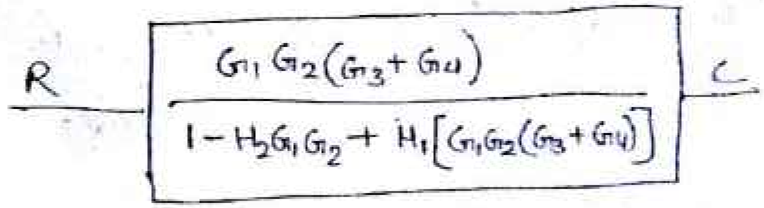


Step 3 :- Cascade.



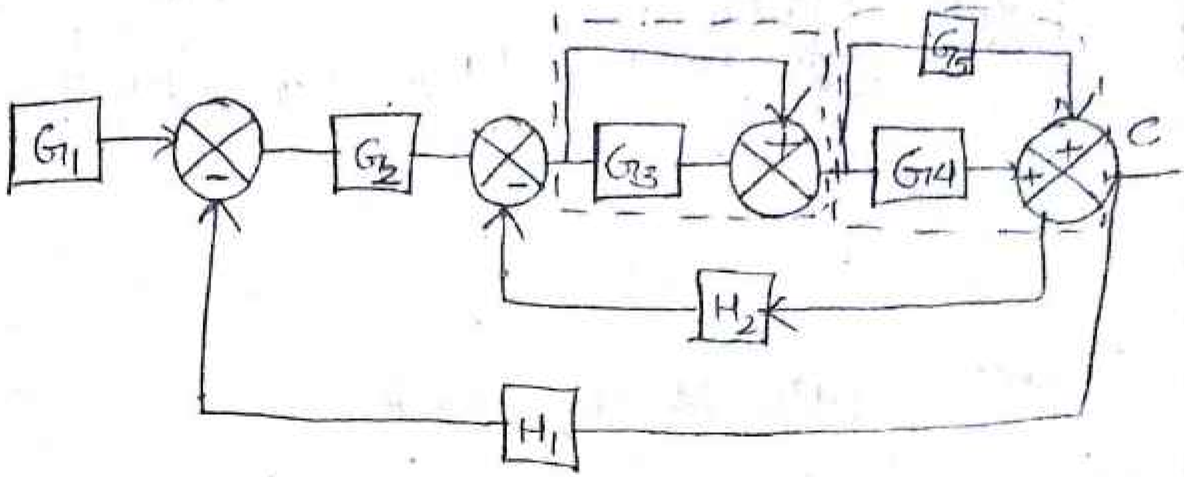
Step 4 :- eliminate feedback loop.



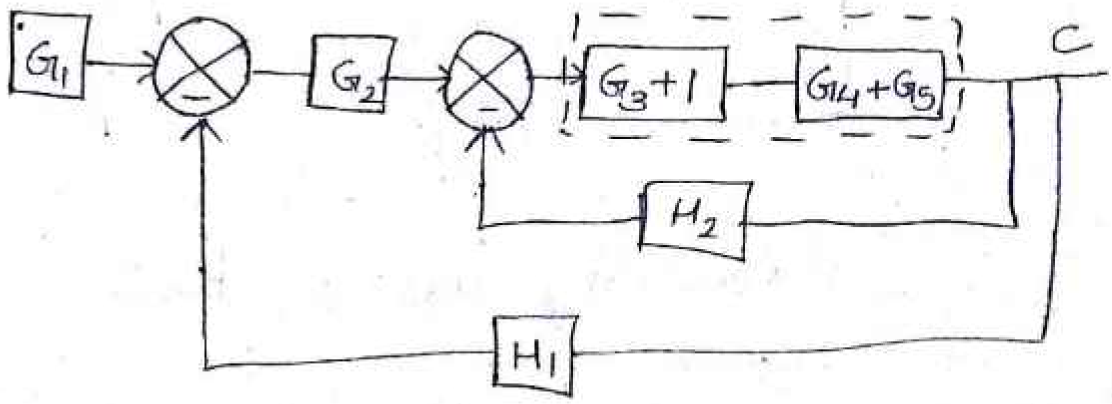


$$\frac{C}{R} = \frac{G_1G_2(G_3+G_4)}{1-H_2G_1G_2+H_1\left[G_1G_2(G_3+G_4)\right]}$$

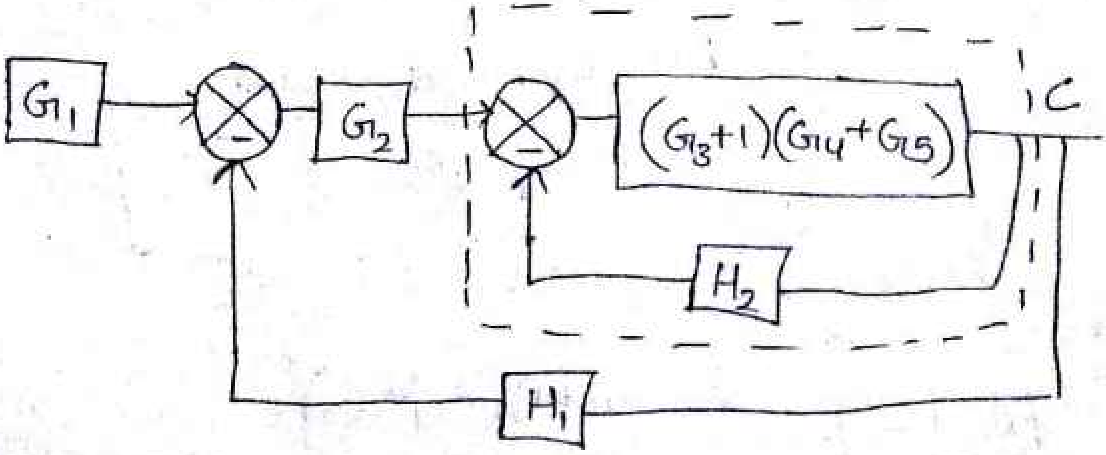
1. consider the block diagram shown in fig. reduce by block diagram reduction method



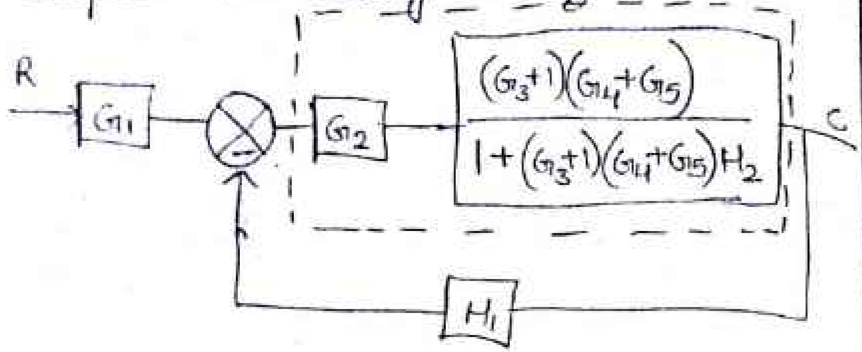
Step 1 :- parallel the blocks



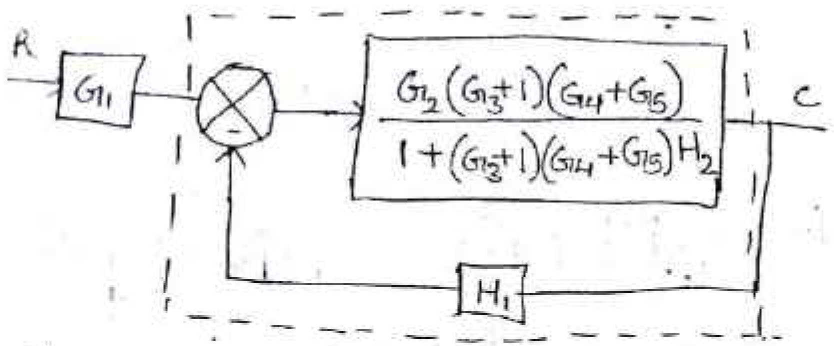
Step 2 :- cascade the blocks



Step 3:- Eliminating the feedback.



Step 4:- Cascade the blocks.



Step 5:- Eliminating the feedback

$$R \rightarrow \boxed{G_1} \rightarrow \boxed{\dfrac{\dfrac{G_2(G_3+1)(G_4+G_5)}{1+(G_3+1)(G_4+G_5)H_2}}{1+\dfrac{G_2(G_3+1)(G_4+G_5)H_1}{1+(G_3+1)(G_4+G_5)H_2}}} \rightarrow C$$

~~St~~

$$R \rightarrow \boxed{G_1} \rightarrow \boxed{\dfrac{G_2(G_3+1)(G_4+G_5)}{1+(G_3+1)(G_4+G_5)H_2+G_2(G_3+1)(G_4+G_5)H_1}}$$

Step 6:- cascade the blocks

$$R \rightarrow \boxed{\dfrac{G_1G_2(G_3+1)(G_4+G_5)}{1+(G_3+1)(G_4+G_5)H_2+G_2(G_3+1)(G_4+G_5)H_1}}$$

Terms used in signal flowgraph:-

1. Node:- A node is a point representing a variable.
2. Branch:- A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of the branch is transmittance.
3. transmittance:- The gain acquired by the signal when it travels from one node to another is called transmittance.
4. i/p node or source:- It is a node that has only outgoing branches.
5. o/p node or sink:- It is a node that has only incoming branches
6. Mixed node:- It is a node that has both incoming & outgoing branches

7. path :- A path is traversal of connected branches in the direction of the branch arrows. path should not cross a node more than once.

8. open path :- A open path starts at a node & ends at another node.

9. closed path :- A closed path starts & ends at the same node.

10. Forward path :- It is a path from i/p node to an o/p node that does not cross any node more than once.

11. Forward path gain :- It is the product of branch transmittances (gains) of a forward path.

12. Individual loop :- It is a closed path starting from a node and after passing through a certain part of a graph arrives at the same node without acrossing any node more than once.

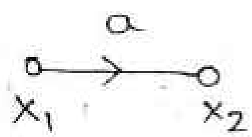
13. Loop gain :- It is the product of branch transmittance of a loop.

14. Non-touching loops :- If the loops doesnot have a common node then they are said to be non-touching loops.

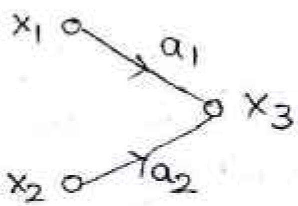
Signal Flow graph algebra :-

Rule 1 :-

Incoming signals to a node through a branch is given by the product of the signal at previous branch & the gain of the branch.



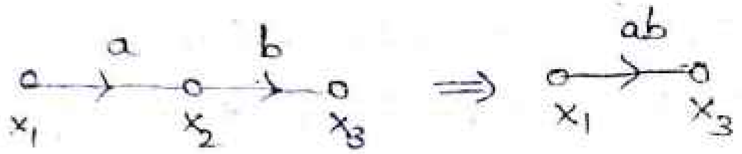
$x_2 = ax_1$



$x_3 = x_1a_1 + x_2a_2$.

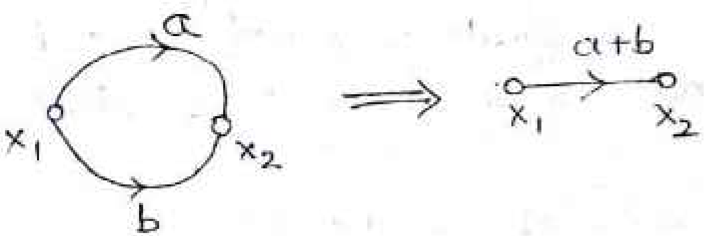
Rule 2 :-

cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual transmittances.



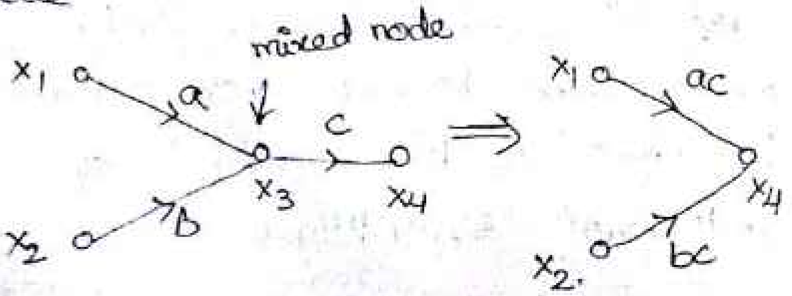
Rule 3:-

Parallel branches may be represented by single branch whose transmittance is sum of individual branch transmittances.



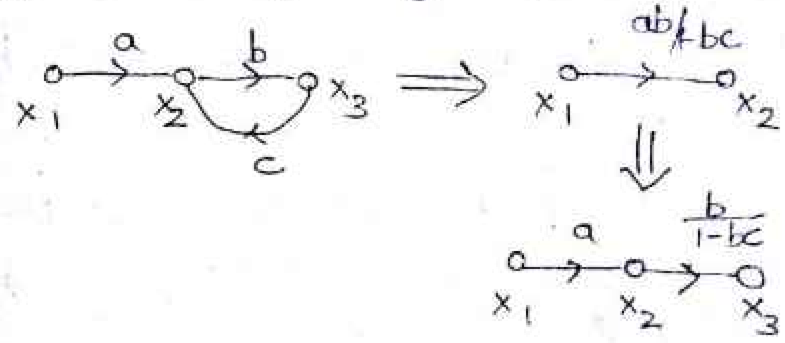
Rule 4:-

A mixed node can be eliminated by multiplying the transmittance of outgoing branch to the transmittance of all incoming branches to the mixed node



Rule 5:-

A loop may be eliminated by writing eqn's at the i/p & o/p node & rearranging the eqn's to find the ratio of o/p to i/p that ratio gives the gain of resultant branch



Mason's Gain formula:-

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

Let $R(s)$ = i/p of the system

$C(s)$ = o/p of the system

then the T.F of the system is

$$T(s) = \frac{C(s)}{R(s)}$$

The overall gain given by Mason is

$$\boxed{T = \frac{1}{\Delta} \sum_K P_K \Delta_K}$$

where

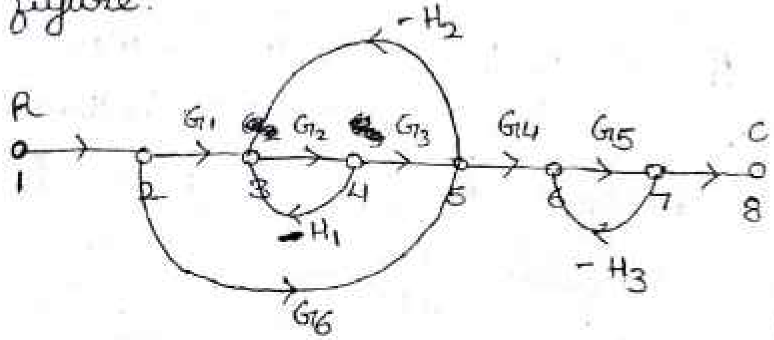
$T = T(s)$ = transfer function of the system

P_K = forward path gain of K'th forward path. k = no. of Frwd paths in signal flow graph

$\Delta = 1 -$ (sum of individual loop gains) + (sum of gain products of all possible combination of two non touching loops) − (sum of gain products of all possible combination of three non touching loops)

$\Delta_K = \Delta$ for the part of the graph which is not touching K^{th} forward path.

1. Find the overall T.F of the system whose signal flow graph is shown in figure.



Sol:- Given signal flowgraph

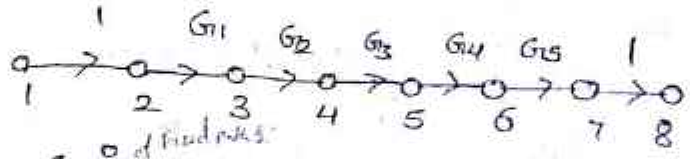
$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

Step 1:-

Forward path gains:-

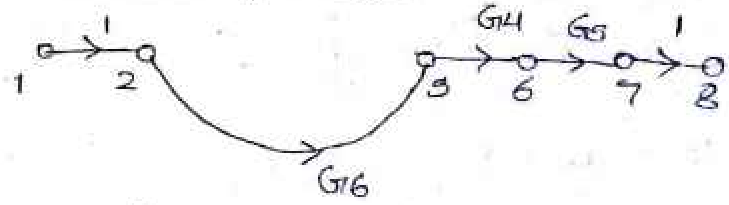
No. of forward paths $K=2$

Forward path 1:-



Gain of Frwd paths $P_1 = G_1G_2G_3G_4G_5$

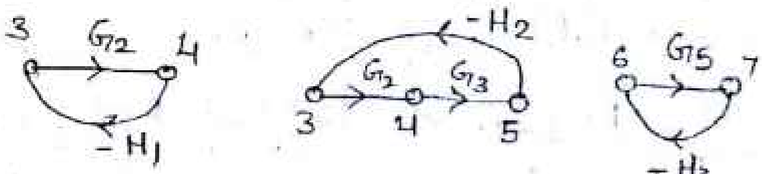
Forward path 2:-



Gain $P_2 = G_4G_5G_6$

Step 2:-

Individual loop Gains



loop ① loop ② loop ③

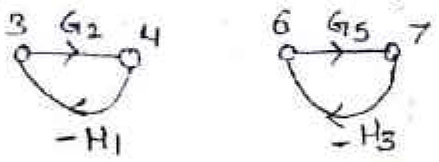
loop gain $P_{11} = -G_2H_1$

$P_{21} = -G_2G_3H_2$

$P_{31} = -G_5H_3$

Gain products of Two Non-touching loops

Step 3:- Non-Touching loop gains:



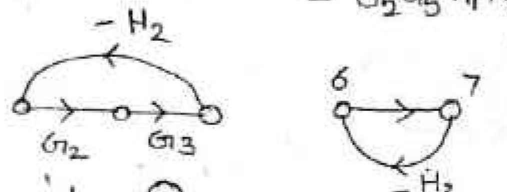
loop ① loop ③

Gain products of two combination of two non touching loops

loop gain $P_{12} = P_{11} \cdot P_{31}$.

$= (-G_2H_1)(-G_5H_3)$

$= G_2G_5H_1H_3$.



loop ② loop ③

loop gain $P_{22} = P_{21} \cdot P_{31}$

$= (-G_2G_3H_2)(-G_5H_3)$

$= G_2G_3G_5H_2H_3$

Step 4

Step 4:-

calculation of Δ & Δ_k

$\Delta = 1 -$ (sum of individual loop gains) + (sum of gain products of all possible combination of two non touching loops).

$= 1 - [P_{11} + P_{21} + P_{31}] + [P_{12} + P_{22}]$

$\Delta = 1 - [-G_2H_1 - G_2G_3H_2 - G_5H_3] + [G_2G_5H_1H_3 + G_2G_3G_5H_2H_3]$

$\Delta = 1 + [G_2H_1 + G_2G_3H_2 + G_5H_3] + [G_2G_5H_1H_3 + G_2G_3G_5H_2H_3]$

Step 4:- calculation of Δ_K

$\Delta_K = 1 - [$sum of individual non-touching loops gain$]$

$\Delta_1 = 1 - 0 = 1$ [ie, which does not touch forward path]

Since the loop of graph is not touching with forward path

$\Delta_2 = 1 - [-G_2H_1]$

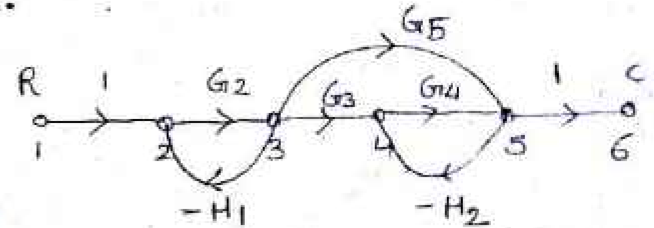
$= 1 + G_2H_1$

Step 5: Transfer function

$T = \frac{1}{\Delta} \sum (P_1\Delta_1 + P_2\Delta_2)$

$T = \dfrac{G_4G_1G_2G_3 + G_4G_5G_6(1+G_2H_1)}{1 + [G_2H_1 + G_2G_3H_2 + G_5H_3] + [G_2G_5H_1H_3 + G_2G_3G_5H_2H_3]}$

2.



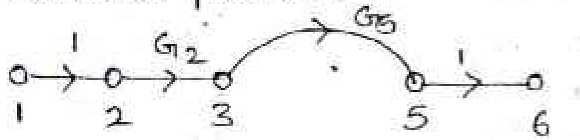
Sol:- Step 1:- Forward path gains

No. of forward paths $K = 2$

Forward path 1:-

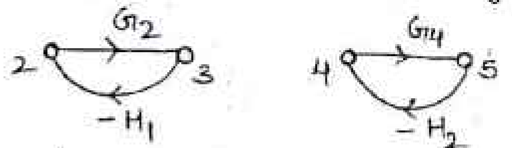
Gain $P_1 = G_2 G_3 G_4$

Forward path 2:-



Gain $P_2 = G_2 G_5$

Step 2: Individual loop gain

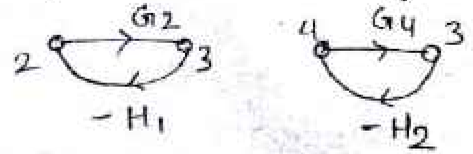


loop ① loop ②

loop gain $P_{11} = -G_2 H_1$

$P_{21} = -G_4 H_2$

Step 3: Non-touching loop gains



loop gain $P_{12} = G_2 G_4 H_1 H_2$

$\Delta = 1 - [-G_2H_1 - G_4H_2] + [G_1 G_4 H_1 H_2]$

Step 4: calculation of Δ_K

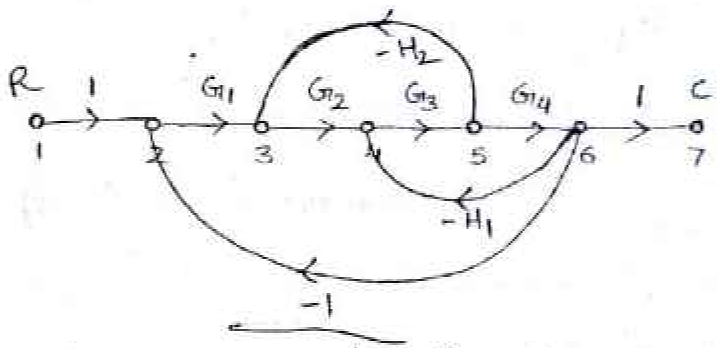
$\Delta_1 = 1$

$\Delta_2 = 1$

$T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$

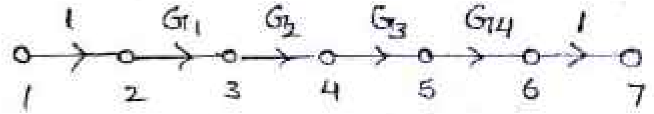
$= \dfrac{G_2G_3G_4 + G_2G_5}{1 + G_2H_1 + G_4H_2 + G_1G_4H_1H_2}$

3.



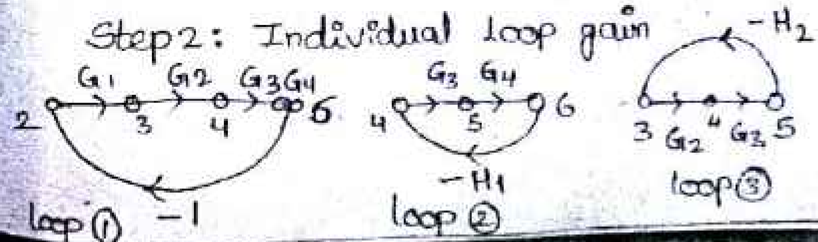
Sol:- Step 1: Forward path gains

No of forward path $K = 1$



Gain $P_1 = G_1 G_2 G_3 G_4$

Step 2: Individual loop gain



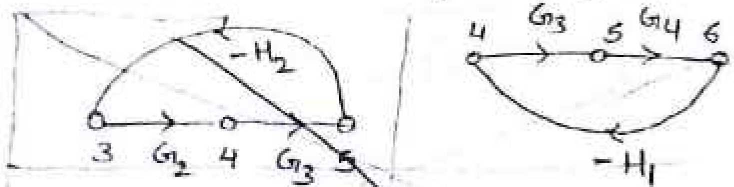
loop ① loop ② loop ③

loop gain $P_{11} = -G_1G_2G_3G_4$

$P_{21} = -G_3G_4H_1$

$P_{31} = -G_2G_3H_2$

Step 3: Non-touching loop gains



loop ③ loop ②

~~loop gain $P_{32} = G_2G_3H_2H_1$~~

There are no non-touching loops.

$$\Delta = 1 + [G_1G_2G_3G_4 + G_3G_4H_1 + G_2G_3H_2]$$

Step 4: Calculation of Δ_K

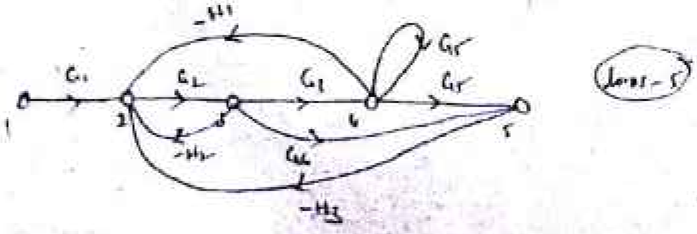
$\Delta_1 = 1$

~~$\Delta_2 = 1$~~

$$T = \frac{1}{\Delta} P_1 \Delta_1$$

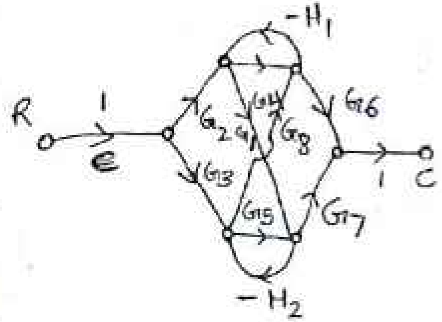
$$= \frac{G_1G_2G_3G_4}{1 + [G_1G_2G_3G_4 + G_3G_4H_1 + G_2G_3H_2]}$$

(5):



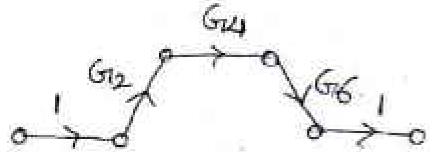
Loops - 5

4. Find the overall gain of system whose signal flow graph.

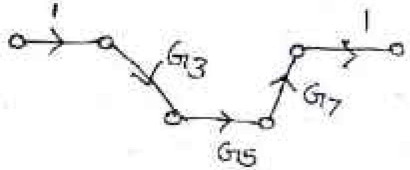


Sol:- Step I:-

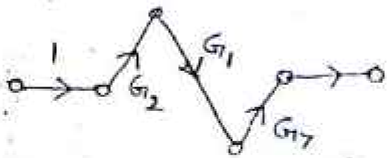
K = 6



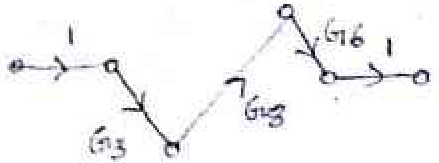
Forward path gain $P_1 = G_2G_4G_6$.



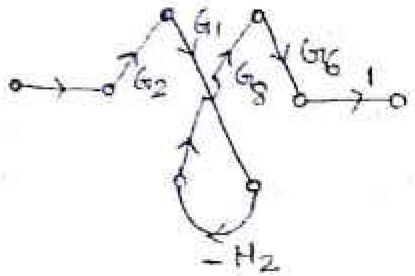
$P_2 = G_3G_5G_7$



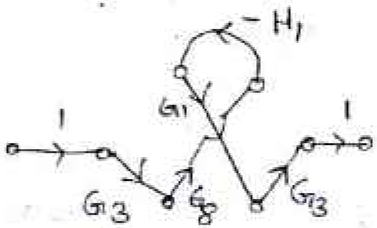
$P_3 = G_1G_2G_7$



$P_4 = G_3 G_6 G_8.$

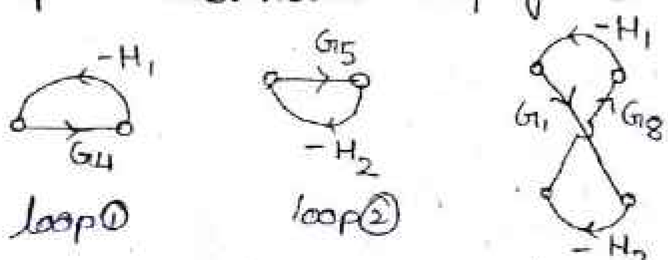


$P_5 = -G_1 G_2 H_2 G_6 G_8.$



$P_6 = -H_1 G_1 G_3 G_8 G_7$

Step 2:- Individual loop gains.

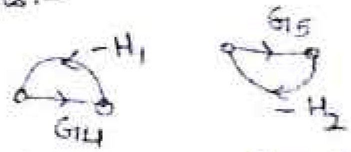


$P_{11} = -G_4 H_1$

$P_{21} = -G_5 H_2$

$P_{31} = G_1 G_8 H_1 H_2$

Step 3:- Gain product of two non touching loops:-



$P_{12} = G_4 G_5 H_1 H_2.$

$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12}).$

$= 1 - (-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2) + G_4 G_5 H_1 H_2$

$= 1 + (G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2) + G_4 G_5 H_1 H_2$

Step 4:- calculation of Δ_K

$\Delta_1 = 1 - [-G_5 H_2] = 1 + G_5 H_2$

$\Delta_2 = 1 - [-G_4 H_1] = 1 + G_4 H_1$

$\Delta_3 = 1 - (0) = 1$

$\Delta_4 = \Delta_5 = \Delta_6 = 1$

$T = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6\right]$

$$T = \frac{(G_2 G_4 G_6)(1 + G_5 H_2) + (G_3 G_5 G_7)(1 + G_4 H_1) + (G_1 G_2 G_7) + (G_3 G_6 G_8) + (-G_1 G_2 G_6 G_8 H_2) + (-H_1 G_1 G_3 G_7 G_8)}{1 + (G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2) + G_4 G_5 H_1 H_2}$$

Procedure for constructing signal flow graph from block diagrams:-

⇒ Assume nodes at i/p, o/p, at every summing points, at every branch points and in b/w cascaded blocks.

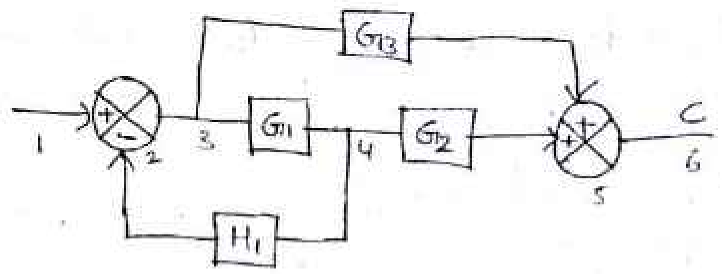
⇒ Draw the nodes seperately as small circles and number the circles in the order 1, 2, 3 ---- etc.

⇒ From the block diagram find the gain b/w each node in the forward path & connect all the corresponding circles by a straight line & mark the gain b/w the nodes. With the direction.

⇒ Draw the feed forward paths b/w various nodes and mark the gain of feed forward paths along with sign.

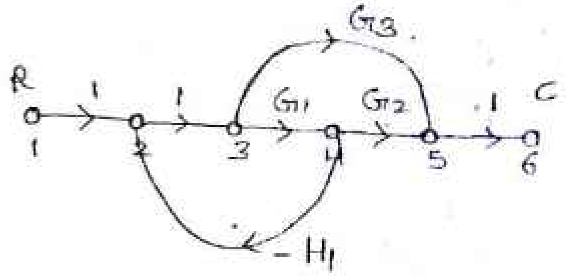
⇒ the feedback paths b/w various nodes and mark the gain of the feedback paths along with sign.

1.



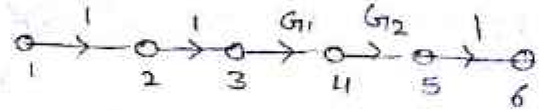
Find the T.F of the given block diagram by using signal flow graph.

Sol:-

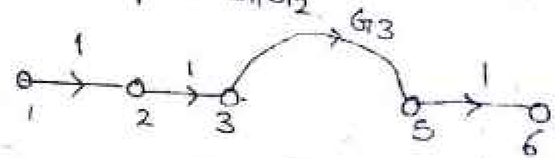


Step 1:- Forward path gain

K = 2

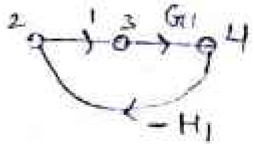


$P_1 = G_1G_2$



$P_2 = G_3$.

Step 2:- Individual loop gains.



$P_{11} = -G_1H_1$

III. Gain product of two non touching loops:-

There are no two non touching loops.

Step4:- calculation of Δ

$$\Delta = 1 - (P_{11}) \cancel{+}$$

$$= 1 + G_1 H_1$$

Step5:- calculation of Δ_K.

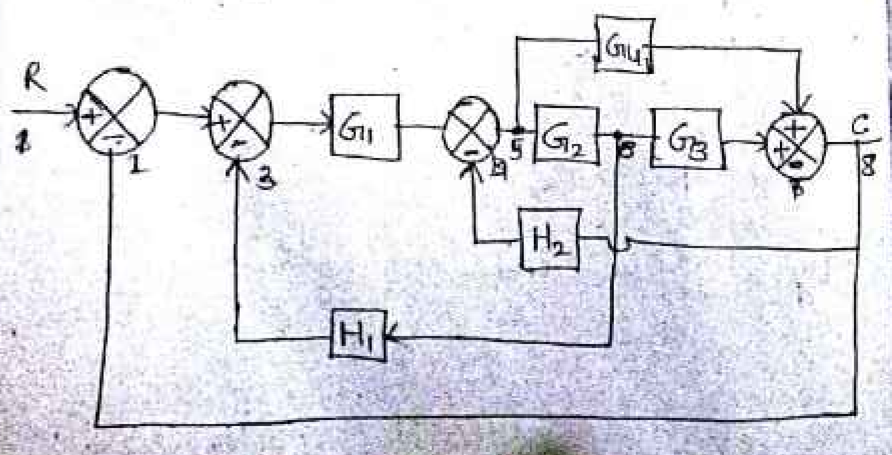
$\Delta_1 = 1$

$\Delta_2 = 1$

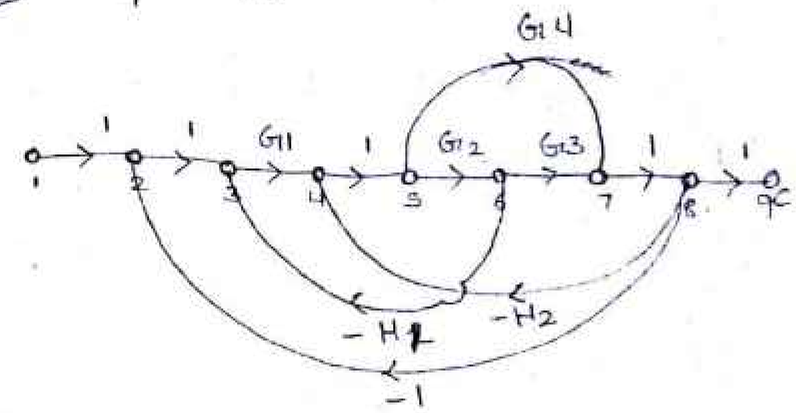
$$T = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right]$$

$$= \frac{G_1G_2 + G_3}{1 + G_1H_1}$$

2. convert the block diagram to signal flow graph & determine the T.F using Mason's gain formula

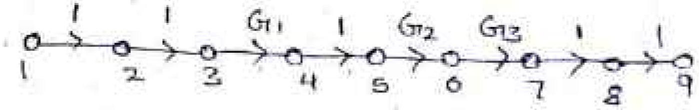


Sol:- Step: SFG

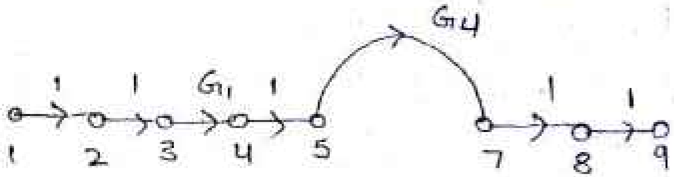


Step:-1 forward path gain.

K = 2.

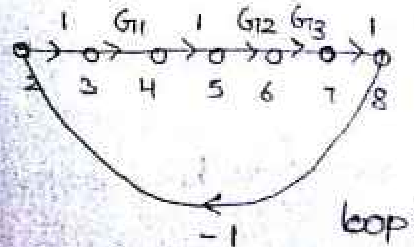


Forward path gain $P_1 = G_1G_2G_3$



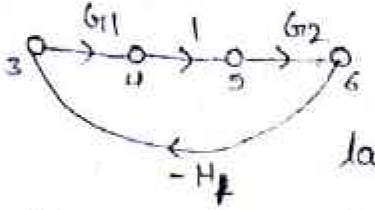
$P_2 = G_1G_4$

Step2:- Individual loop gains -



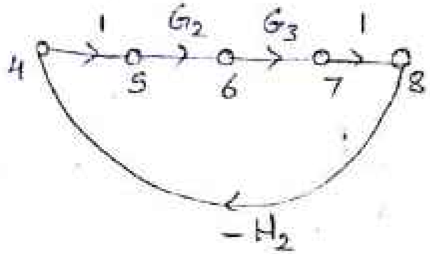
loop ①

$P_{11} = -G_1G_2G_3$



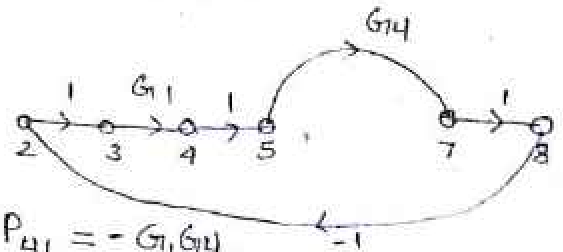
loop (2)

$P_{21} = -G_1G_2H_1$

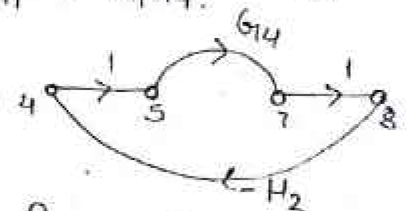


loop (3)

$P_{31} = -G_2G_3H_2$



$P_{41} = -G_1G_4$



$P_{51} = -G_4H_2$

Step 3:- Gain product of two non-touching loops:-

There are no two non-touching loops.

Step 4:- calculation of Δ.

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$$

$$\Delta = 1 + [G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2]$$

Step 5:- calculation of Δ_K

$\Delta_1 = 1$

$\Delta_2 = 1$

$$T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + [G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2]}$$

(i)