NATL
Unit-I
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Circuit Elements:

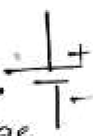
Any individual circuit component [Resistor, Inductor, capacitor and generator, etc) with two terminals by which it may be connected to other electric components.

Branch:

A group of elements, usually in series, and having two terminals.

Voltage or potential source:—

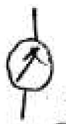
A hypothetical generator which maintains it's value of potential independent of the output current.



+ VDC
DC Voltage Source

+ VAc → AC Voltage Source

Current Source:— A hypothetical generator which maintain an o/p current independent of the Voltage across it's terminals. The current source is indicated by a circle enclosing an arrow. Here the direction of the current is usually indicated by the Arrow.



symbol

Network:— It is an interconnection of electric circuit elements or branches.

Lumped Network: It is one in which physically separate resistors, inductors and capacitors can be represented.

Distributed Networks:—

A network in which the resistors, inductors and capacitors cannot be separated electrically and isolated individually as separate elements. A transmission line is an example for Distributed networks.

Passive Network:— A network containing circuit elements without energy sources.

Active network:— A network containing energy sources or generators as well as other elements.

Linear Element:—

A Circuit element is linear if the relation between Current and Voltage involves a constant coefficient as in

$$V = RI, \quad V = L\frac{di}{dt}, \quad V = \frac{1}{C}\int i\,dt$$

[When R is constant, 'V' is directly varied with 'I'. 'V' is varied in same proportion with the 'I'.]

→ In Non-linear element, 'V' is varied in a non-linear variation with 'I'. Ex:- Iron core Reactor and Incandescent lamp, etc.

Linear Networks:—

The networks in which the differential equation relating the instantaneous current and voltage is a linear equation with constant coefficients.

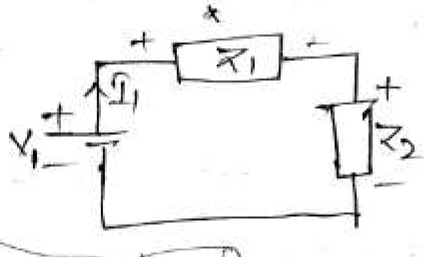
Mesh or Loop:— A set of branches forming a closed path in a network, provided that if one branch is removed, the remaining branches do not form a closed path.

Node or Junction:— A terminal of any branch of a network or a terminal common to two or more branches.

Kirchhoff's Laws:- These are used to simplify any network.

Kirchhoff Voltage Law:— It states that the algebraic sum of all the potentials around a closed loop is zero at all instants of time.

Ex:-

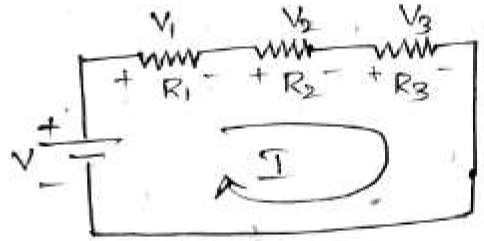


⇒ By KVL,

$+V_1 - I_1Z_1 - I_1Z_2 = 0$ → By considering the polarity of leaving terminal

$-V_1 + I_1Z_1 + I_1Z_2 = 0$ → By considering the polarity of entering terminal

{→ The current direction is always from +Ve to −Ve of the battery since the direction of the e is opposite.}



By KVL, $-V+V_1+V_2+V_3=0$

$\Rightarrow \boxed{V = V_1+V_2+V_3}$

Where $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$

$\therefore V = I[R_1+R_2+R_3] \Rightarrow \boxed{I = \frac{V}{R_1+R_2+R_3}}$

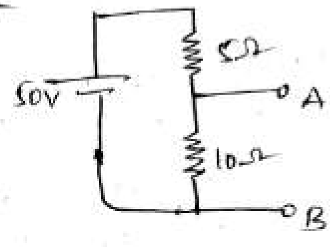
Since

$$\left.\begin{aligned} V_1 &= IR_1 = \left(\frac{R_1}{R_1+R_2+R_3}\right)V \\ V_2 &= IR_2 = \left[\frac{R_2}{R_1+R_2+R_3}\right]V \\ V_3 &= IR_3 = \left(\frac{R_3}{R_1+R_2+R_3}\right)V \end{aligned}\right\} \text{Voltage - Division Rule.}$$

*) The series circuit acts as a Voltage Divider. Since the same current flows through each resistor, the Voltage drops are proportional to the Values of resistors.

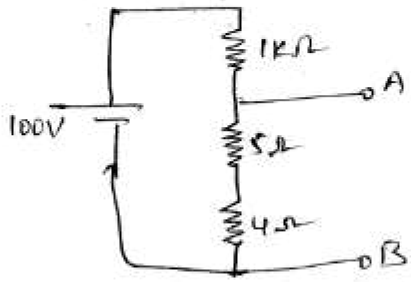
In a Voltage Divider, different voltages can be obtained from a single Source.

Ex:-



Voltage across 10Ω = V_{10}

$= 50 \times \frac{10}{10+5} = \frac{500}{15} = 33.3V$



Voltage across AB is

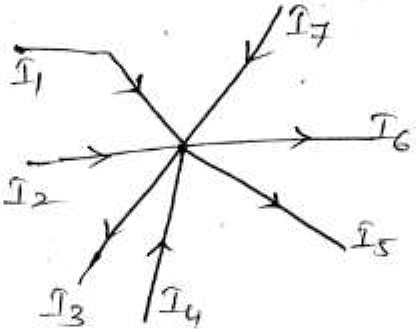
$V_{AB} = \frac{9}{10} \times 100 = 90V$

Kirchhoff's Current Law:-

It states that the sum of the Currents entering into any node is equal to the Sum of the Currents leaving that the node. The node may be an interconnection of two or more branches.

Conceptually, the KCL can also be defined as the algebraic sum of all the currents entering or leaving a node is zero.

Ex:-



* By considering the entering currents the KCL at a node is written as

$$\boxed{I_1 + I_2 - I_3 + I_4 - I_5 - I_6 + I_7 = 0}$$

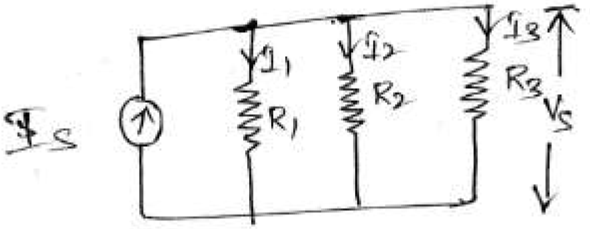
* By considering all the leaving currents at a node, it can be written as

$$\boxed{-I_1 - I_2 + I_3 - I_4 + I_5 + I_6 - I_7 = 0}$$

Current Division:-

In parallel circuits, the current divides in all branches. Thus, a parallel circuit acts as a 'Current Divider'.

* The total current entering into the parallel branches is divided into the branches currents according to the resistance values.



By KCL,

$$I_s = I_1 + I_2 + I_3$$

where

$$I_1 = \frac{V_s}{R_1}, \quad I_2 = \frac{V_s}{R_2}, \quad I_3 = \frac{V_s}{R_3}$$

$$\therefore I_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$

$$\Rightarrow I_s = V_s\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\therefore V_s = I_s\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

where $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

According to Current Division Rule, ~~it is noticed~~

$$I_1 = \frac{V_s}{R_1}, \quad I_2 = \frac{V_s}{I_2}, \quad R_T \text{ is the total resistance,}$$

which is given by $R_T = R_1 // R_2 // R_3$

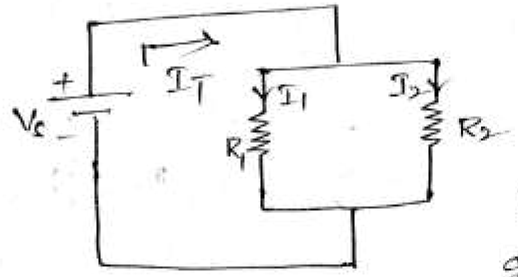
Total current $I_s = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2 R_3}\left[R_1 R_2 + R_2 R_3 + R_3 R_1\right]$

$$\therefore I_s = \frac{I_1 R_1}{R_1 R_2 R_3}\left[R_1 R_2 + R_2 R_3 + R_3 R_1\right] \Rightarrow$$

$$\Rightarrow \boxed{I_1 = I_s \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}}$$

$$I_2 = \frac{I_S R_1 R_3}{R_1R_2+R_2R_3+R_3R_1} \quad \text{and} \quad I_3 = I_S\left[\frac{R_1R_2}{R_1R_2+R_2R_3+R_3R_1}\right]$$

*→



from the Current Division Rule.

$$I_1 = \frac{V_s}{R_1}, \quad I_2 = \frac{V_s}{R_2}$$

If R_T is the total resistance, which is given by $R_T = R_1 // R_2 = \frac{R_1R_2}{R_1+R_2}$

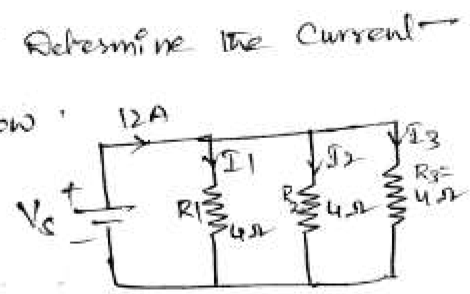
Total Current $I_T = I_1 + I_2 = \frac{V_s}{R_T} = \frac{V_s}{R_1R_2}\left[R_1+R_2\right]$

$$I_T = I_1\left[\frac{\cancel{R_1}}{\cancel{R_1}R_2}(R_1+R_2)\right] = I_1\left[\frac{R_1+R_2}{R_2}\right]$$

$$\Rightarrow \boxed{I_1 = I_T\left[\frac{R_2}{R_1+R_2}\right]}$$

Similarly $\boxed{I_2 = I_T\left[\frac{R_1}{R_1+R_2}\right]}$ //

Ex:- Determine the Current through each resistor in the Circuit shown below:



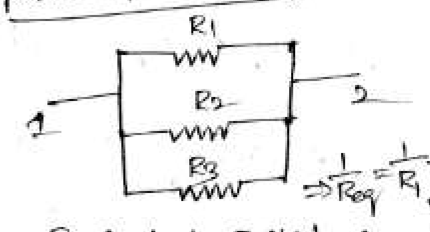
Solution:-
$$I_1 = I_T \times \frac{R_T}{R_1+R_T} \quad \text{where } R_T = \frac{R_2R_3}{R_2+R_3} = 2\,\Omega$$

$R_1 = 4\,\Omega$, $I_T = 12\,A$.

$$\therefore I_1 = 12 \times \frac{2}{2+4} = 4A,$$

Similarly
$$I_2 = 12 \times \frac{2}{2+4} = 4A.$$
$$I_3 = 12 \times \frac{2}{2+4} = 4A.$$

Parrallel Connection:-



$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Equivalent Resistance =

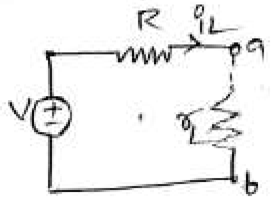
$$R_{eq} = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3}$$ //

Series Connection:-

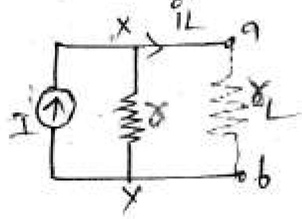
Equivalent Resistance, $R_{eq} = R_1+R_2+R_3$.

Source Transformation:—

Any practical voltage source consists of an ideal voltage source in series with its internal resistance. Similarly, any practical current source consists of an ideal current source in parallel with it's internal resistance.



Practical Voltage Source



Practical Current Source

The voltage and current sources are mutually transferable.

If a load resistance R_L is connected across the terminals A&B, i_L and I_L' can be obtained from the figures as

$$i_L = \frac{V}{R+r_L} \quad \& \quad I_L' = \frac{Ir}{r+r_L}$$

The two sources will be identical if and only if $i_L = i_L'$ i.e.,

$$\boxed{\frac{V}{R+r_L} = \frac{Ir}{r+r_L}}$$

$$\therefore V = Ir$$

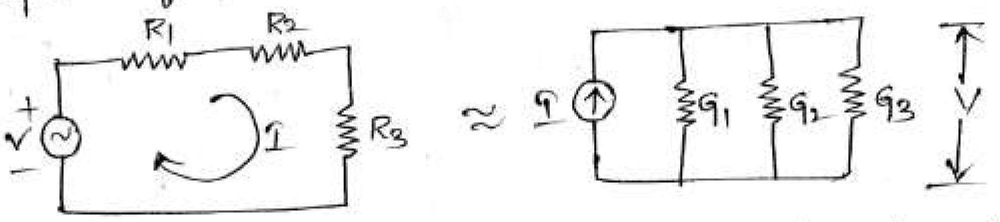
$$\therefore R+r_L = r+r_L$$

$$\therefore \boxed{R=r}$$

Therefore, a practical voltage source can be replaced by an equivalent practical current source.

Principle of Duality:—

A voltage is considered the driving force, and a current is the response of the circuit.



Here the interchange of the dependent and independent variables in the problem, and leads to the concept of duality in networks.

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Some dual relations involving exchange of current for voltage are illustrated by the pairs of following equations:

$$V = RI \quad , \quad I = GV$$

$$V = L\frac{di}{dt} \quad , \quad i = C\frac{dv}{dt}$$

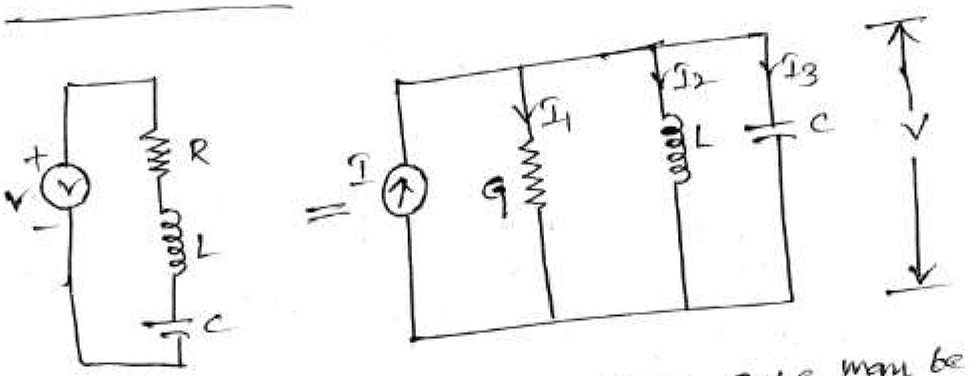
$$V = \frac{1}{C}\int i\,dt \quad , \quad i = \frac{1}{L}\int v\,dt$$

from figure, the loop equation is

$$I\left[Z_1 + Z_2 + Z_3\right] = V$$

whereas the node voltage equation for the figure is

$$V\left[Y_1 + Y_2 + Y_3\right] = I$$

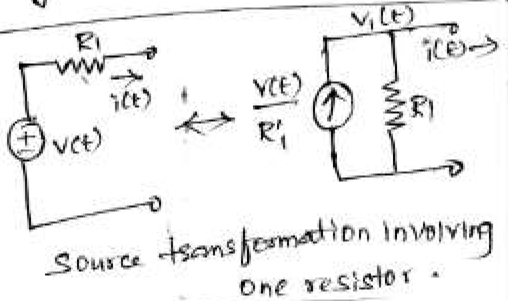


The Mesh equations for the series case may be written as

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$

and for the dual-parallel circuit,

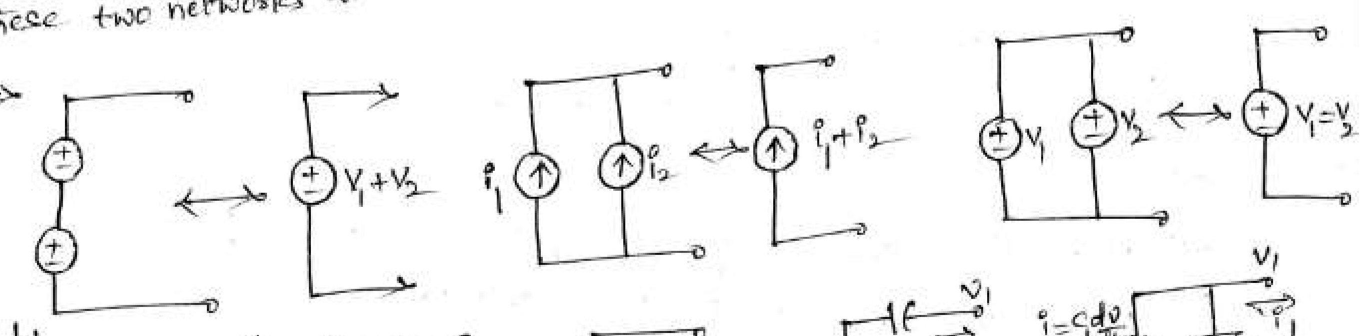
$$GV + C\frac{dV}{dt} + \frac{1}{L}\int v\,dt = I$$

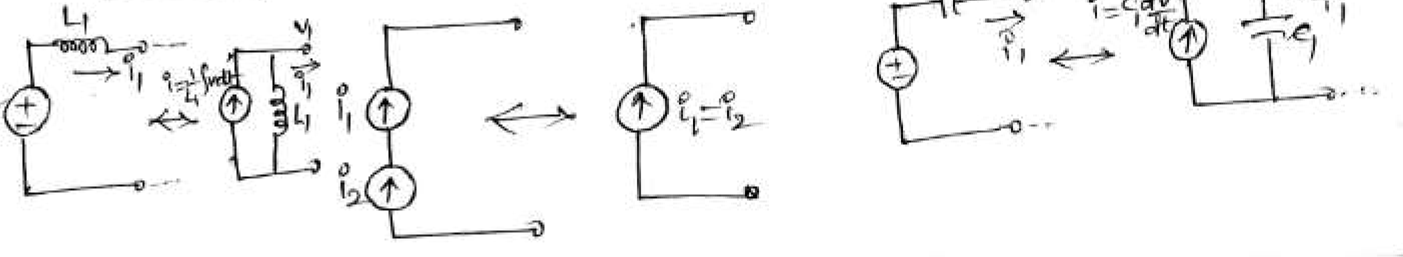


Source transformation involving one resistor.

$\ast$ If two electric networks are governed by the same type of equations, these two networks are known as 'Dual-Networks'.

$\Rightarrow$





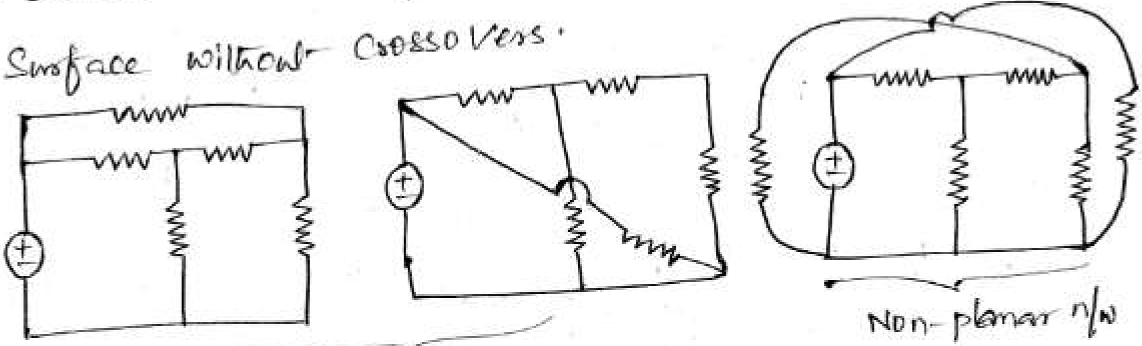
Mesh and Nodal Analysis :-

* Mesh and Nodal Analysis are two basic important techniques to solve a network.
* The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources.
* The mesh analysis is used if a network has a large number of voltage sources. This requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources.
* The Nodal analysis is used when a network has more number of current sources.

Mesh Analysis :-

* Mesh analysis is applicable only for planar networks which has no crossovers. A non-planar circuit cannot be drawn on a plane surface without crossovers.

Ex:-



planar n/w's

Non-planar n/w

* A mesh is defined as a loop which doesnot contain any other loops within it.
* Steps to solve mesh analysis:
  (i) Check whether the circuit is planar or not.
  (ii) Select mesh currents.
  (iii) Write Kirchhoff's Voltage law equations in terms of unknown and solving them leads to the final solution.

NATL Unit - I

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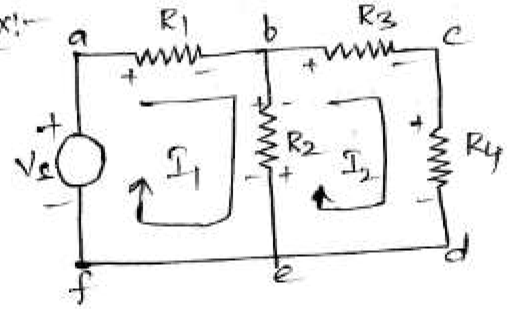
Ex:-



Solution:-

In the figure, there are two loops. I_1 and I_2 are the two loop currents passes in two loops of the network.

Considering loop (1):-

By KVL,

$-V_s + I_1R_1 + [I_1 - I_2]R_2 = 0$

$-V_s + I_1[R_1+R_2] - I_2R_2 = 0$

$\therefore \boxed{V_s = I_1R_1 + R_2[I_1 - I_2]} \rightarrow (1)$

Considering loop (2):-

By KVL,

$\boxed{(I_2 - I_1)R_2 + I_2R_3 + I_2R_4 = 0} \rightarrow (2)$

By Re-arranging the above equations, the corresponding mesh current equations are

$I_1[R_1+R_2] - I_2R_2 = V_s$

$-I_1R_2 + (R_2+R_3+R_4)I_2 = 0$

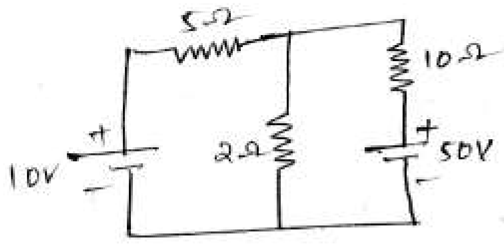
By solving the above equations, it can be found the currents I_1 and I_2.

Conclusions:-

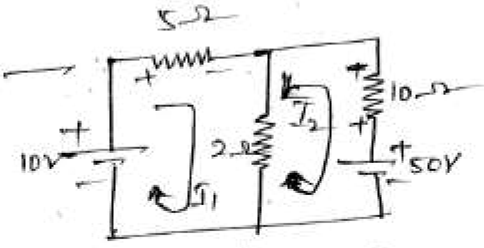
* The number of mesh currents is equal to the number of mesh equations.
* The number of mesh equations of a network = Branches − (nodes − 1)

** $\boxed{M = B - (N-1)}$

Ex:- Write the mesh current equations of the following circuit, and determine the currents?



Solution:-



The mesh current equations are

$-10 + 5I_1 + 2[I_1 - I_2] = 0 \Rightarrow 5I_1 + 2(I_1 - I_2) = 10 \rightarrow (1)$

$-50 + 10I_2 + [I_2 - I_1]2 = 0 \Rightarrow -2I_1 + 12I_2 = 50 \rightarrow (2)$

from (2),

$-2I_1 = 50 - 12I_2$

$\Rightarrow \boxed{I_1 = 6I_2 - 25}$

from (1)

$5[6I_2 - 25] + 2[6I_2 - 25 - I_2] = 10$

$\Rightarrow 30I_2 - 125 + 12I_2 - 50 - 2I_2 = 10$

$40I_2 = 185 \Rightarrow I_2 = \frac{18.5}{4} = 4.625\,A$ //

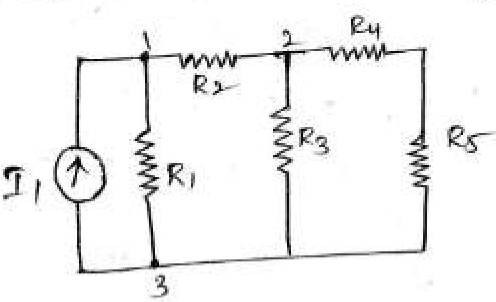
$I_1 = 6[4.625] - 25 = 27.750 - 25 = 2.75\,A$ //

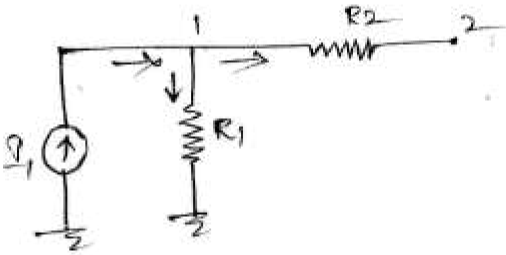
→ If any of the branches in the network has a Current Source, then it is slightly difficult to apply mesh analysis directly. In this case, a supermesh analysis approach is best suited.

Nodal Analysis :—

→ If a circuit has 'N' number of nodes, then it is possible to write (N-1) nodal equations by assuming (N-1) node voltages.

→ The node voltage is the voltage of a given node with respect to one particular node, called the "reference node", which is assume at zero potential.



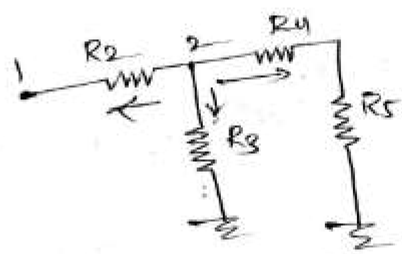


→ The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3.

where V_1 & V_2 → Voltages at node 1 & node 2.

At node 1 →

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$



At node 2 →

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

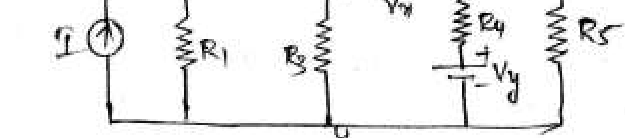
$$\Rightarrow V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] = I_1$$

$$-V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0$$

from the above equations, we can find the voltages at each node.

→ In any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. In this case, a supernode approach is best suited.

Ex:-



$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

Due to the presence of voltage source V_x in between nodes '2' & '3'.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_y}{R_4} + \frac{V_3}{R_5} = 0$$

& $V_2 - V_3 = V_x$ //

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Star - Delta Transformation :-

Basic circuit elements can also be connected in Star (or Y) connection or Delta [or Δ] connection.

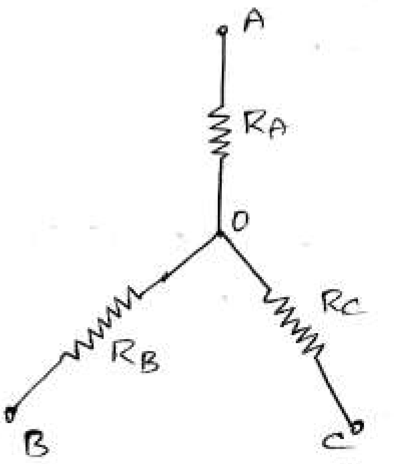


fig: Star or Y connection

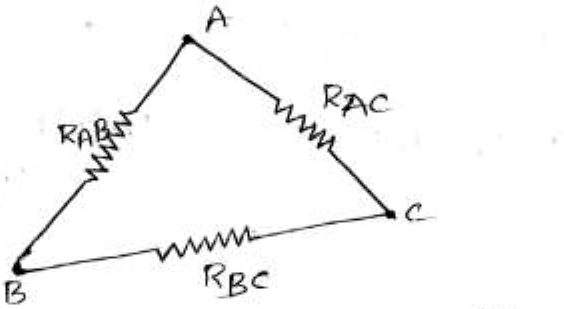
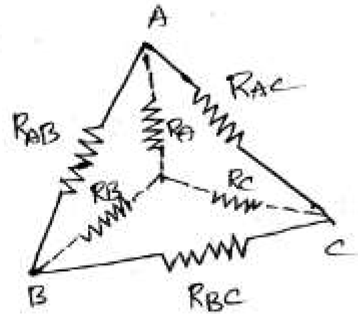


fig: Delta or Δ Connection.

The above two circuits are equal if their respective resistances from the terminals AB, BC and CA are equal.



The resistance from the terminals AB, BC and CA respectively are from the star connected circuit are

$$R_{AB}(Y) = R_A + R_B$$

$$R_{BC}(Y) = R_B + R_C$$

$$R_{CA}(Y) = R_C + R_A$$

Similarly in the delta connected network, the resistances seen from the terminals AB, BC and CA, respectively are

$$R_{AB}(\Delta) = R_{AB} \| (R_{AC} + R_{BC}) = \frac{R_{AB}(R_{AC} + R_{BC})}{R_{AB} + R_{BC} + R_{AC}} \; //$$

$$R_{BC}(\Delta) = R_{BC} \| (R_{AB} + R_{AC}) = \frac{R_{BC}(R_{AB} + R_{AC})}{R_{AB} + R_{BC} + R_{AC}} \; //$$

$$R_{CA}(\Delta) = R_{AC} \| (R_{AB} + R_{BC}) = \frac{R_{AC}(R_{AB} + R_{BC})}{R_{AC} + R_{AB} + R_{BC}} \; //$$

Now, if we equate the resistances of star and delta circuits, we get

$$R_A + R_B = \frac{R_{AB}(R_{AC} + R_{BC})}{R_{AB} + R_{BC} + R_{AC}} \Rightarrow \boxed{R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}} \; //$$

$$R_B + R_C = \frac{R_{BC}[R_{AB} + R_{AC}]}{R_{AB} + R_{BC} + R_{AC}} \Rightarrow \boxed{R_B = \frac{R_{BC} R_{AB}}{R_{AB} + R_{AC} + R_{BC}}} \; //$$

$$R_C + R_A = \frac{R_{AC}\left[R_{AB}+R_{BC}\right]}{R_{AB}+R_{BC}+R_{AC}} \Rightarrow \boxed{R_C = \frac{R_{BC}R_{AC}}{R_{AB}+R_{BC}+R_{AC}}}$$

Thus, a delta connection of R_{AB}, R_{BC} and R_{CA} may be replaced by a star connection of R_A, R_B and R_C as determined above.

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_{AB}^2 R_{BC} R_{AC} + R_3^2 R_1 R_2 + R_2^2 R_1 R_3}{(R_{AB}+R_{BC}+R_{CB})^2}$$

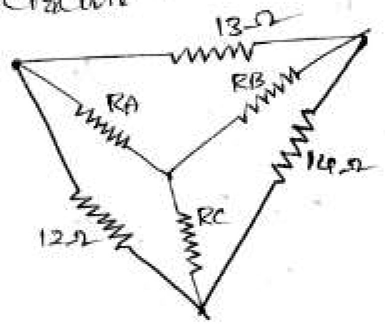
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \qquad R_{AC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

** Any resistance of the delta circuit is equal to the sum of the products of all possible pairs of star resistances divided by the opposite resistance of the star circuit.

** Any resistance of the star circuit is equal to the product of two adjacent resistances in the delta connected circuit divided by the sum of all resistances in delta connected circuit.

Prob:- obtain the star connected equivalent for the delta connected circuit.

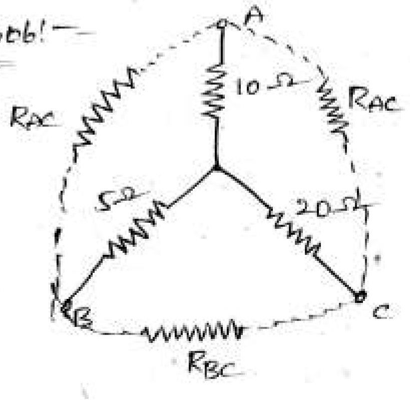


$$R_A = \frac{13\times 12}{13+12+14} = \frac{156}{39} = 4\,\Omega.$$

$$R_B = \frac{13\times 14}{13+12+14} = \frac{182}{39} = 4.66\,\Omega$$

$$R_C = \frac{12\times 14}{13+12+14} = \frac{168}{39} = 4.31\,\Omega$$

Prob:-



$$R_{AC} = \frac{5\times 10 + 10\times 20 + 20\times 5}{20} = \frac{350}{20} = 17.5\,\Omega$$

$$R_{BC} = \frac{5\times 10 + 10\times 20 + 20\times 5}{10} = \frac{350}{10} = 35\,\Omega$$

$$R_{AC} = \frac{5\times 10 + 10\times 20 + 20\times 5}{5} = \frac{350}{5} = 70\,\Omega$$

NATL Unit-I (7)

Basic Cut-set and Tie-set Matrices for planar networks :-

Network topology:

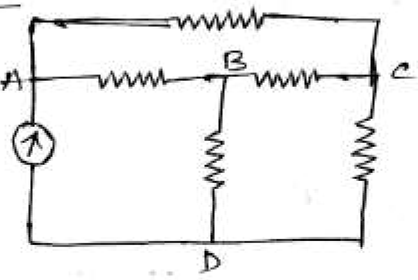
It is the study of the network properties by investigating the interconnection between branches and the nodes, it mainly concentrate on the geometry of the network.

→ In the network topology, any network is replaced by a Simple graph. While developing the graph, each element is replaced by a straight line or arc of the semicircle, voltage source is replaced by a short circuit and the current source is replaced by an open circuit and graph retains all the nodes of the original network.

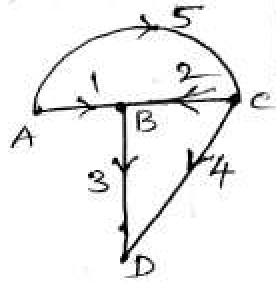
** Nodes can be incident to one or more elements.

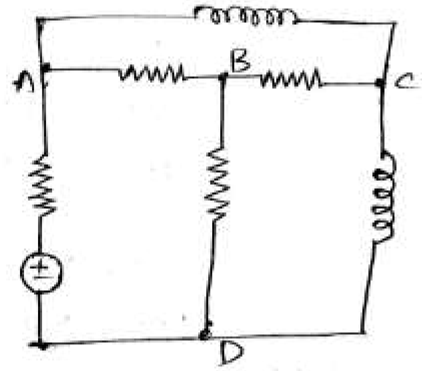
*) The number of branches incident at a node of a graph indicates the degree of the node.

EX:-

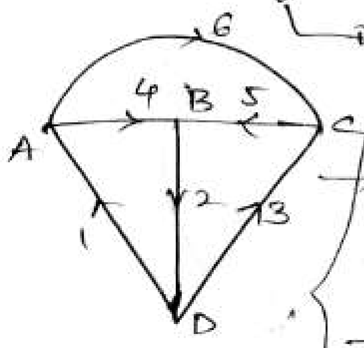


⇒





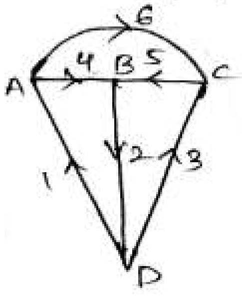
⇒



Here in this, every branch of a graph has a direction, then this graph is called a "Directed Graph".

→ If there is no direction then it is "undirected Graph".

⇒ Branches in Graph ≤ Branches in the network



	1	2	3	4	5	6
A	+1	0	0	−1	0	−1
B	0	−1	0	+1	+1	0
C	0	0	+1	0	−1	+1
D	−1	+1	−1	0	0	0

This is an Augmented Incidence Matrix

*) For a given graph, incidence matrix is unique.

*) All the network information can be represented mathematically in concised form is called "Incidence Matrix".

*) Size of the incidence matrix is $n \times b$. $n \to$ no. of nodes, $b \to$ branches.

Tree:— It is a connected subgraph. It connects all the nodes of the network but doesn't contain any closed loop.

Co-Tree:— The set of branches which are disconnected to form a tree is called a "co-tree".

Tree = {1,2,3}

CO Tree = {4,5,6}

⇒ The number of nodes in a tree = Number of nodes in a graph.

*) The branch which form a tree is called "Tree Branch" or "Twig". Generally it is represented by a solid line or Thick line.

$$\boxed{\text{Total number of Tree branches} = \text{Nodes}(N) - 1}$$

*) The branch which is disconnected to form a tree is called as a "Link" or "chord". In general, it is represented by a dotted line.

$$\boxed{\text{Total number of links} = b-(N-1)}$$ b → Branches, N → nodes.

*) Tree is not unique.

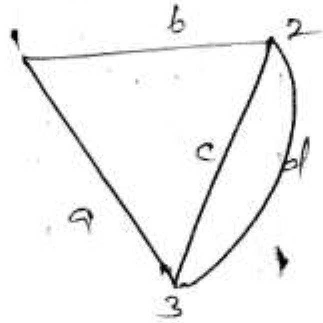
*) The number of branches in a tree is less than the number of branches in a graph.

*) A graph is connected if and only if there is a path between every pair of nodes.

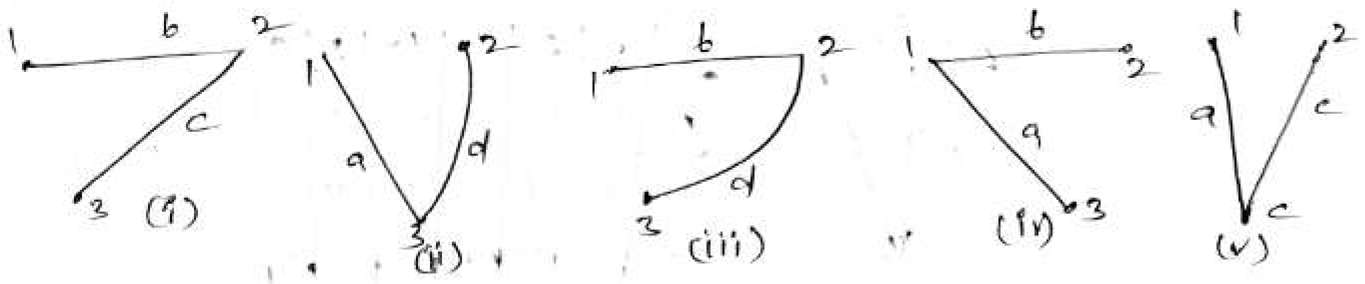
*) A graph can be drawn if there exists a path between any pair of nodes.

*) A loop exists, if there is more than one path between two nodes.

Ex:—



Different Trees formed by the above graph are



*) The number of Twigs (n-1) is known as the tree value of the graph. It is also called the 'rank of the tree'.

*) Loops which contain only one link are independent and are called "Basic Loops".

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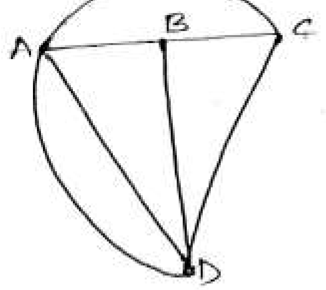
→ Total number of possible trees when no repetitive branches are exists between any two nodes, is $N^{(n-2)}$. Here in this, all the nodes must be connected together. Also, all the nodes must be connected together.

*→ When ~~a tree~~ repetitive branches are exists between any two branches then the number of possible trees is calculated by $Det(AA^T)$. Where A is reduced incidence matrix.

Ex:-



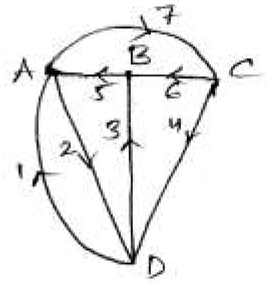
Here, the number of trees = $Det(AA^T)$

Since two branches are connected between 'A' and 'D'.

Here node 'D' is the "reference node" or 'Datum node'.

* For the redundant incidence matrix, we may delete the row corresponding to the datum node.

Ex:-



Incidence Matrix:-

	I_1	I_2	I_3	I_4	I_5	I_6	I_7
A	+1	−1	0	0	+1	0	−1
B	0	0	+1	0	−1	+1	0
C	0	0	0	−1	0	−1	+1
D	−1	+1	−1	+1	0	0	0

This is due to the datum node.

Redundant Incidence Matrix

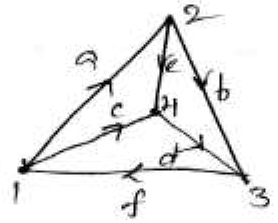
$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix}_{3\times 7}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}_{7\times 3}$$

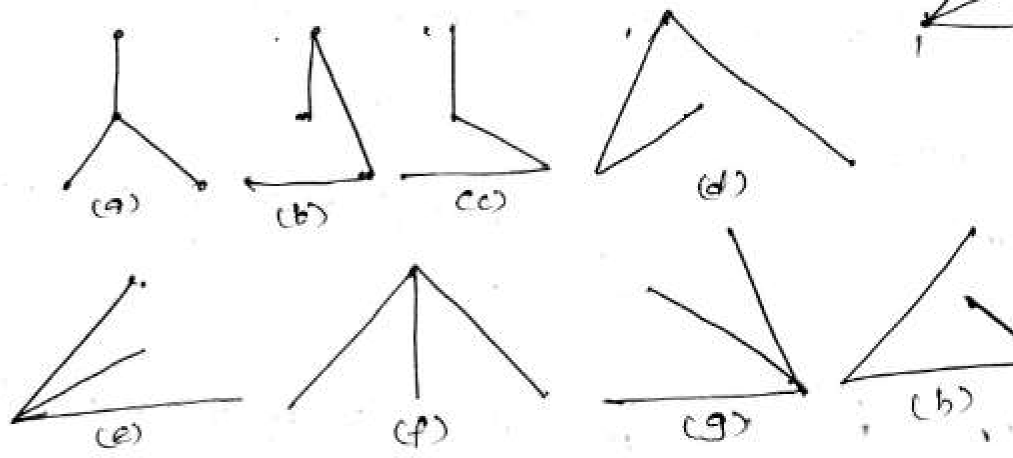
$$AA^T = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$Det(AA^T) = 4[9-1]+1[-3-1]-1[1+3] = 4[8]+1[-4]-1[4]$$
$$= 32-8 = \underline{\underline{24}}$$

Prob:- For the given graph, draw the number of possible trees.



Sol:-



$\therefore$ Total number of possible trees $= n^{n-2} = 4^2 = \underline{16}$

Tie-Set Matrix:

→ For a given tree of a graph, addition of each link between any two nodes forms a loop called the "fundamental loop".

→ The reference current direction in the loop is considered the link current direction of the corresponding loop.

→ The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is also called "f-loop or a tie-set".

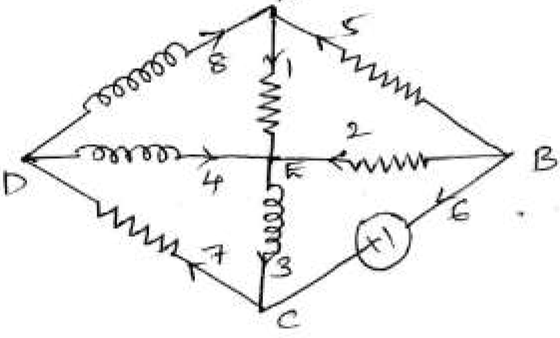
→ Kirchhoff's voltage law can be applied to the f-loops to get a set of linearly independent equations.

→ Tie-set matrix is usually represented as

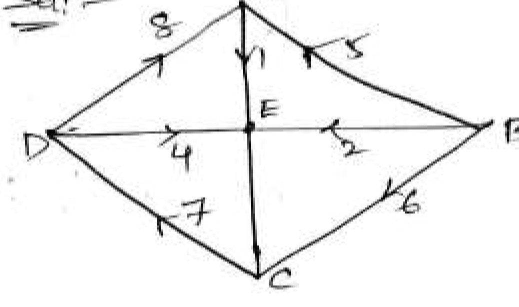
$$[I \times b][V_b] = 0$$

where I → loop currents, b → branches, V_b → column vector of branch voltages.

Ex:-



Sol:-



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** Steps to be followed for creating a Tie-set Matrix

(i) Develop a graph for a given network.

(ii) Develop a tree for a graph

(iii) (a) Identify the number of basic loops

(b) The basic or fundamental loop should consists of only one link

(c) Total number of basic loops = Total no. of links = $b-(N-1)$

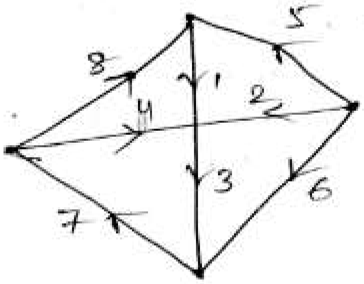
(d) choose the f-loop current direction is same as the link current direction.

(e) Apply KVL in each f-loop based on link current direction

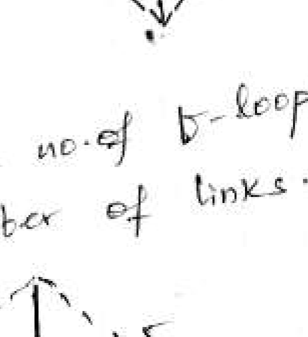
(f) Develop a Tie-set matrix based on KVL equations in the form of $[I \times b][V_b] = 0$

Continuation problem :-

(i)

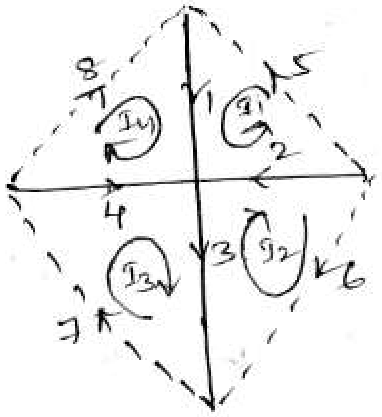


Step (2) :-



Step (3) :-

(a) The no. of f-loops are 4 which is equivalent to number of links. $= b-(n-1) = 8-(5-1) = 8-4 = 4$



	b1	b2	b3	b4	b5	b6	b7	b8
I_1	+1	−1	0	0	+1	0	0	0
I_2	0	−1	−1	0	0	+1	0	0
I_3	0	0	1	1	0	0	1	0
I_4	1	0	0	−1	0	0	0	1

$$
= \begin{array}{c} I_1 \\ I_2 \\ I_3 \\ I_4 \end{array}
\begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = 0.
$$

→ The branch currents can also be expressed as a linear combination of link current.

If I_B and I_L represents the branch current matrix and loop current matrix respectively and A is the Tie-set Matrix then

$$[I_b] = [A^T][I_L]$$

$$
A = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad
A^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & +1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}
$$

$$
I_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}, \quad
I_L = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}
$$

$$
\underset{8\times1}{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}} =
\underset{8\times4}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
\underset{4\times1}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}
$$

The branch currents are

$i_1 = I_1 + I_4$, $i_2 = -I_1 - I_2$, $i_3 = -I_2 + I_3$, $i_4 = I_3 - I_4$

$i_5 = I_1$, $I_6 = I_2$, $I_7 = I_3$, $I_8 = \cancel{I_3} + I_4$ //

NATL Unit-I (10)

Cut-Set and Tree Branch Voltages

→ A Cut-set is a minimal set of branches of a connected graph such that the removal of these branches causes the graph to be cut into exactly two parts.

*→ The Cut-set consists of one and only one branch of the tree network together with any links.

→ KCL is ~~also~~ applicable to a cut-set of a network. While writing the KCL equation for a cut-set, we assign positive sign for the current in a branch if its direction coincides with the orientation of the cut-set and a negative sign to the current in a branch whose direction is opposite to the orientation of the cut-set.

→ Total number of possible Cut-set Matrices = N^{N-2}

Rank of the Cut-set Matrix = $N-1$.

→ Total no. of basic Cut-sets = Total no. of Tree branches = $N-1$.

*→ The fundamental Cut-sets are defined for a given tree of the graph. From a connected graph, first a tree is selected, and then twig is selected. Removing this twig from the tree separates the tree into two parts. All the links which go from one part of the disconnected tree to the other, together with the twig of the selected tree will constitute a Cut-set. This Cut-set is called a "fundamental Cut-set" or "f-cut set".

*→ The f-cut-set of a graph with respect to a tree is a Cut-set that is formed by one twig and a unique set of links.

*) For each branch of the tree, i.e. for each twig, there will be a f-cut-set. So for a connected graph having N-nodes, there will be (N-1) twigs in a tree, the number of f-cut-sets is equal to (N-1).

→ Fundamental Cut-set matrix is one in which each row represents a cut-set with respect to a given tree of the graph.

→ The rows of Cut-set Matrix corresspond to the fundamental Cut-sets and the column corresspond to the branches of the graph.

✱ **Steps for Developing a Cut-set Matrix:**

(i) Develop a graph for a given network

(ii) Develop a Cut-set for a graph.

(iii) (a) identify total number of basic Cut-sets (or) fundamental Cutsets (or) f-Cut sets.

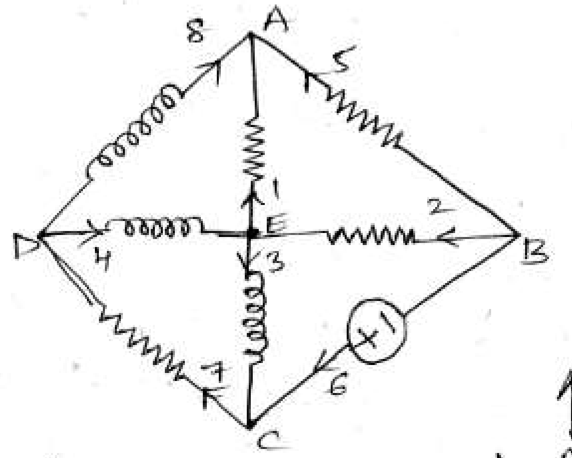
(b) Basic Cut-set should consists of only one tree branch.

(c) Cut-set may consists of one tree branch or more than one tree branch.

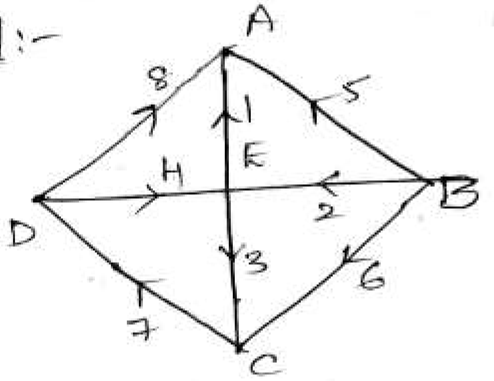
(d) Total number of basic Cut sets = total no. of tree branches = N−1

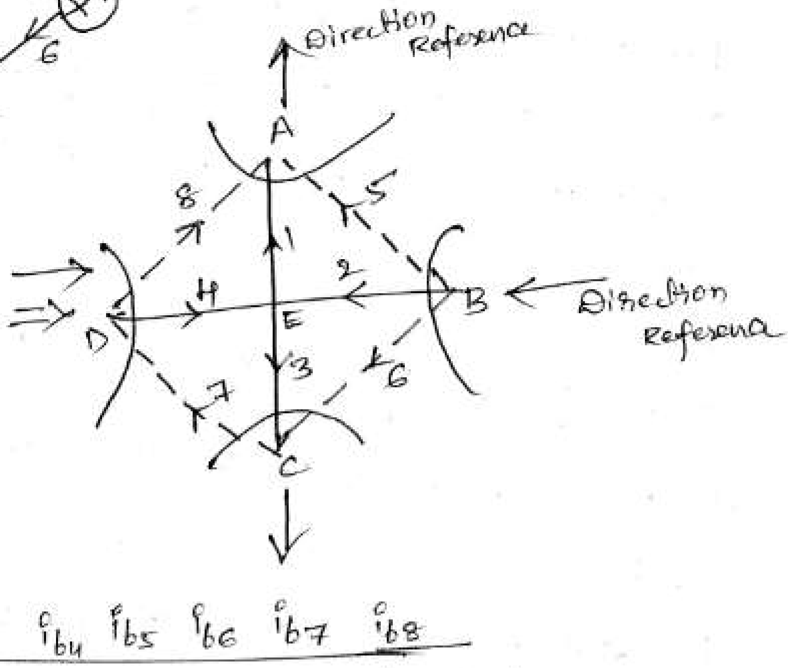
(e) Basic Cut set direction is same as the tree branch direction.

prob! –



Sol:-





Cut-set Matrix:

Cut sets \ Branches	i_{b1}	i_{b2}	i_{b3}	i_{b4}	i_{b5}	i_{b6}	i_{b7}	i_{b8}
A	1	0	0	0	1	0	0	1
B	0	1	0	0	1	1	0	0
C	0	0	1	0	0	1	−1	0
D	0	0	0	+1	0	0	−1	+1

//

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The Cut-set matrix is

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Magnetic Circuits

A magnetic circuit is made up of one or more closed loop paths containing a "Magnetic flux".

→ The two circuits are said to be "Coupled" when energy transfer takes place from one circuit to the other when one of the circuit is energised.

→ Inductance is the property of the circuit element which allows energy to be stored in the form of magnetic flux. Such components known as "Inductors".

→ Inductors are made of many turns of fine wire often wound on a magnetic material that is capable of storing more energy.

→ Self inductance is the inductance created in the coil due to the current passes through the same coil.

→ Mutual inductance is the inductance created in one coil due to the current passes through another coil (or) the inductance between two coils is called "Mutual-Inductance".

Self-Inductance:-

→ When the current in a coil is changing, the resulting changing flux linked with the coil induces an emf in the coil.

→ If the permeability of the coil is assumed to be constant, the emf induced in the coil is proportional to the rate of change of current i.e.,

$e \propto \frac{di}{dt}$ or $e = L\frac{di}{dt}$ $\longrightarrow$ (1)   L → constant of proportionality known as the "self-inductance" of the coil.

According to faraday's law of electromagnetic induction, the induced emf in a coil having 'N' turns is given by

$$e = N\frac{d\phi}{dt} \longrightarrow (2)$$

from (1) & (2),

$$L\frac{di}{dt} = N\frac{d\phi}{dt} \Rightarrow \boxed{L = N\frac{d\phi}{di}} \longrightarrow (3)$$

If 'ϕ' versus 'i' relationship is assumed linear, eqn (3) can be written as

$$\boxed{L = \frac{N\phi}{i}} \longrightarrow (4)$$

Now, $\phi = \frac{Ni}{S} = \frac{Ni}{l/\mu a} = \frac{N\mu a\, i}{l} \longrightarrow (5)$

where 'S' is the reluctance of the coil and is equal to $l/\mu a$.

from (5),

$$\frac{\phi}{i} = \frac{N\mu a}{l} \longrightarrow (6)$$

Substituting eqn (6) into eqn (4) gives,

$$\boxed{L = \frac{\mu N^2 a}{l}} \longrightarrow (7)$$

* The reluctance of a magnetic material is its ability to oppose the flow of magnetic flux.

Equation (7) suggests that the inductance is proportional to the length and the area of cross section of the coil.

→ Self inductance is defined as the flux linkage of the coil per ampere current flowing through it.

→ The unit of self-inductance is the henry (H).

Ex:- If a coil of 100 turns is linked with a flux of 0.01 Wb when carrying current of 10A, calculate the inductance of the coil. If this current is uniformly reversed in 0.01s, calculate the induced emf?

Sol:-

$L = \frac{N\phi}{I} = \frac{100 \times 0.01}{10} = 10 \times 0.01 = 0.1\,H$

$e = L\frac{di}{dt}$; $di = 10 - (-10) = 20A$, $dt = 0.01\,s$

$e = L\,di/dt = 0.1 \times \frac{20}{0.01} = 200V$ //

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Mutual Coupling: —

When interchanging of energy takes place from one circuit to other, the circuits are said to be "Mutually Coupled". The coupling between between the two circuits may be conductive, electromagnetic or electrostatic.

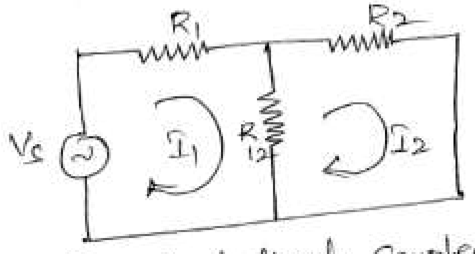

fig: Conductively Coupled

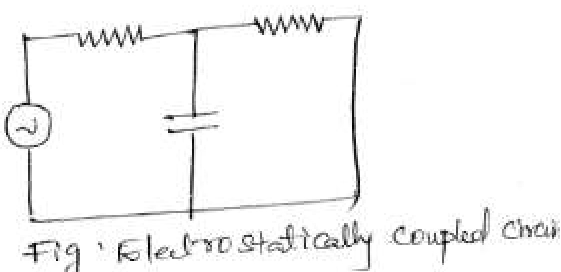

Fig: Electrostatically coupled circuit

Magnetic coupling: —

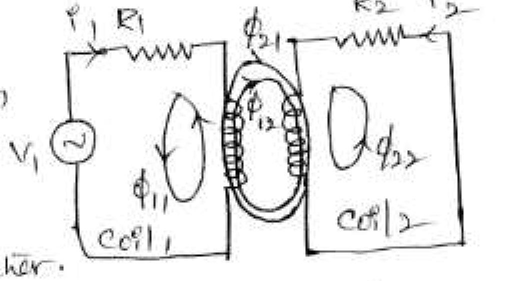

figure shows the magnetic coupling between two coils. Here, a portion of the magnetic flux established by one coil interlinks the other.

In this case, the energy is transferred from one circuit to the other through the medium of magnetic flux that is common to both.

If ϕ_1 is the total flux produced by i_1 and ϕ_2 is the total flux produced by i_2 then ϕ_1 and ϕ_2 can be written as

$$\phi_1 = \phi_{11} + \phi_{12} \quad \text{and} \quad \phi_2 = \phi_{21} + \phi_{22}$$

where ϕ_{11} & ϕ_{22} are the fluxes which are linked to coil 1 and coil 2 respectively. ϕ_{12} and ϕ_{21} are the mutual fluxes that are linked to the turns of coil 2 and coil 1 respectively.

Mutual Inductance: —

The induced voltage in coil 2 due to a change in flux can be written as (By Faraday's law)

$$e_2 = N_2 \frac{d\phi_{12}}{dt} \longrightarrow (1)$$

Here ϕ_{12} is related to i_1. So e_2 is proportional to the rate of change of i_1 i.e.

$$e_2 \propto \frac{di_1}{dt} \quad \text{or} \quad e_2 = M_{12}\frac{di_1}{dt} \longrightarrow (2)$$

M_{12} is the Mutual inductance between two coils. The unit for the Mutual Inductance is in Henry's.

from ① & ②, we can write

$$M_{12}\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt} \quad \text{or} \quad \boxed{M_{12} = N_2\frac{d\phi_{12}}{di_1}} \longrightarrow (3)$$

As mutual coupling is bilateral and a change in current i_2 in circuit 2 causes a mutually induced voltage in circuit 1. It can be said that

$$\boxed{M_{21} = N_1\frac{d\phi_{21}}{di_2}} \longrightarrow (4)$$

$M_{12} \rightarrow$ Inductance created in coil 2 due to the current passes in coil 1.

$M_{21} \rightarrow$ Inductance created in coil 1 due to the current passes in coil 2.

→ If the permeability of the mutual flux is assumed to be constant, then $M_{12} = M_{21} = M$.

* Permeability is a measure of the ability of a material to support the formation of a magnetic field within the material.

Coefficient of Coupling :-

This coefficient gives an idea of how much of the flux produced by one coil is linked with the other.

→ The coefficient of coupling indicates the extent to which the two inductors are coupled independently of the size of the inductors themselves.

→ Two inductively coupled coils 1 and coil 2 have the number of turns N_1 and N_2 respectively.

$$L_1 = \frac{\mu N_1^2 a}{l}, \quad L_2 = \frac{\mu N_2^2 a}{l},$$

→ ϕ_1 is the total flux established in coil 1 due to flow of current i_1 through it. Let a part of it, i.e., $\phi_{12} = K_1\phi_1$ be linked with coil 2.

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It can then be written as

$$\phi_1 = \frac{N_1 i_1 \mu a}{l}, \quad M = \frac{K_1 \phi_1 N_2}{i_1} \quad \text{where } K_1 \le 1.$$

'M' can also be written as

$$M = \frac{K_1 N_1 N_2 \mu a}{l} \longrightarrow (1)$$

Similarly, $\phi_2 = \dfrac{N_2 i_2 \mu a}{l} \Rightarrow \cancel{M = K_2 \phi_2 N}$

$$M = \frac{K_2 \phi_2 N_1}{i_2} = \frac{K_2 N_2 N_1 \mu a}{l} \longrightarrow (2)$$

where $K_2 \le 1$

from (1) & (2),

$$M^2 = K_1 K_2 \left(\frac{N_1^2 \mu a}{l}\right)\left(\frac{N_2^2 \mu a}{l}\right) = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2}\sqrt{L_1 L_2} = K\sqrt{L_1 L_2} \quad \text{where } K = \sqrt{K_1 K_2}$$

$$\therefore \boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

The constant 'K' is called the coefficient of coupling.

*> If the flux due to one coil is fully linked with the other, the value of 'K' is unity, which is called 'Tightly or closely coupled'.

*> If the flux in one coil does not link the another coil, $K = 0$, which is called 'Magnetically isolated or loosely coupled' coils.

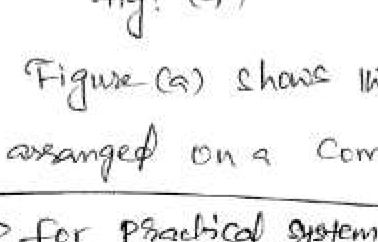
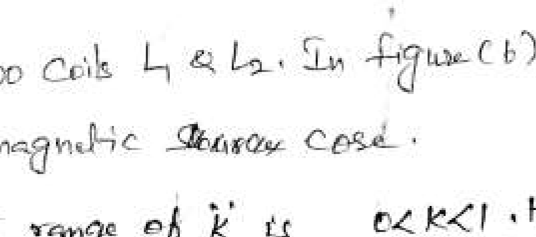


fig: (a)



(b)

*> $K = \dfrac{\text{useful flux}}{\text{Total flux}} = \dfrac{\phi_{12}}{\phi_1} = \dfrac{\phi_{21}}{\phi_2}$

Figure (a) shows the two coils L_1 & L_2. In figure (b), two coils are arranged on a common magnetic ~~structure~~ core.

*> For practical system, the range of 'K' is $0 < K < 1$. Here leakage factor is more.

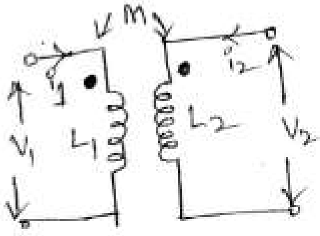
*> For an ideal circuit, $K = 1$.

Dot Convention: —

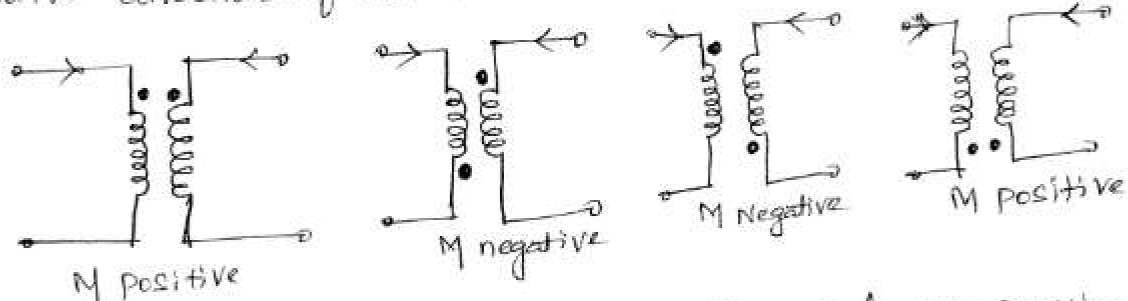
It is used to establish the choice of correct sign for the mutually induced voltages in coupled circuits.

→ Circular dot marks and/or special symbols are placed at one end of each of two coils which are mutually coupled to simplify the diagramatic representation of the winding around its core.

→ The emf induced due to Mutual Inductance may aid or oppose the emf induced due to self-inductance in a circuit. Actually it depends on the relative directions of currents.



→

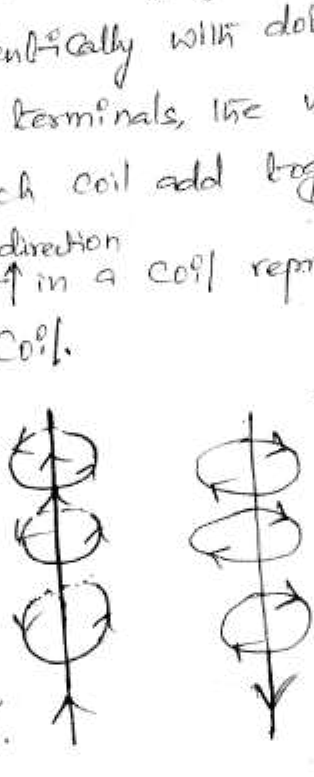


→ Although the self-induced voltages are designated with positive sign, mutually induced voltages can be either positive or negative depending on the direction of currents in the windings of the coil and can be decided by the presence of the dots placed at one end of each of the two coils.

→ If two terminals belonging to two different coils in a coupled circuit are marked identically with dots then for the same direction of current relative to like terminals, the magnetic flux of self and mutual induction in each coil add together.

→ The current direction in a coil represents the orientation of the flux around the coil.

→ When current is in upward direction, flux lines are rotating in anti-clockwise direction. otherwise, clockwise direction.



→ The orientation of the flux lines, represented by the Right-hand Thumb Rule.

According to right-hand thumb rule, If the thumb represents the current direction the curled fingers represents the orientation of the flux lines.

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Series Connection of Coupled Inductors:-

When two inductors are coupled in series and mutual inductance exists between them, the equivalent inductance of this series coupling can be calculated for series aiding and series opposing connections as follows:

(i) Series Aiding:-

→ The fluxes produced by the two coils are additive in nature as per dot convention.

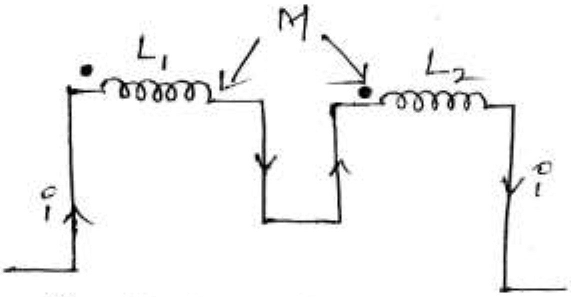

fig: Inductive Coupling in Series (flux Aiding)

Let
L_1 → self-inductance of coil 1
L_2 → self-inductance of coil 2
M → Mutual inductance between coils 1 and 2.

for coil 1:

Self-induced emf, $e_1 = -L_1 \frac{di_1}{dt}$ ⟶ (1)

Mutually-induced emf, $e_1' = -M_{21} \frac{di_2}{dt}$ (This is due to change in current in coil 2) ⟶ (2)

for coil 2:

Self-induced emf, $e_2 = -L_2 \frac{di_2}{dt}$ ⟶ (3)

Mutually-induced emf, $e_2' = -M_{12} \frac{di_1}{dt}$ (This is due to change in current in coil 1) ⟶ (4)

Therefore, the total induced emf of the above combination can be written as,

$$e = -L_1 \frac{di_1}{dt} - L_2 \frac{di_2}{dt} - M_{21}\frac{di_2}{dt} - M_{12}\frac{di_1}{dt} \longrightarrow (5)$$

If $M_{12} = M_{21} = M$, and $i_1 = i_2 = i$ [as per the figure shown]

$$\therefore \boxed{e = -(L_1 + L_2 + 2M)\frac{di}{dt}} \longrightarrow (6)$$

If 'Leq is the equivalent inductance of the combination, then

$$\boxed{e = -L_{eq}\frac{di}{dt}} \longrightarrow (7)$$

from eqns (6) & (7), $-L_{eq}\frac{di}{dt} = -(L_1 + L_2 + 2M)\frac{di}{dt}$

$$\therefore \boxed{L_{eq} = L_1 + L_2 + 2M}$$

The equivalent inductance in series-aiding case,

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

(ii) Series-opposing :—

When two coils are connected in such a way that their fluxes or emf's are in opposite direction as per dot convention, the coils are said to be in series opposing.

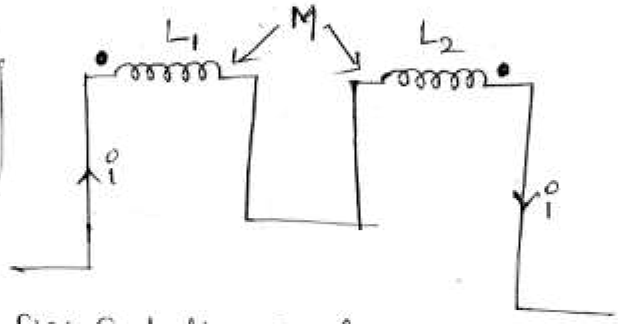

fig: Inductive coupling in series (flux opposing)

for coil 1:

Self-induced emf, $\boxed{e_1 = -L_1 \frac{di}{dt}} \rightarrow (1)$

Mutually-induced emf in coil 1 due to change in current in coil 2 is

$$\boxed{e_1' = M\frac{di}{dt}} \rightarrow (2)$$

for coil 2:

Self-induced emf, $\boxed{e_2 = -L_2 \frac{di}{dt}} \rightarrow (3)$

Mutually-induced emf in coil 2 due to change in current in coil 1 is

$$\boxed{e_2' = M\frac{di}{dt}} \rightarrow (4)$$

The total emf induced in the combination can be written as

$$e = -L_1\frac{di}{dt} - L_2\frac{di}{dt} + 2M\frac{di}{dt} = -\left[L_1 + L_2 - 2M\right]\frac{di}{dt} \rightarrow (5)$$

If L_{eq}' is the equivalent inductance of the combination, it can be said that

$$e = -L_{eq}\frac{di}{dt} \rightarrow (6)$$

from (5) and (6), we get

$$-L_{eq}\frac{di}{dt} = -(L_1 + L_2 - 2M)\, di/dt$$

$$\Rightarrow \boxed{L_{eq} = +(L_1 + L_2 - 2M)}$$

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(iii) Inductive Coupling in parallel :-

→ When two coils are coupled in parallel and there is mutual inductance between them, the equivalent inductance of the parallel combination can be calculated as follows.

(i) Parallel - Aiding :-

figure shows the parallel connection of two coils where the fluxes are additive as per dot convention. Using KVL, we can write as

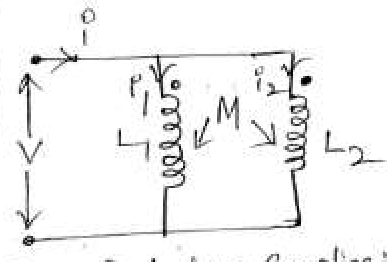


fig: Inductive Coupling in parallel (Flux Aiding)

$$V = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \longrightarrow (1) \quad \text{and} \quad V = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \longrightarrow (2)$$

from eqn's (1) & (2).

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \longrightarrow (3)$$

Here $i = i_1 + i_2$, or $i_2 = i - i_1$ $\longrightarrow (4)$

Substituting Eqn (4) into eqn (3),

$$L_1\frac{di_1}{dt} + M\frac{d}{dt}(i - i_1) = L_2\frac{d}{dt}(i - i_1) + M\frac{di_1}{dt}$$

or $$\boxed{\frac{di_1}{dt} = \left[\frac{L_2 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt}} \longrightarrow (5)$$

Similarly. $$\boxed{\frac{di_2}{dt} = \left[\frac{L_1 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt}} \longrightarrow (6)$$

Substituting eqns (5) and (6) into eqn (1),

$$V = L_1\left[\frac{L_2 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt} + M\left[\frac{L_1 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt}$$

or $$V = \left[\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right]\frac{di}{dt} \longrightarrow (7)$$

If 'L' is the equivalent inductance of the parallel combination of two inductors.

$$V = L\frac{di}{dt} \longrightarrow (8)$$

from equations (7) and (8), we get

$$L(di/dt) = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\left(\frac{di}{dt}\right) \Rightarrow \boxed{L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}} \longrightarrow (9)$$

//

(iv) parallel opposing :-

In a parallelly connected network, the mutually induced emf in one coil is observed due to variation of current in the other ~~coil~~ coil is negative.

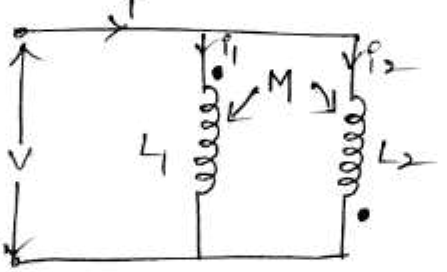


fig: Inductive Coupling in parallel (Parallel opposing)

By a KVL equation,

$$V = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} \longrightarrow (1)$$

$$V = L_2\frac{di_2}{dt} - M\frac{di_1}{dt} \longrightarrow (2)$$

from equations (1) and (2),

$$L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = L_2\frac{di_2}{dt} - M\frac{di_1}{dt} \longrightarrow (3)$$

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \text{or} \quad \frac{di_1}{dt} = \frac{di}{dt} - \frac{di_2}{dt} \longrightarrow (4)$$

substituting eqn (4) into eqn (3),

$$L_1\frac{di_1}{dt} - M\left[\frac{di}{dt} - \frac{di_1}{dt}\right] = L_2\left(\frac{di}{dt} - \frac{di_1}{dt}\right) - M\frac{di_1}{dt}$$

$$\text{or} \quad \boxed{\frac{di_1}{dt} = \left(\frac{L_2 + M}{L_1 + L_2 + 2M}\right] di/dt} \longrightarrow (5)$$

Similarly,

$$\boxed{\frac{di_2}{dt} = \left[\frac{L_1 + M}{L_1 + L_2 + 2M}\right] di/dt} \longrightarrow (6)$$

substituting eqns (5) and (6) into eqn (1),

$$V = L_1\left[\frac{L_2 + M}{L_1 + L_2 + 2M}\right]\frac{di}{dt} - M\left[\frac{L_1 + M}{L_1 + L_2 + 2M}\right] di/dt$$

$$\text{or} \quad \boxed{V = \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}\right] di/dt} \longrightarrow (7)$$

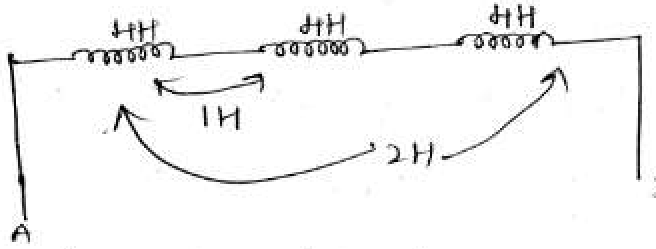
If 'L' is the equivalent inductance, then

$$V = L\frac{di}{dt} \longrightarrow (8)$$

from eqns (8) and (9),

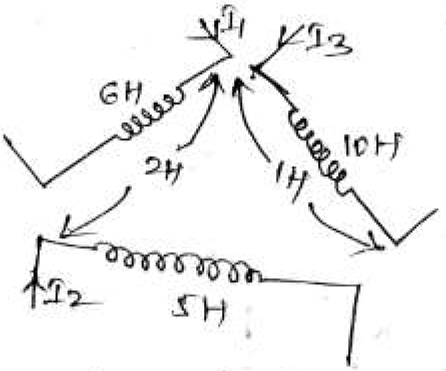
$$L * \frac{di}{dt} = \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}\right]\frac{di}{dt} \Rightarrow \boxed{L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Prob:- find the equivalent inductance with respect to the terminals A and B.



Sol:- $L_{eq} = L_1 + L_2 + L_3 \pm 2M_1 \pm 2M_2 \pm 2M_3$
$= 4 + 4 + 4 + (2\times1) + 0 - (2\times2) = 14 - 4 = 10$

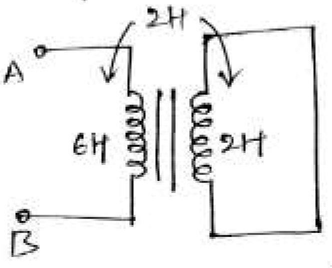
Prob:- Develop an inductance matrix of the circuit shown.



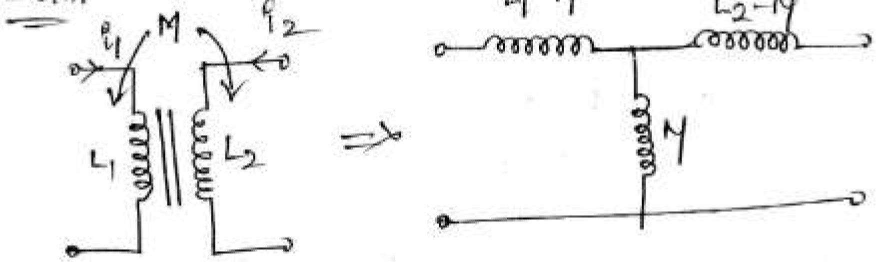
Sol:-

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 10 \end{bmatrix}$$

Prob:- find equivalent inductance with respect to terminals A & B.

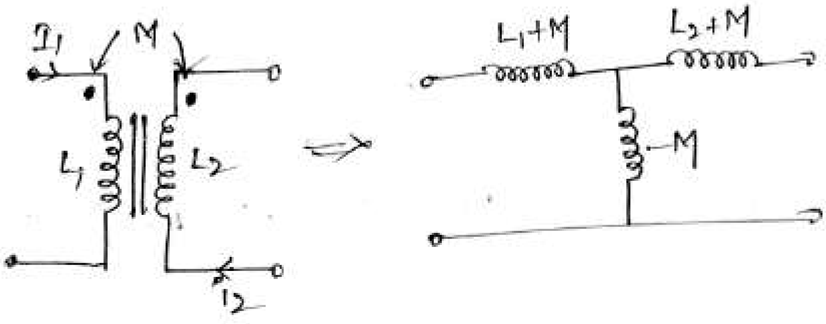


Soln:-



$L_{eq} = (L_1 - M) + \frac{M(L_2-M)}{L_2 - M + M}$

$\boxed{L_{eq} = L_1 - \frac{M^2}{L_2}}$



$L_{eq} = (L_1+M) + \frac{(L_2+M)(-M)}{L_2+M-M}$

$= \frac{(L_1+M)L_2 + [-L_2M - M^2]}{L_2}$

$\boxed{L_{eq} = L_1 - \frac{M^2}{L_2}}$

Impedance Transformation :—

The impedance transformation approach is used to match the differing impedance values of source and load to transfer maximum amount of source power to the load according to the maximum power transfer theorem. Various impedance transformation approaches are discussed as follows.

(i) Transformation of impedances with tapped resonant circuits:

→ A parallel LC circuit represents a resistance at anti-resonance and may therefore be used as a power-absorbing load on a generator of large internal resistance.

The value of anti-resonant resistance R_{ar} is dependent on the L/C ratio chosen for the circuit

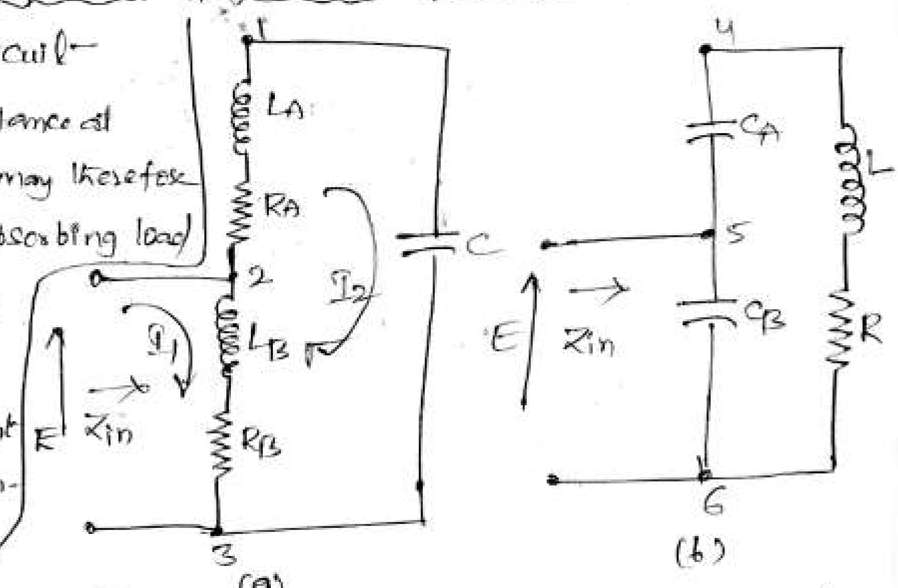


fig: forms of tapped antiresonance circuits for impedance transformation.

→ The mutual inductance between L_A and L_B is such that the total inductance is

$$\boxed{L = L_A + L_B + 2M} \longrightarrow (1)$$

Then anti-resonance will be found to occur between terminals 1 and 3. if

$$\omega(L_A + L_B + 2M) = \frac{1}{\omega C} \longrightarrow (2)$$

considering the terminals 2,3 of (a), antiresonance will occur if

$$\omega(L_B + M) = \frac{1}{\omega C} - \omega(L_A + M) \longrightarrow (3)$$

The circuit equations for figure (a) circuit may be written in the form

$$\begin{bmatrix} E \\ 0 \end{bmatrix} = \begin{bmatrix} R_B + jX_{LB} & -(R_B + jX_{LB} + jX_M) \\ -(R_B + jX_{LB} + jX_M) & R_A + R_B + jX_{LA} + jX_{LB} + j2X_M - jX_C \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

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and Since $Z_{in} = Z_{2,3} = \frac{\Delta}{\Delta_1}$, then

$$Z_{2,3} = \frac{(R_B+jX_{LB})(R_A+R_B+jX_{LA}+jX_{LB}+j2X_M-jX_c) - (R_B+jX_{LB}+jX_M)^2}{R_A+R_B+jX_{LA}+jX_{LB}+j2X_M-jX_C}$$

In view of equation (1), and letting $R = R_A+R_B$,

$$Z_{2,3} = R_B+jX_{LB} - \frac{(R_B+jX_{LB}+jX_M)^2}{R+j(X_L-X_C)} \longrightarrow (4)$$

If the circuit is antiresonant and of reasonable Q, then $X_L = X_C$; also $X_{LB} >> R_B$, and R_B may be dropped, giving

$$Z_{23} = jX_{LB} + \frac{(X_{LB}+X_M)^2}{R} \longrightarrow (5)$$

Here the magnitude of the first term will be small with respect to the second, for reasonable Q values, and the reactive term may be dropped. If the value of impedance Z_{28} is then compared with the resonant impedance across the 1,3 terminals, where

$$Z_{13} = \frac{R^2+\omega^2L^2}{R} = \frac{R^2+X_L^2}{R}$$

then

$$\frac{Z_{23}}{Z_{13}} = \frac{(X_{LB}+X_M)^2}{R^2+X_L^2} \longrightarrow (6)$$

Again, $R << X_L$, for reasonable Q, and

$$\frac{Z_{23}}{Z_{13}} = \frac{(X_{LB}+X_M)^2}{X_L^2} = \frac{(L_B+M)^2}{L^2} \longrightarrow (7)$$

for values of L_B considerably greater than M, the effect of tapping down on the inductance varies as the square of the fraction of the inductance across which the generator is connected. This would make the impedance vary approximately as the fraction of the total turns.

If the circuit capacitance is split into two capacitors in series, equivalent in capacitance to the single capacitor C and if the external generator is tapped between the two capacitors, and since $\omega L = \frac{1}{\omega C}$,

$$Z_{56} = \frac{(X_{C_2})^2}{R} \quad \text{and} \quad Z_{46} = \frac{(X_{C_1}+X_{C_2})^2}{R}$$

The effect of tapping down on the capacitive side of the circuit is then given by,

$$\boxed{\frac{Z_{56}}{Z_{46}} = \frac{X_{C_2}^2}{(X_{C_1}+X_{C_2})^2} = \frac{C_1^2}{(C_1+C_2)^2}} \longrightarrow (8)$$

also showing a reduction of impedance. These methods became Very convenient at high frequencies.

(ii) Reactance L-Sections for Impedance Transformation:

Two reactances of opposite sign may be arranged to transform at one frequency a load resistance R to provide a desired load R_{in} for the generator, where $R < R_{in}$. Such a reactance circuit as that between the terminals a,b and c,d as shown in figure below, is called an "L-section" because of its appearance when drawn in the circuit diagram.

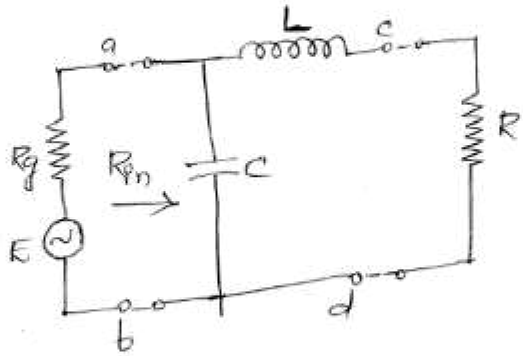


fig 1: Reactance L-Section for impedance Transformation ($R < R_g$)

The value of the load resistance is a function of the L/C ratio chosen for the reactance matching section.

The conditions to be realised are that the circuit to the right of a,b be in antiresonance and have an antiresonant impedance equal to R_{in}. These conditions may be expressed by

Angular frequency $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \longrightarrow$ (1)

$$R_{ar} = R_{in} = \frac{L}{CR} \longrightarrow (2)$$

Where $R_{ar} \rightarrow$ Antiresonance resistance

from eqn (2), $\boxed{L = R_{in} RC}$

which is inserted in eqn (1) leads to

$$\omega^2 = \frac{1}{R_{in} RC^2} - \frac{1}{R_{in}^2 C^2} \Rightarrow \omega C = \sqrt{\frac{R_{in} - R}{R_{in}^2 R}}$$

which gives for the value of capacitance 'C' needed for the L-section.

$$\boxed{C = \frac{1}{WR_{in}}\sqrt{\frac{R_{in}}{R} - 1}} \longrightarrow (3)$$

Similarly from equation (2), $\boxed{C = \frac{L}{R_{in}R}}$

from equation (1),

$$W^2 = \frac{R_{in}R}{L^2} - \frac{R^2}{L^2}, \quad WL = \sqrt{R_{in}R - R^2}$$

$$\boxed{L = \frac{R}{W}\sqrt{\frac{R_{in}}{R} - 1}} \longrightarrow (4)$$

where 'L' is the value of inductance needed for the L-section to ensure the desired value of load R_{in}, where $R < R_{in}$.

⇒ for the case in which $R > R_g$, the L-section may be reversed, as shown in figure 2. It can then be recognised that the equations just developed will apply if R_{in} is substituted for 'R' and 'R' for R_{in}.

If $R > R_g$, the transforming L-section should containing the following components:

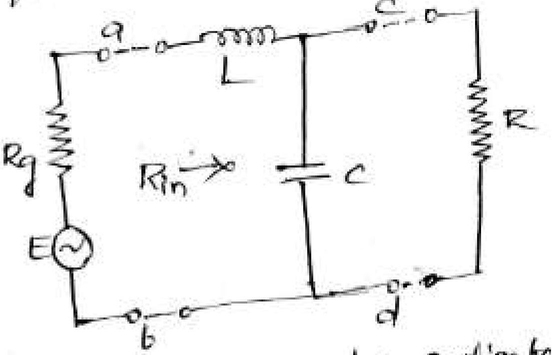


fig. 2: The reversed L-section for impedance transformation ($R > R_g$)

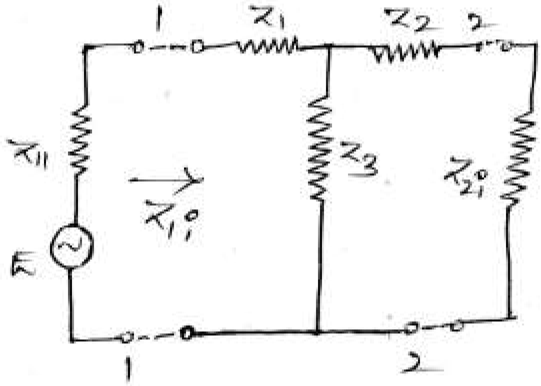
$$\boxed{C = \frac{1}{WR}\sqrt{\frac{R}{R_{in}} - 1}} \longrightarrow (5)$$

$$\boxed{L = \frac{R_{in}}{W}\sqrt{\frac{R}{R_{in}} - 1}} \longrightarrow (6)$$

The value of 'R_{in}' normally being equal to 'R_g' for matched conditions.

(iii) <u>Reactance Matching</u>:—

Consider a T-section of impedances interposed between a generator having internal impedance Z_{1i} and a load impedance Z_{2i}, as shown in figure.



It is desired that the impedance at the 11 terminals, into which the generator supplies power, be equal to the generator impedance, and that the

impedance looking into the 22 terminals be equal to the load Z_{2i}. H
Here Z_{1i} and Z_{2i} are the image impedances because the impedance looking in one-direction is the same as that looking in the other.

The impedance Z_{1in} at the 11 terminals is required to be Z_{1i} and is

$$Z_{1in} = Z_{1i} = Z_1 + \frac{Z_3(Z_2+Z_{2i})}{Z_2+Z_3+Z_{2i}} \quad , \text{Similarly } Z_{2i} = Z_2 + \frac{Z_3(Z_1+Z_{1i})}{Z_1+Z_{1i}+Z_3}$$

upon solving for Z_{1i} and Z_{2i},

$$Z_{1i} = \sqrt{\frac{(Z_1+Z_3)(Z_1Z_2+Z_2Z_3+Z_3Z_1)}{Z_2+Z_3}}, \quad Z_{2i} = \sqrt{\frac{(Z_2+Z_3)(Z_1Z_2+Z_2Z_3+Z_3Z_1)}{Z_1+Z_3}}$$

Making an impedance measurement on the T-section at the 11 terminals with the 22 terminals open gives

$$Z_{1oc} = Z_1 + Z_3$$

A Similar measurement at the 11 terminals with the 22 terminals short circuited gives

$$Z_{1sc} = Z_1 + \frac{Z_2Z_3}{Z_2+Z_3} = \frac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_2+Z_3}$$

$$\therefore \boxed{Z_{1i} = \sqrt{Z_{1oc} * Z_{1sc}}}, \quad \text{similarly, } \boxed{Z_{2i} = \sqrt{Z_{2oc} * Z_{2sc}}}$$

These equations show that Z_{1i} and Z_{2i} are obtained by simple open- and short-circuit measurements on any network.

Thus, a properly designed T-network may have the property of transformation of an impedance to produce matching of a load and source.

→ If, in a network of pure reactances, terminated in a dissipative load and supplied from a generator having an internal resistance, a conjugate impedance match occurs at one pair of terminals the impedances will have a conjugate match at every other pair of terminals.

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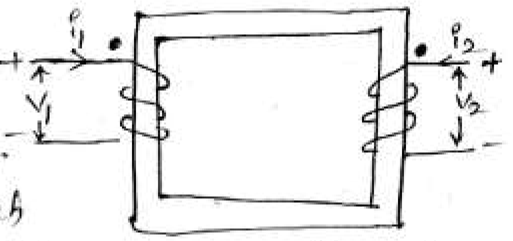
Ideal Transformers:-

Transformers are used to transfer of energy from one Circuit to another Circuit through Mutual induction. Most often, they transform energy at one Voltage (or Current) into energy at some other voltage (or Current).

A transformer has two or more windings or coils arranged on a common magnetic core. The transformer winding to which the supply source is connected is called the "primary winding", while the winding connected to load is called the "secondary winding". Accordingly, the voltage across the primary is called the "primary Voltage", and that across the secondary, the "secondary Voltage".

The vertical lines between the coils represent the iron core, the currents are assumed such that the mutual inductance is positive.



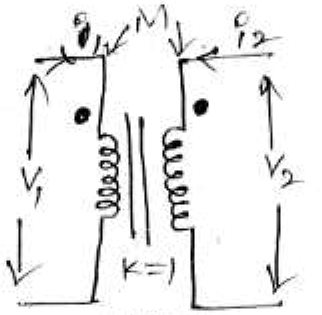


figure 1

→ An ideal transformer is characterised by assuming

(i) zero power dissipation in the primary and secondary windings, i.e. resistances in the coils are assumed to be zero,

(ii) the self inductances of the primary and secondary are extremely large in comparison with the load impedance, and

(iii) the coefficient of coupling is equal to unity, i.e. the coils are tightly coupled without having any leakage flux.

If the flux produced by the current flowing in a coil links all the turns, the self inductance of either the primary or secondary coil is proportional to the square of the number of turns of the coil.

The magnitude of the self-induced emf is given by

$$V = L\frac{di}{dt} \rightarrow (1)$$

If the flux linkages of the coil with N turns and current are known, then the self-induced emf can be expressed as

$$V = N\,d\phi/dt \longrightarrow (2)$$

from ① & ②, $L\frac{di}{dt} = N\frac{d\phi}{dt} \Rightarrow \boxed{L = N\frac{d\phi}{di}} \longrightarrow (3)$

But $\phi = \frac{Ni}{\text{reluctance}}$

$$\therefore L = N\frac{d\phi}{di} = N\frac{d}{di}\left[\frac{Ni}{\text{reluctance}}\right] = \frac{N^2}{\text{reluctance}}$$

$$L \propto N^2$$

from the above relation, it follows that

$$\boxed{\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2} \longrightarrow (4)$$

where $N_2/N_1 = a$ is called the "turns ratio" of the transformer. The turns ratio, a, can be expressed in terms of primary and secondary voltages.

→ If ϕ is the flux through a single turn coil on the core and N_1, N_2 are the number of turns of the primary and secondary, respectively, then the total flux through windings 1 and 2, respectively are

$$\phi_1 = \phi\,[N_1]; \quad \phi_2 = N_2\phi$$

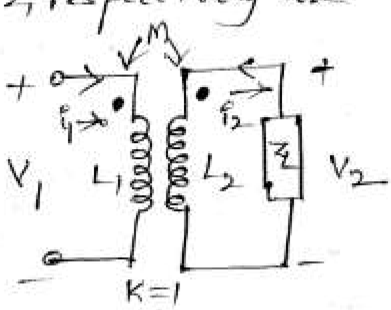


figure 2

Also, we have $V_1 = \frac{d\phi_1}{dt}$ and $V_2 = \frac{d\phi_2}{dt}$

so that,

$$\boxed{\frac{V_2}{V_1} = \frac{N_2\frac{d\phi}{dt}}{N_1 d\phi/dt} = \frac{N_2}{N_1}} \longrightarrow (5)$$

figure shows an ideal transformer to which the secondary is connected to a load impedance Z_L. The turns ratio $\frac{N_2}{N_1} = a$.

→ Let us analyze the ideal transformer with sinusoidal excitations. When the excitations are sinusoidal voltages or currents, the steady state response will also be sinusoidal. We can use phasor for represent these voltages and currents.

→ The input impedance of the transformer can be determined by

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \longrightarrow (6)$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 \longrightarrow (7)$$

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V_1, V_2 are the voltage phasors, and I_1 and I_2 are the current phasors in the two windings.

from eqn (7), $I_2 = \dfrac{j\omega M I_1}{(Z_L + j\omega L_2)}$

Substituting eqn (8) in eqn (6), we have

$$V_p = I_1 j\omega L_1 + \frac{I_1 \omega^2 M^2}{Z_L + j\omega L_2}$$

The input impedance $Z_{in} = V_1/I_1$

$$\therefore Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{(Z_L + j\omega L_2)}$$

When the coefficient of coupling is assumed to be equal to unity.

$$M = \sqrt{L_1 L_2}$$

$$\therefore Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{(Z_L + j\omega L_2)}$$

Since $\dfrac{L_2}{L_1} = a^2$ where 'a' is the transformer turns ratio

$$\therefore Z_{in} = j\omega L_1 + \frac{\omega^2 L_1^2 a^2}{Z_L + j\omega L_2}$$

further simplification leads to

$$Z_{in} = \frac{(Z_L + j\omega L_2)(j\omega L_1) + \omega^2 L_1^2 a^2}{(Z_L + j\omega L_2)} = \frac{j\omega L_1 Z_L}{(Z_L + j\omega L_2)}$$

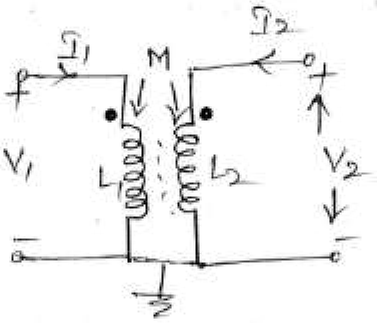
As L_2 is assumed to be infinitely large compared to Z_L.

$$Z_{in} = \frac{j\omega L_1 Z_L}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = \left(\frac{N_1}{N_2}\right)^2 Z_L.$$

→ The ideal transformer changes the impedance of a load, and can be used to match the circuits with different impedances in order to achieve maximum power transformer.

Equivalent T and π networks for Magnetically Coupled Circuits:-

Transformers works on Mutual induction principle to transfer energy from one winding to another winding connected to the same core.



Assume that there is a common ground connected between two windings for better understanding.

By applying KVL in both the loops,

$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Since $d/dt = j\omega$ in phasor representation.

$$\therefore V_1 = j\omega L_1 i_1 + j\omega M i_2 \quad , \quad V_2 = j\omega L_2 i_2 + j\omega M i_1$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longrightarrow (1)$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{(j\omega L_1)(j\omega L_2) - (j\omega)^2 M^2} \begin{bmatrix} j\omega L_2 & -j\omega M \\ -j\omega M & j\omega L_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \frac{1}{(j\omega)^2[L_1L_2 - M^2]} \begin{bmatrix} j\omega L_2 & -j\omega M \\ -j\omega M & j\omega L_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{L_2}{j\omega[L_1L_2 - M^2]} & \frac{-M}{j\omega[L_1L_2 - M^2]} \\ \frac{-M}{j\omega[L_1L_2 - M^2]} & \frac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \longrightarrow (2)$$

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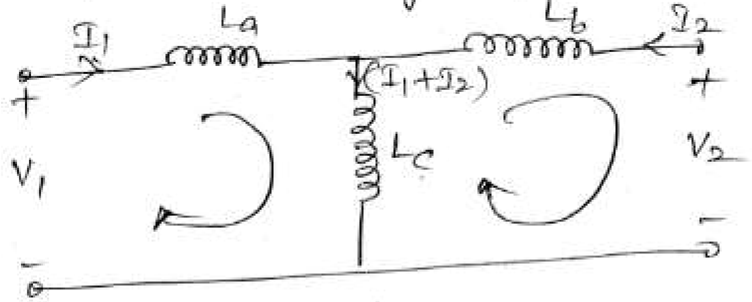
The equivalent T-model of the two inductively coupled circuits is



By KVL,

$$V_1 = L_a\frac{di_1}{dt} + L_c\frac{d(i_1+i_2)}{dt}, \quad V_2 = L_b\frac{di_2}{dt} + L_c\frac{d(i_1+i_2)}{dt}$$

Since $d/dt = j\omega$

$$\therefore V_1 = j\omega L_a i_1 + j\omega L_c i_1 + j\omega L_c i_2 \qquad V_2 = j\omega L_b i_2 + j\omega L_c(i_1+i_2)$$

$$V_1 = j\omega[L_a+L_c)i_1 + j\omega L_c i_2 \qquad V_2 = j\omega L_c i_1 + j\omega[L_b+L_c)i_2$$

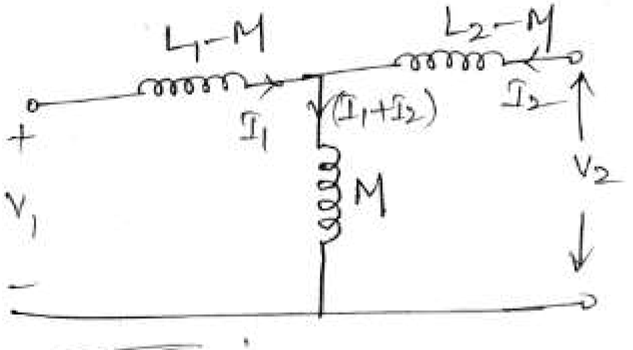
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a+L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b+L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longrightarrow (3)$$

By comparing (1) & (3),

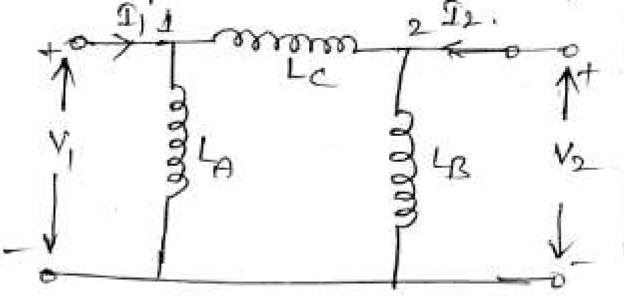
$$L_1 = L_a + L_c, \quad L_c = M, \quad L_b + L_c = L_2$$

$$\Rightarrow \boxed{L_a = L_1 - M \quad L_b = L_2 - M, \quad L_c = M}$$

$\therefore$



$\Rightarrow$ The equivalent π-model of the two inductively coupled circuits is



By applying nodal analysis at nodes (1) & (2),

$$I_1 = \frac{V_1 - V_2}{d/dt(L_C)} + \frac{V_1}{d/dt(L_A)}$$

$$I_2 = \frac{V_2 - V_1}{d/dt(L_C)} + \frac{V_2}{d/dt(L_B)}$$

Since $d/dt = j\omega$ in phasor representation.

$$I_1 = \frac{V_1 - V_2}{j\omega L_C} + \frac{V_1}{j\omega L_A}, \quad I_2 = \frac{V_2 - V_1}{j\omega L_C} + \frac{V_2}{j\omega L_B}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_C} + \frac{1}{j\omega L_A} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \longrightarrow (4)$$

By comparing equations (2) & (4).

$$\frac{L_2}{\cancel{j\omega}[L_1L_2 - M^2]} = \frac{1}{\cancel{j\omega}}\left[\frac{1}{L_C} + \frac{1}{L_A}\right] \Rightarrow \frac{L_2}{L_1L_2 - M^2} = \frac{1}{L_A} + \frac{1}{L_C} \longrightarrow (5)$$

$$\frac{\cancel{-}M}{\cancel{j\omega}(L_1L_2 - M^2)} = \cancel{-}\frac{1}{\cancel{j\omega}L_C} \Rightarrow \frac{M}{(L_1L_2 - M^2)} = \frac{1}{L_C} \Rightarrow \boxed{L_C = \frac{L_1L_2 - M^2}{M}} \longrightarrow (6)$$

$$\frac{L_1}{\cancel{j\omega}[L_1L_2 - M^2]} = \frac{1}{\cancel{j\omega}}\left[\frac{1}{L_B} + \frac{1}{L_C}\right] \Rightarrow \frac{L_1}{L_1L_2 - M^2} = \frac{1}{L_B} + \frac{1}{L_C} \longrightarrow (7)$$

from eqns (5) & (6).

$$\frac{1}{L_A} + \frac{1}{L_C} = \frac{L_2}{L_1L_2 - M^2} \Rightarrow \frac{1}{L_A} = \frac{L_2}{L_1L_2 - M^2} - \frac{M}{L_1L_2 - M^2}$$

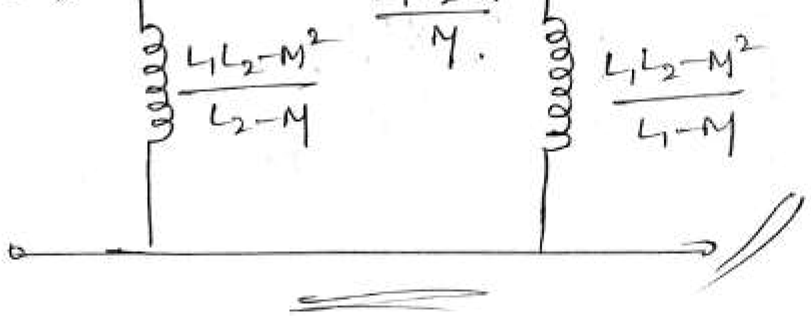
$$\therefore \frac{1}{L_A} = \frac{L_2 - M}{L_1L_2 - M^2} \Rightarrow \boxed{L_A = \frac{L_1L_2 - M^2}{L_2 - M}}$$

from (6) & (7)

$$\frac{1}{L_B} + \frac{1}{L_C} = \frac{L_1}{L_1L_2 - M^2} \Rightarrow \frac{1}{L_B} + \frac{M}{L_1L_2 - M^2} = \frac{L_1}{L_1L_2 - M^2}$$

$$\Rightarrow \frac{1}{L_B} = \frac{L_1 - M}{L_1L_2 - M^2} \Rightarrow \boxed{L_B = \frac{L_1L_2 - M^2}{L_1 - M}}$$

$\therefore$



Network Analysis and Transmission Lines

Unit-II

by

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Unit-II (1)

UNIT-II

Transient and Steady State Response:—

A Circuit having Constant Sources is said to be in steady state if the Currents and Voltages do not change with time.

→ The behaviour of the Voltage or Current when it is changed from one state to other state is called the "transient state". Also defin as the time taken for the Circuit to change from one steady stat to another steady state. ~~It's called~~.

→ The application of KVL and KCL to Circuits containing energy storag elements results in differential, rather than algebraic equations.

→ When we Consider a circuit containing storage elements which are independent of the Sources, then the response depends upon the nature of the circuit and is called the "Natural Response". Storage elements deliv their energy to the resistances. Hence the response changes with time, gets saturated after some time, and is referred to as the "Transient Response".

→ When the Sources acting on a Circuit, the response depends on the nature of the Source. This response is called "forced Response".

→ Complete response of the Circuit consists of two parts: the forced response and the transient response.

→ The Complete response of a differential equation comprises of two parts: complementary function and the particular Solution.

→ Complementary function is referred to as the "Transient" or "Source-free" response.

→ The particular Solution is the steady state response or forced response.

DC - Response of an R-L Circuit :-

The inductor in the circuit is initially uncharged and is in series with the resistor, as shown in figure 2.1.

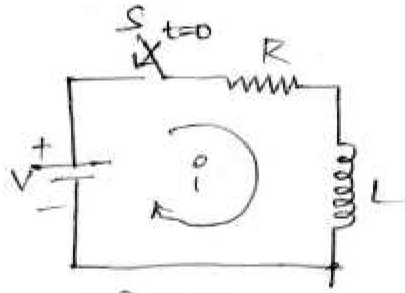


fig. 2.1

→ When the switch 'S' is closed, it is possible to find the complete solution for the current.

By KVL,

$$V = Ri + L\frac{di}{dt} \longrightarrow (1)$$

(or)

$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}} \longrightarrow (2)$$

Here, current 'i' is the solution to be found and 'V' is the applied constant voltage. The voltage is applied to the circuit when switch is closed.

from (2), $\left(D + \frac{R}{L}\right)i = V/L$

It is the linear differential equation of first order, then the solution is

$$i = ce^{-(R/L)t} + e^{-(R/L)t}\int \frac{V}{L} e^{(R/L)t}\,dt$$

$$= ce^{-\frac{R}{L}t} + e^{-\frac{R}{L}t} * \frac{V}{L} \cdot \frac{e^{\frac{R}{L}t}}{R/L}$$

$$i = ce^{-\frac{R}{L}t} + \frac{V}{L} * \frac{L}{R}\, e^{-\frac{R}{L}t + \frac{R}{L}t}$$

$$\boxed{i = ce^{-(R/L)t} + \frac{V}{R}} \longrightarrow (3)$$

Note :-

$$\frac{dx}{dt} + px = K$$

Solution is

$$x = ce^{-pt} + e^{-pt}\int K e^{pt}\,dt$$

To determine the value of 'c', initial conditions are used.

In the circuit, the switch 'S' is closed at $t=0$. At $t=0^-$ i.e., just before closing the switch 'S', the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at $t=0^+$ just after the switch is closed, the current remains zero.

Thus at, $t=0, i=0$

substituting in eqn ③, we have

$$0 = C + V/R \Rightarrow \boxed{C = -V/R}$$

substituting the value of 'c' in eqn ③, we get

$$i = \frac{V}{R} - \frac{V}{R} e^{-[R/L]t} \Rightarrow i = \frac{V}{R}\left[1 - e^{-(R/L)t}\right] \longrightarrow ④$$

Equation ④ consists of two parts, the steady state part V/R, and the transient part $\frac{V}{R} e^{-\frac{R}{L}t}$.

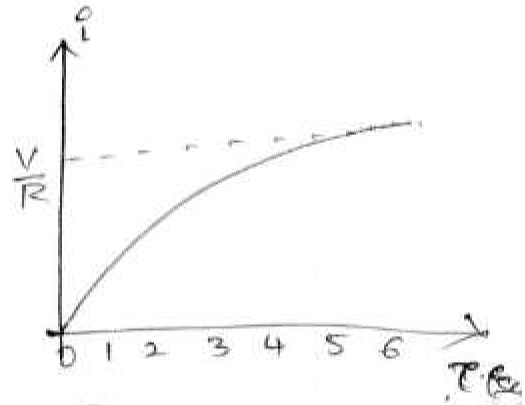


Fig. 2(2)

When switch 's' closed, the response reaches a steady state value after a time interval.

→ the time taken for the current to reach its final or steady state value from its initial value.

Time constant (τ) is the time required for the current to reach from initial value of zero to the final value V/R.

$$\tau = L/R \text{ sec.}$$

∴ The transient part of the solution is

$$i = -\frac{V}{R} e^{(-R/L)t} = -\frac{V}{R} e^{-t/\tau}$$

At one time constant, the transient reaches 36.8% of its initial value.

$$i(\tau) = -\frac{V}{R} e^{-t/\tau} = -\frac{V}{R} e^{-1} = -0.368\ V/R$$

Similarly, $i(2\tau) = -\frac{V}{R} e^{-2} = -0.135\ V/R$

$$i(3\tau) = -V/R\ e^{-3} = -0.0498\ V/R$$

$$i(5\tau) = -V/R\ e^{-5} = -0.0067\ V/R$$

After 5τ, the transient part reaches more than 99% of its final value

Voltage across the resistor is

$$V_R = Ri = R \times \frac{V}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right] = V\left[1-e^{-\frac{R}{L}t}\right]$$

Similarly, the voltage across the inductance is

$$V_L = L\,di/dt = \frac{LV}{R} \times \frac{R}{L} e^{\left[-\frac{R}{L}t\right]} = Ve^{-\frac{R}{L}t}$$

Power in the resistor is

$$P_R = V_R i = V\left[1-e^{-\frac{R}{L}t}\right]\left[1-e^{-\frac{R}{L}t}\right] V/R = \frac{V^2}{R}\left[1-2e^{-\frac{R}{L}t} + e^{-\frac{2R}{L}t}\right]$$

Power in the inductor is

$$P_L = V_L i = V e^{-\frac{R}{L}t} \times \frac{V}{R}\left[1-e^{-\frac{R}{L}t}\right] = \frac{V^2}{R}\, e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}.$$

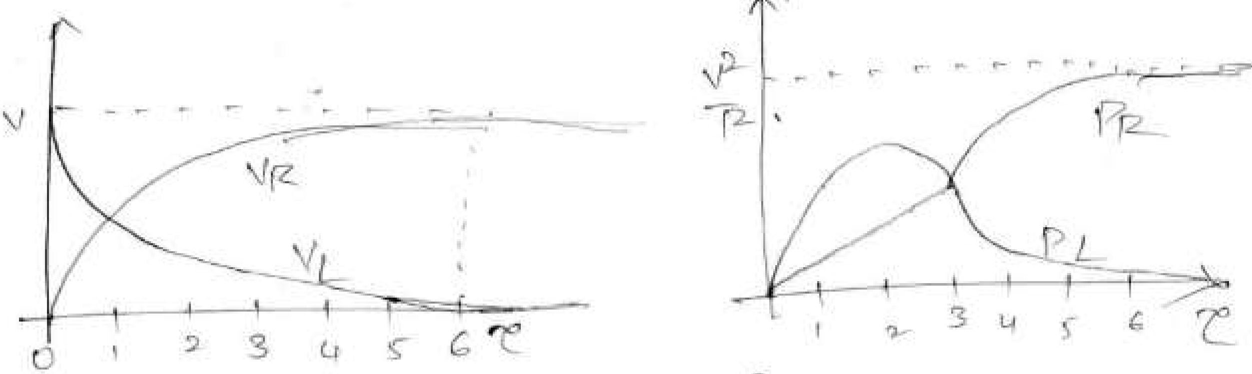


fig. 2(3)

DC Response of an RC-Circuit :

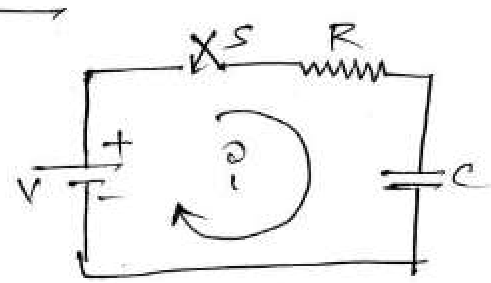


fig. 2(4)

When the switch is closed at $t=0$, it is possible to determine the complete solution for the current.

By KVL,

$$V = Ri + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

By differentiating with respect to 't', we get

$$0 = R\frac{di}{dt} + \frac{i}{C} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{RC}\, i = 0 \longrightarrow (2)$$

Eqn (2) is a linear differential equation with only the complementary function. The particular solution for the above equation is zero. Therefore, the solution is

$$\boxed{i = Ce^{-t/RC}} \longrightarrow (3)$$

U-II (3)

To find the value of c in eqn(3), initial conditions are applied.

In the circuit, the switch 'S' is closed at $t=0$. Since the capacitor never allows sudden changes in voltage, it will act as a short-circuit at $t=0^+$. So, the current in the circuit at $t=0^+$ is V/R.

At $t=0$, the current $i = V/R$

from eqn(3), we get

$$\frac{V}{R} = c$$

$\therefore$ The current equation becomes

$$\boxed{i = \frac{V}{R} e^{-t/RC}}$$

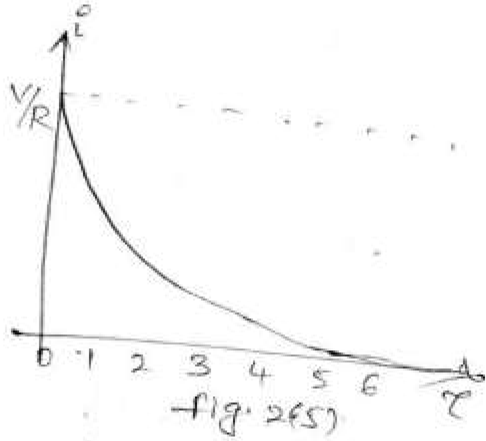


fig. 2(5)

When switch 'S' is closed, the response decays with time as shown in fig. 2(5).

The quantity RC is the time constant, and is denoted by τ.

where $\tau = RC$ sec.

→ After 5τ, the curve reaches 99% of its final value.

The voltage across the resistor is

$$V_R = Ri = R \times \frac{V}{R} e^{-(1/RC)t} \quad ; \quad V_R = V e^{-t/RC}$$

Similarly, the voltage across the capacitor is

$$V_C = \frac{1}{C}\int i\, dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC}\, dt$$

$$V_C = -\left(\frac{V}{RC} \times RC\, e^{-t/RC}\right) + c = -V e^{-t/RC} + c$$

At $t=0$, voltage across the capacitor is zero since capacitor doesnot allow sudden change of voltg

$$0 = -V + c \Rightarrow \boxed{c = V}$$

$$\therefore V_C = V - V e^{-t/RC} = V\left[1 - e^{-t/RC}\right]$$

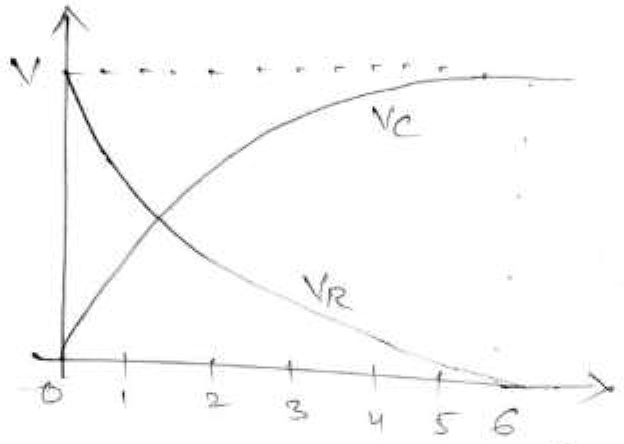
Power in the capacitor

$$P_C = V_C i = \cancel{V_C(1-e^{-t/RC})}\; V e^{-t/RC} \times \frac{V}{R} e^{-t/RC}$$

$$\boxed{P_C = \frac{V^2}{R} e^{-2t/RC}}$$

power in the capacitor.

$$P_C = V_C i = V\left[1-e^{-t/RC}\right]\left[\frac{V}{R}e^{-t/RC}\right] = \frac{V^2}{R}\left[e^{-t/RC} - e^{-2t/RC}\right]$$



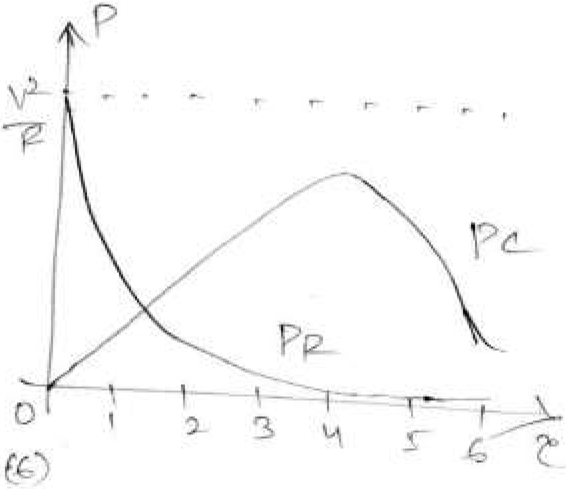


Figure 2(6)

DC Response of an RLC Circuit :-

The capacitor and inductor are initially uncharged, and are in series with a resistor. The complete solution for the current can be determined when the switch 's' is closed at $t=0$.

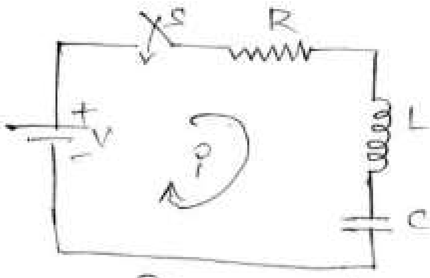


Figure 2(7).

By KVL, $V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$

By differentiating w.r.t. 't' the above equation, we have

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

or $\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0} \longrightarrow (2)$

The above equation is a second order differential equation, with only complementary function.

The particular solution for the above equation is zero. characteristic equation for the above differential equation is

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) = 0 \longrightarrow (3)$$

The roots for the above equation are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

By assuming $K_1 = -\frac{R}{2L}$ and $K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

NOTE U-II (H)

$D_1 = K_1 + K_2$ and $D_2 = K_1 - K_2$

Here K_2 may be positive, negative or zero.

⇒ K_2 is positive, when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal, and give the over damped response as shown in figure 2(7). The equation ② becomes

$$[D-(K_1+K_2)][D-(K_1-K_2)]i = 0$$

The solution for the above equation is

$$i = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t}$$

The current curve for the overdamped case is shown in figure 2(8).

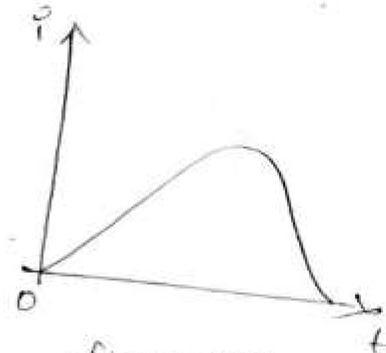


figure 2(8)

⇒ K_2 is negative, when $(R/2L)^2 < 1/LC$. The roots are complex conjugate, and give the underdamped response as shown in figure 2(9). Then equation ② becomes

$$[D-(K_1+jK_2)][D-(K_1-jK_2)]i = 0$$

The solution for the above equation is

$$i = e^{K_1 t}[C_1 \cos K_2 t + C_2 \sin K_2 t]$$

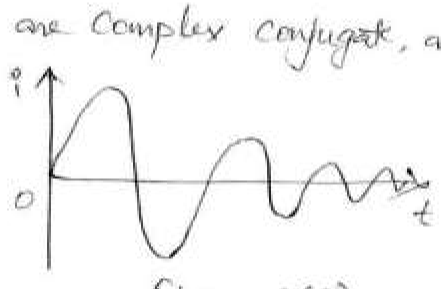


figure 2(9)

The current curve for the underdamped case is shown in figure 2(9).

⇒ K_2 is zero, when $(R/2L)^2 = 1/LC$

The roots are equal, and give the critically damped response as shown in figure 2(10).

Then eqn (2) becomes

$$(D-K_1)(D-K_1)i = 0$$

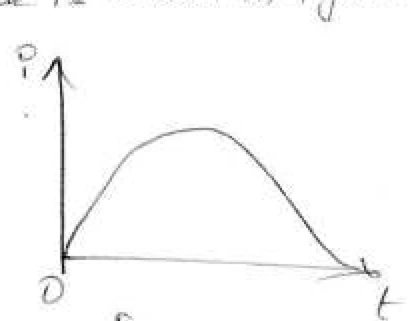


figure 2(10)

The solution for the above equation is

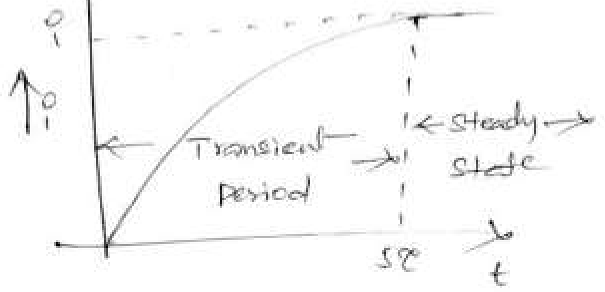
$$i = e^{K_1 t}(C_1 + C_2 t)$$

The current curve for the critically damped case is shown in fig. 2(

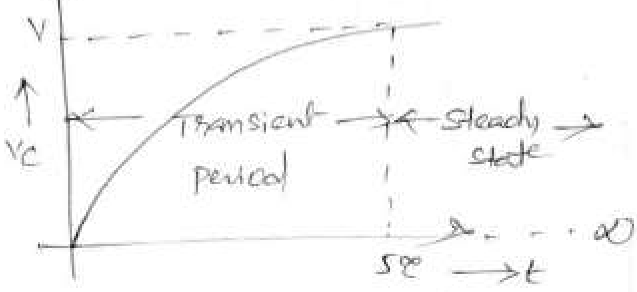
Time Constant :—

The time taken for the response to raise to 63.2% of the maximum value and it is given by

$$\tau = \frac{L}{R} \text{ sec} \qquad \tau = RC \text{ sec}.$$

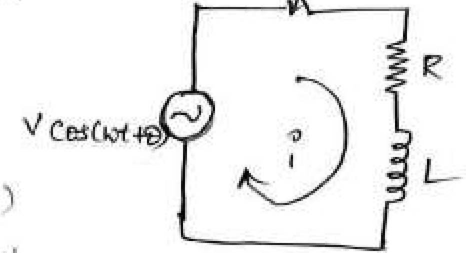


$$\boxed{i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$



$$\boxed{V_C = V\left(1 - e^{-t/RC}\right)}$$

Sinusoidal Response of RL Circuit :-



The Switch 'S' is closed at $t=0$. At $t=0$, a Sinusoidal voltage $V\cos(\omega t+\theta)$ is applied to the series RL Circuit, where 'V' is the amplitude and 'θ' is the phase angle.

By applying KVL,

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt} \Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta) \longrightarrow (1)$$

The corresponding characteristic equation is

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L}\cos(\omega t+\theta) \longrightarrow (2)$$

The complete Solution Consists of two parts, viz. Complementary function and particular integral.

The Complementary function of the solution 'i' is

$$\boxed{i_c = c\,e^{-t(R/L)}} \longrightarrow (3)$$

The particular Solution can be obtained by using undetermined Coefficients.

Assume

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \longrightarrow (4)$$

therefore, $$\frac{di}{dt} = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) \longrightarrow (5)$$

Substituting equations (4) & (5) in equation (2), we get

$$-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) + \frac{R}{L}\left\{A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right\} = \frac{V}{L}\cos(\omega t+\theta)$$

NATL unit-II (5)

(or)

$$\left[-A\omega+\frac{BR}{L}\right]\sin(\omega t+\theta)+\left[B\omega+\frac{AR}{L}\right]\cos(\omega t+\theta)=\frac{V}{L}\cos(\omega t+\theta)$$

Comparing sine and cosine terms on both sides.

$$-A\omega+\frac{BR}{L}=0 \Rightarrow A=\frac{BR}{\omega L}$$

$$B\omega+\frac{AR}{L}=\frac{V}{L}$$

$$\Rightarrow B\omega+\left[\frac{BR}{\omega L}\right]\left[\frac{R}{L}\right]=\frac{V}{L}$$

$$\Rightarrow B\left[\omega+\frac{R^2}{\omega L^2}\right]=\frac{V}{L} \Rightarrow B\left[\frac{\omega^2L^2+R^2}{\omega L^2}\right]=\frac{V}{L}$$

$$\Rightarrow \boxed{B=V\frac{\omega L}{L^2\omega^2+R^2}}$$

Since $A=\frac{BR}{\omega L} \Rightarrow A=\left(\frac{V\omega L}{\omega^2L^2+R^2}\right)\frac{R}{\omega L}=\frac{VR}{R^2+(\omega L)^2}$

$$\therefore \boxed{A=V\frac{R}{(\omega L)^2+R^2}}, \quad \boxed{B=V\frac{\omega L}{(\omega L)^2+R^2}}$$

Substituting the values of 'A' and 'B' in equation (4), we get

$$i_p=\frac{VR}{(\omega L)^2+R^2}\cos(\omega t+\theta)+\frac{V\omega L}{(\omega L)^2+R^2}\sin(\omega t+\theta) \longrightarrow (6)$$

the particular solution for the above equation is

$$i_{p(find)}=\sqrt{\frac{V^2R^2}{[(\omega L)^2+R^2]^2}+\frac{V^2\omega^2L^2}{[(\omega L)^2+R^2]^2}}\cos\left(\omega t+\theta-\tan^{-1}\left[\frac{V\omega L/(\omega L)^2+R^2}{VR/(\omega L)^2+R^2}\right]\right)$$

$$=\sqrt{\frac{V^2[\omega^2L^2+R^2]}{[(\omega L)^2+R^2]^2}}\cos\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$\boxed{i_{p(find)}=\frac{V}{\sqrt{(\omega L)^2+R^2}}\cos\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)} \longrightarrow (7)$$

* the solution for $x=A\cos(\omega t+\theta)+B\sin(\omega t+\theta)$ is

$$x=\sqrt{A^2+B^2}\cos(\omega t+\theta-\tan^{-1}B/A)$$

The Complete solution for the Current $i = i_c + i_p$

$$\therefore i = Ce^{-t(R/L)} + \frac{V}{\sqrt{(\omega L)^2+R^2}} \cos\left[\omega t+\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

Since the inductor doesnot allow sudden changes in currents.

At $t=0$, $i=0$.

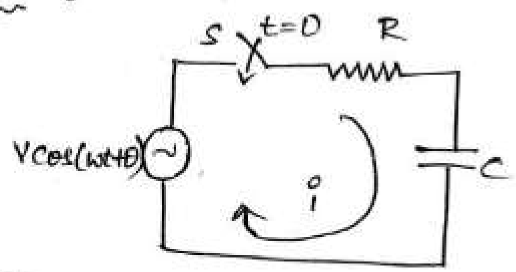
$$C = -\frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left(\theta - \tan^{-1}(\omega L/R)\right)$$

The complete solution for the Current is

$$i = e^{-\frac{R}{L}t}\left[\frac{-V}{\sqrt{R^2+(\omega^2 L^2)}} \cos\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)\right] +$$

$$\frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left(\omega t+\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

Sinusoidal Response of RC Circuit:—



At $t=0$, a sinusoidal voltage ~~across~~ $V\cos(\omega t+\theta)$ is applied to the RC circuit, where 'V' is the amplitude of the wave and 'θ' is the phase angle.

By KVL,

$$V\cos(\omega t+\theta) = Ri + \frac{1}{C}\int i\,dt \longrightarrow (1)$$ By differentiating eqn(1),

$$R\frac{di}{dt} + \frac{i}{C} = -V\omega \sin(\omega t+\theta)$$

$$\left(D+\frac{1}{RC}\right)i = -\frac{V\omega}{R}\sin(\omega t+\theta) \longrightarrow (2)$$

The complementary function, $i_c = Ce^{-t/RC} \longrightarrow (3)$.

The particular solution can be obtained by using undetermined coefficients,

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \longrightarrow (4)$$

$$i_p' = \frac{di_p}{dt} = -A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta) \longrightarrow (5)$$

By substituting equations (4),(5) in equation(2). we get

$$\left\{-A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta)\right\} + \frac{1}{RC}\left\{A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right\} = -\frac{V\omega}{R}\sin(\omega t+\theta) \longrightarrow (6)$$

NAT
Unit-II (6)

Comparing, Sine and Cosine terms on both Sides,

$$-A\omega + \frac{B}{RC} = \frac{-V\omega}{R} \longrightarrow 6(i) \qquad B\omega + \frac{A}{RC} = 0 \Rightarrow B = \frac{-A}{\omega RC} \longrightarrow 6(ii)$$

Substituting B in equation 6(i),

$$-A\omega + \frac{1}{RC}\left[\frac{-A}{\omega RC}\right] = \frac{-V\omega}{R} \Rightarrow A\left[\omega + \frac{1}{\omega R^2C^2}\right] = \frac{V\omega}{R} \Rightarrow A = \frac{V\omega}{R[\omega^2 R ...}$$

$$\therefore \boxed{A = \frac{VR\omega^2C^2}{1+\omega^2R^2C^2}} \text{ and } B = \frac{-V}{\omega C\left[R^2+(1/\omega C)^2\right]}$$

Substituting the values of A & B in equation (4), we have

$$i_p = \frac{VR\omega^2C^2}{1+\omega^2R^2C^2}\cos(\omega t+\theta) + \frac{-V}{\omega C\left[R^2+(1/\omega C)^2\right]}\sin(\omega t+\theta)$$

The final particular integral can be written as

$$i_p(\text{final}) = \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left(\omega t+\theta+\tan^{-1}(1/\omega CR)\right) \longrightarrow (7)$$

The complete solution for the current $i = i_c + i_p$

$$\therefore i = Ce^{-t/RC} + \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left[\omega t+\theta+\tan^{-1}(1/\omega CR)\right]$$

Since the capacitor doesnot allow sudden changes in voltage:

At $t=0$, $i = \frac{V}{R}\cos\theta$.

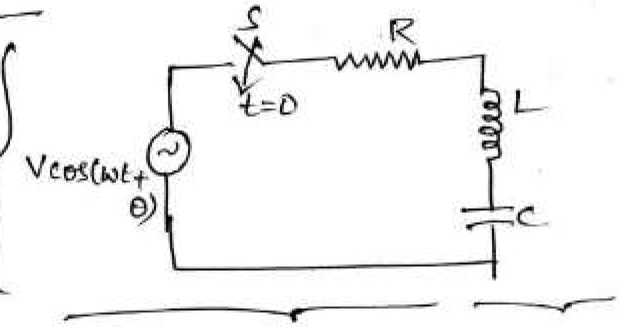
$$\therefore \frac{V}{R}\cos\theta = C + \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left[\theta+\tan^{-1}(1/\omega CR)\right]$$

$$\Rightarrow C = \frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left[\theta+\tan^{-1}(1/\omega CR)\right]$$

The complete solution for the current is

$$i = e^{-t/RC}\left[\frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left[\theta+\tan^{-1}(1/\omega CR)\right]\right] + \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left[\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right]$$

Sinusoidal Response of RLC Circuit :—



Consider a Circuit consisting of resistance, inductance and capacitance in series, as shown in figure.

Switch 's' is closed at t=0. At t=0, a sinusoidal Voltage $V\cos(\omega t+\theta)$ is applied to the RLC circuit. Where 'V' is the amplitude of the wave and 'θ' is the phase angle.

According to the KVL,

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

It is difficulty to get the solution with integral factor. Hence, the above equation is differentiated with respect to 't',

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = -V\omega\sin(\omega t+\theta)$$

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right]i = -\frac{V\omega}{L}\sin(\omega t+\theta) \longrightarrow (2)$$

The particular solution can be obtained by using undetermined coefficients.

Assume
$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \longrightarrow (3)$$

$$i_p' = \frac{di_p}{dt} = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) \longrightarrow (4)$$

$$i_p'' = \frac{di_p'}{dt} = -A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta) \longrightarrow (5)$$

Substituting equations (3), (4), (5) in eqn (2)

$$\left[-A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta)\right] + \frac{R}{L}\left[-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\right]$$
$$+\frac{1}{LC}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right] = -\frac{V\omega}{L}\sin(\omega t+\theta)$$

$$\Rightarrow \cos(\omega t+\theta)\left[-A\omega^2 + \frac{B\omega R}{L} + \frac{A}{LC}\right] + \sin(\omega t+\theta)\left[-B\omega^2 - \frac{A\omega R}{L} + \frac{B}{LC}\right]$$
$$= -\frac{V\omega}{L}\sin(\omega t+\theta) \longrightarrow (6)$$

Comparing sine and cosine terms on both sides.

Sine terms $\Rightarrow$ $-B\omega^2 - A\frac{\omega R}{L} + \frac{B}{LC} = -\frac{V\omega}{L} \Rightarrow A\left[\frac{\omega R}{L}\right] + B\left[\omega^2 - \frac{1}{LC}\right] = \frac{V\omega}{L} \rightarrow (7)$

NATL
Unit II (7)

Cosine terms ⇒

$$-AW^2 + BW\frac{R}{L} + \frac{A}{LC} = 0 \Rightarrow A\left[W^2 - \frac{1}{LC}\right] - B\left[\frac{WR}{L}\right] = 0 \longrightarrow (8)$$

By solving equations (7) & (8), we get — from (8) ⇒ $A = \frac{BWRC}{W^2LC-1}$

$$A = \frac{BWRC}{W^2LC-1}$$

By substituting 'A' in equation (7), we get —

$$\left(\frac{BWRC}{W^2LC-1}\right)\left[\frac{WR}{L}\right] + B\left[W^2 - \frac{1}{LC}\right] = \frac{VW}{L}$$

$$\Rightarrow B = \frac{\left(W^2 - \frac{1}{LC}\right)VW}{L\left[\left(\frac{WR}{L}\right)^2 - \left(W^2 - \frac{1}{LC}\right)^2\right]}$$

Since $A = \frac{BWRC}{W^2LC-1}$

hence, $A = \frac{VW^2R/L^2}{\left[\left(\frac{WR}{L}\right)^2 - \left(W^2 - \frac{1}{LC}\right)^2\right]}$

Substituting the values of 'A' & 'B' in eqn (3), we get —

$$i_p = \frac{VW^2R/L^2}{\left[\left(\frac{WR}{L}\right)^2 - \left(W^2 - \frac{1}{LC}\right)^2\right]}\cos(Wt+\theta) + \frac{\left(W^2 - \frac{1}{LC}\right)VW}{L\left[\left(\frac{WR}{L}\right)^2 - \left(W^2 - \frac{1}{LC}\right)^2\right]}\sin(Wt+\theta)$$

The particular solution becomes

$$\boxed{i_p = \frac{V}{\sqrt{R^2 + \left[\frac{1}{WC} - WL\right]^2}}\cos\left[Wt + \theta + \tan^{-1}\frac{\left[\frac{1}{WC} - WL\right]}{R}\right]}$$

⇒ To find the complementary function, we have the characteristic equation

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] = 0.$$

The roots for the equation are, $D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

By assuming $K_1 = -\frac{R}{2L}$, $K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$\therefore D_1 = K_1 + K_2$ and $D_2 = K_1 - K_2$.

Case (1):- K_2 is positive, when $(R/2L)^2 > 1/LC$

The roots are real and unequal. which gives an overdamped response.

$$[D-(K_1+K_2)][D-(K_1-K_2)]i=0$$

The complementary function for the above equation is

$$i_c = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t}$$

Therefore, the complete solution is

$$i_c = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \cos\left[\omega t+\theta+\tan^{-1}\left[\frac{1}{\omega CR}-\frac{\omega L}{R}\right]\right]$$

Case (2):-

K_2 is negative when $\left[\frac{R}{2L}\right]^2 < \frac{1}{LC}$

Then the roots are complex conjugate, which gives an underdamped response.

Therefore, the equation becomes

$$[D-(K_1+jK_2)][D-(K_1-jK_2)]i=0$$

The solution for the above equation is

$$i_c = e^{K_1 t}\left[C_1 \cos K_2 t + C_2 \sin K_2 t\right]$$

Therefore, the complete solution is

$$i = i_c + i_p$$

$$i = e^{K_1 t}\left[C_1 \cos K_2 t + C_2 \sin K_2 t\right] + \frac{V}{\sqrt{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}} \cos\left[\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}-\frac{\omega L}{R}\right)\right]$$

Case (3):- K_2 is zero when $\left[\frac{R}{2L}\right]^2 = 1/LC$

Then the roots are equal which gives critically damped response.

The equation becomes $(D-K_1)(D-K_1)i=0$

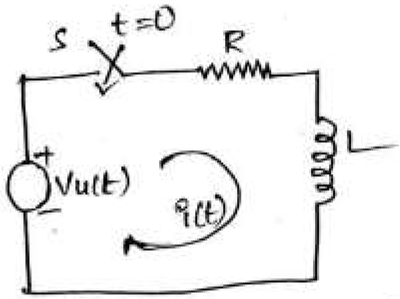
The complementary function is $i_c = e^{K_1 t}[C_1 + C_2 t]$

Therefore, the complete solution is $i = i_c + i_p$

$$\therefore i = e^{K_1 t}[C_1 + C_2 t] + \frac{V}{\sqrt{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}} \cos\left[\omega t+\theta+\tan^{-1}\left[\frac{1}{\omega CR}-\frac{\omega L}{R}\right]\right]$$

NATL Unit-II (8)

Step Response of RL Circuit :-



Series RL Circuit

To obtain the step response of the series RL Circuit, the input $Vu(t)$ is applied to the circuit.

By applying KVL,

$$Vu(t) = i(t)R + L\frac{di(t)}{dt}$$

Taking the Laplace transform on both sides, we get

$$\frac{V}{s} = RI(s) + L[sI(s) - i(0)]$$

The current in the inductor cannot change abruptly. Therefore

$$\frac{V}{s} = RI(s) + sLI(s) \Rightarrow \frac{V}{s} = (R+sL)I(s)$$

$$\therefore \boxed{I(s) = \frac{V}{s[R+sL]}}$$

$$\Rightarrow I(s) = \frac{V}{L}\left[\frac{1}{s[s+R/L]}\right] = \frac{V}{L}\cdot\frac{L}{R}\left[\frac{1}{s} - \frac{1}{(s+R/L)}\right]$$

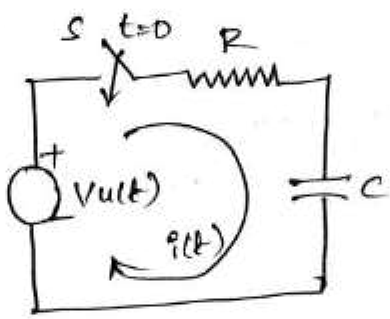
$$\Rightarrow I(s) = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

By applying inverse Laplace transform of the above equation

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]}$$

This is the step response of RL (series) Circuit.

Step Response of RC Circuit :-



To obtain the step response of the series RC Circuit, the input $Vu(t)$ is applied to the circuit.

By applying KVL,

$$Vu(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

This equation can be written as,

$$Vu(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

* Taking Laplace transform on both sides.

$$L\{Vu(t)\} = L[Ri(t)] + L\left[\frac{1}{C}\int_{-\infty}^{0} i(t)dt\right] + L\left[\frac{1}{C}\int_{0}^{t} i(t)dt\right]$$

$$\Rightarrow \frac{V}{s} = RI(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right] + \frac{1}{C}\left[\frac{q(0^+)}{s}\right]$$

where, $q(0^+)$ is the charge on the capacitor at $t=0^+$ i.e. it is the initial charge. If the initial conditions are neglected, then,

$$\frac{V}{s} = RI(s) + \frac{I(s)}{Cs} = I(s)\left[R + \frac{1}{Cs}\right]$$

Therefore, the current in the circuit is given by,

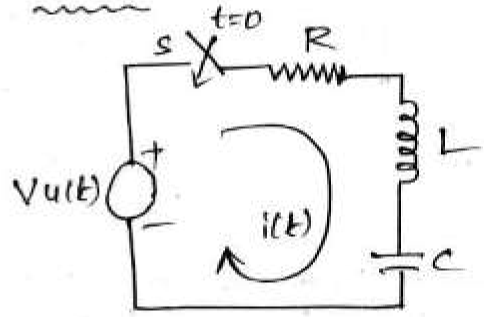
$$I(s) = \frac{V}{s\left[R + 1/Cs\right]} = \frac{V}{s}\left(\frac{sC}{sRC+1}\right) = \frac{VC}{RC\left[s + 1/RC\right]}$$

$$\therefore I(s) = \frac{V}{R}\left[\frac{1}{s + 1/RC}\right]$$

Taking the inverse Laplace transform on both sides. we get

$$\boxed{i(t) = \frac{V}{R} e^{-(1/RC)t}}$$

Step Response of Series RLC Circuit:-



To obtain the step response of series RLC circuit, the input $Vu(t)$ is applied to the circuit.

By KVL,

$$V u(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

$$\Rightarrow V u(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{t} i(t)dt$$

Taking Laplace transform of above equation on both sides, we get

$$L\{Vu(t)\} = L[Ri(t)] + L\left[L\frac{di(t)}{dt}\right] + L\left[\frac{1}{C}\int_{-\infty}^{0} i(t)dt\right] + L\left[\frac{1}{C}\int_{0}^{t} i(t)dt\right]$$

NATL
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$$\Rightarrow \frac{V}{s} = RI(s) + L\left[sI(s) - I(0^+)\right] + \frac{1}{C}L\left[q(0^+)\right] + \frac{1}{C}\frac{I(s)}{s}$$

Where $i(0^+)$ is the initial current through the inductor and $q(0^+)$ is the initial charge on the capacitor

By neglecting the initial conditions of inductor and capacitor, we can write the equation as

$$\frac{V}{s} = RI(s) + LsI(s) + \frac{1}{C}\frac{I(s)}{s} \Rightarrow \frac{V}{s} = \left[R + Ls + \frac{1}{Cs}\right]I(s)$$

Therefore, the current through the circuit is given by,

$$I(s) = \frac{V}{sR + s^2L + 1/C} = \frac{V}{L\left[s^2 + \frac{R}{L}s + 1/LC\right]} = \frac{V}{L(s-\alpha_1)(s-\alpha_2)}$$

where α_1 and α_2 are the roots of equation $\left[s^2 + (R/L)s + 1/LC\right]$ and are given by

$$\alpha_1, \alpha_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Therefore,

$$i(t) = L^{-1}\left[I(s)\right] = L^{-1}\left[\frac{V}{L(s-\alpha_1)(s-\alpha_2)}\right]$$

$$\therefore i(t) = \frac{V}{L(\alpha_1 - \alpha_2)}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right]$$

$$\alpha_1 - \alpha_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC} - \left[-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}\right]$$

$$= 2\sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}$$

$$\boxed{i(t) = \frac{V}{2L\sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}}\left[e^{\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}\right]t} - e^{-\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}\right]t}\right]}$$

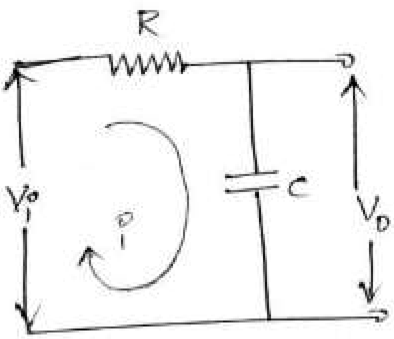
RC Circuits as Integrator:—

A circuit in which the output voltage is proportional to the integral of the input voltage is known as the "Integrating Circuit". An RC circuit can acts as an integrator. In general, the capacitor is a circuit element whose function is to store charge between two conductors in the form of electric field. A Capacitor is defined by it's ability to hold Charge, which is proportional to the applied Voltage,

$$Q = CV \qquad C \rightarrow \text{Capacitance.}$$

The presence of dielectric between two Conductors doesnot allow for the flow of DC Current; therefor a Capacitor acts as an open Circuit in the presence of DC Current i.e. no Current can flow through the Capacitor if the Voltage across it is constant, a time varying voltage will cause charge to vary in time. Thus if the charge is changing in time, the Current in the Circuit is given by

$$i(t) = \frac{dq(t)}{dt} = C\frac{dV(t)}{dt}$$



RC integrator Circuit

[shown in figure]

* The RC Circuit passes low frequencies but attenuates high frequency signals because the reactance of the capacitor decreases with increasing frequency.

At Very high frequencies, the capacitor acts as a short circuit then the output falls to zero.

→ from the circuit,

V_i is the input Voltage, $V_o \rightarrow$ output Voltage

$$V_i = iR + \frac{1}{C}\int_0^t i\,dt$$

$$\Rightarrow CV_i = iRC + \int_0^t i\,dt$$

The condition for integrating circuit is, RC Value must be much greater than the time period of the input wave i.e.

$$\boxed{RC >> T}$$

As $RC >> T$, the term $\int_0^t i\,dt$ may be neglected.

$$\therefore CV_i = iRC$$

Explanation! – RC Circuit as LPF

$$V_o(s) = \frac{1}{Cs} \cdot I(s)$$

$$V_i(s) = R + \frac{1}{Cs} \cdot I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1+RCs}$$

$$S = j\omega = j2\pi f$$

if $f = 0 \Rightarrow S = 0$ [low freq's]

$$\therefore \frac{V_o(s)}{V_i(s)} = 1$$

if $f = \infty \Rightarrow S = \infty$ [high freq's]

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+\infty} = 0$$

Allows low freq's but not high frequency signals

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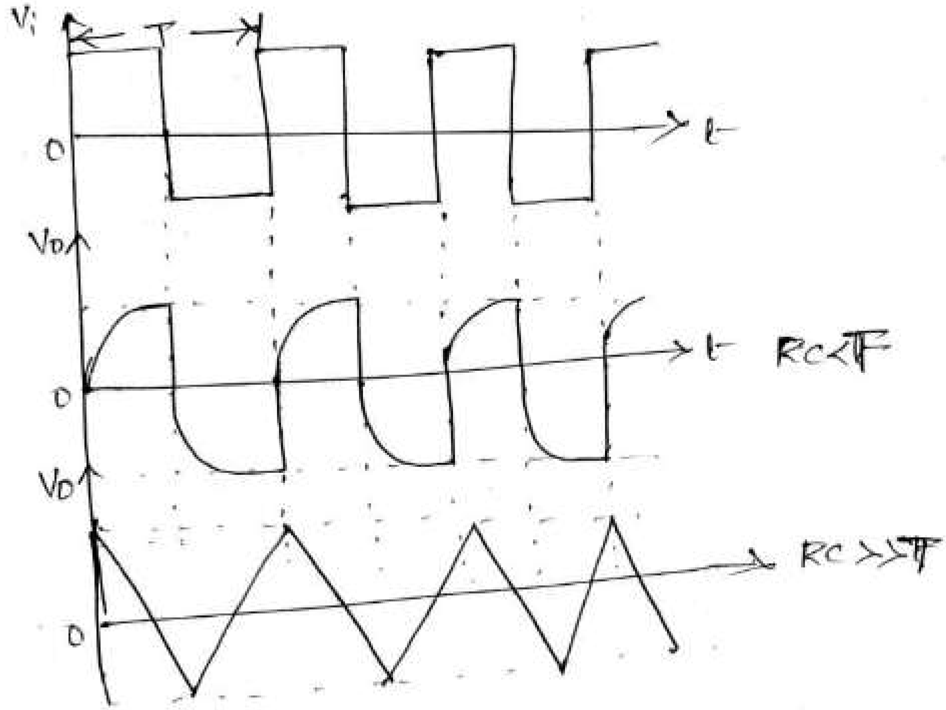
By integrating the above equation with respect to t on both sides, we get

$$\int_0^t C V_i \, dt = RC \int_0^t i \, dt$$

$$\Rightarrow \frac{1}{RC}\int_0^t V_i \, dt = \frac{1}{C}\int_0^t i \, dt \qquad \left[\frac{1}{C}\int_0^t i\,dt = V_0\right]$$

$$\therefore \boxed{V_0 = \frac{1}{RC}\int_0^t V_i \, dt} \quad (or) \quad \boxed{V = \frac{1}{\tau}\int_0^t V_i \, dt}$$

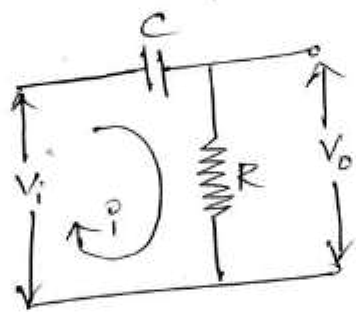
Expected Graph:—



*) RC Lowpass filter acts as an Integrator.

RC Circuit as a differentiator:—

RC circuit also works as a differentiator when the input is connected to the capacitor while the output voltage is taken across the resistor, as shown in figure.



→ A passive RC differentiator is simply a capacitance in series with a resistance, that is frequency dependent device which has reactance in series with a fixed resistance.

The output voltage depends on the circuits RC time constant and frequen

Thus, at low input frequencies, the reactance X_c of the capacitor is high which blocks any DC voltage or slowly varying input signals. While at high frequencies, the capacitor's reactance is low allowing rapidly varying pulses to pass directly from the input to the output. Hence the circuit acts as a high pass filter.

From the circuit, the output is equal to the voltage across the resistor. That is: V_{out} equals V_R and being a resistance, the output voltage can change instantaneously. However, the voltage across the capacitor cannot change instantly but depends on the value of capacitance C as it tries to store an electric charge, Q across its plates. Then the current flowing into the capacitor (i_t) depends on the rate of change of the charge across its plates. Thus, the capacitor current is not proportional to the voltage but its time variation giving:

$$i = \frac{dQ}{dt}.$$

$$\text{i.e. } i(t) = \frac{dQ}{dt} = \frac{d}{dt}(CV_c) = C\frac{dV_c}{dt} = C\frac{dV_{in}}{dt}$$

Since input is connected to capacitor. $\therefore V_c = V_{in}$

$\therefore$ Capacitor current can be written as:

$$\boxed{i_c(t) = C\frac{d V_{in}(t)}{dt}}$$

Explanation:–
from the circuit,
$V_o(S) = RI(S)$, $V_i(S) = \left[R + \frac{1}{CS}\right]I(S)$

$$T/F = \frac{V_o(S)}{V_i(S)} = \frac{R\,I(S)}{(R + 1/SC)\,I(S)}$$

$$T/F = \frac{RSC}{SRC+1} \quad \text{Since } S = j\omega = j2\pi f$$

$\left.\begin{array}{l} f=0 \Rightarrow T/F = 0 \\ f=\infty \Rightarrow T/F = 1 \end{array}\right\} \Rightarrow$ High pass filter

Since $V_{out} = i_R \times R = V_R$

$\&\ i_c = C\frac{dV_{in}}{dt}$

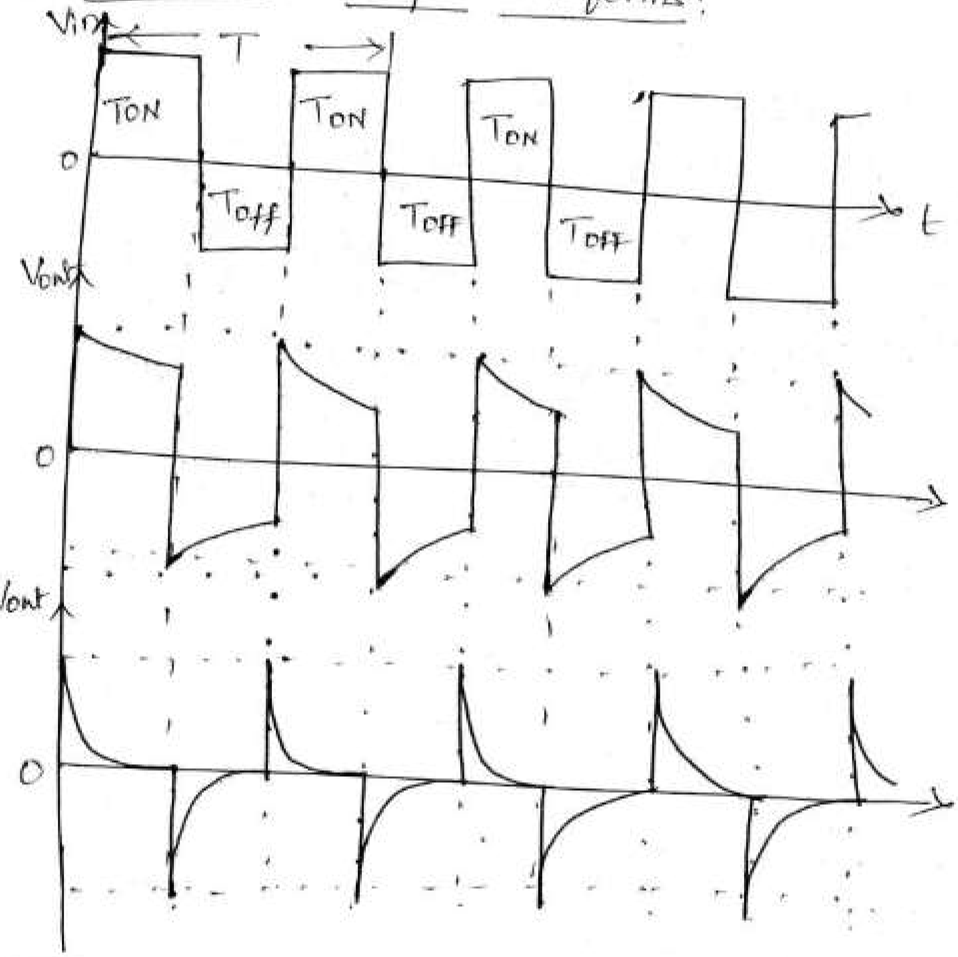
as $i_R = i_c$.

$$\boxed{V_{out} = RC\frac{d V_{in}}{dt}} \quad \text{(or)} \quad \boxed{V_{out} = \tau\frac{d V_{in}}{dt}}$$

* RC differentiator acts as a high pass filter that allows high frequency signals and blocks low frequency signals.

Unit-II (11)

RC differentiator output waveforms:



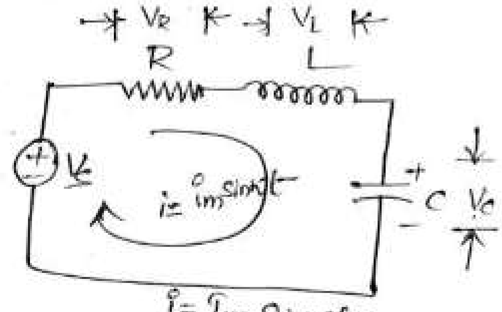
Rise Time and fall Time:—

→ Rise time (t_r) is defined as the time interval between the first 10% point and the first 90% in the rising step response of a system where the percentages are to the base of the final step response value.

→ fall time (t_f) is defined as the time interval between the first 90% point and the first 10% point in the step response of a system where the response variable is such that it starts at a non-zero initial value and decays to zero value in the long run.

2nd order Series RLC Circuit:—

A Series RLC Circuit comprises of resistor (R), inductor (L) and capacitor (C) is connected to the input voltage source (V_s).



In the series circuit, the current in each element remains the same.

By KVL,

$$V_s = V_R + V_L + V_C$$

where $V_R = IR$, $V_L = L\frac{di}{dt}$, $V_C = \frac{1}{C}\int i\,dt$

$$\therefore\ V_s = IR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

Since $i = I_m \sin\omega t \longrightarrow (2)$

$$\Rightarrow \boxed{\frac{di}{dt} = \omega I_m \cos\omega t} \longrightarrow (3)$$

and $\int i\,dt = \int I_m \sin\omega t\,dt = -\frac{I_m}{\omega}\cos\omega t \longrightarrow (4)$

Using (2), (3) and (4) equations in (1), we obtain

$$V_s = R I_m \sin\omega t + \omega L I_m \cos\omega t - \frac{I_m}{\omega C}\cos\omega t$$

$$= I_m\left[R\sin\omega t + \left(\omega L - \frac{1}{\omega C}\right)\cos\omega t\right] \longrightarrow (5)$$

Let

$$R = a\cos\phi,\ \text{and}\ (\omega L - 1/\omega C) = a\sin\phi \longrightarrow (6)$$

Substituting eqn (6) in eqn (5), we have

$$V_s = I_m\left[a\cos\phi\sin\omega t + a\sin\phi\cos\omega t\right] = a I_m \sin(\omega t + \phi) \longrightarrow (7)$$

from eqn (6),

$$R^2 + (\omega L - 1/\omega C)^2 = a^2\cos^2\phi + a^2\sin^2\phi = a^2 \Rightarrow a = \sqrt{R + (\omega L - 1/\omega C)^2} \longrightarrow (8)$$

Again from eqn (6), we get

$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \Rightarrow \boxed{\phi = \tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]} \longrightarrow (9)$$

Using eqn (8) in eqn (7), we obtain

$$\boxed{V_s = \sqrt{R + (\omega L - 1/\omega C)^2}\ I_m \sin(\omega t + \phi)} \longrightarrow (10)$$

$$\because\ V_c = V_m \sin(\omega t + \theta)$$

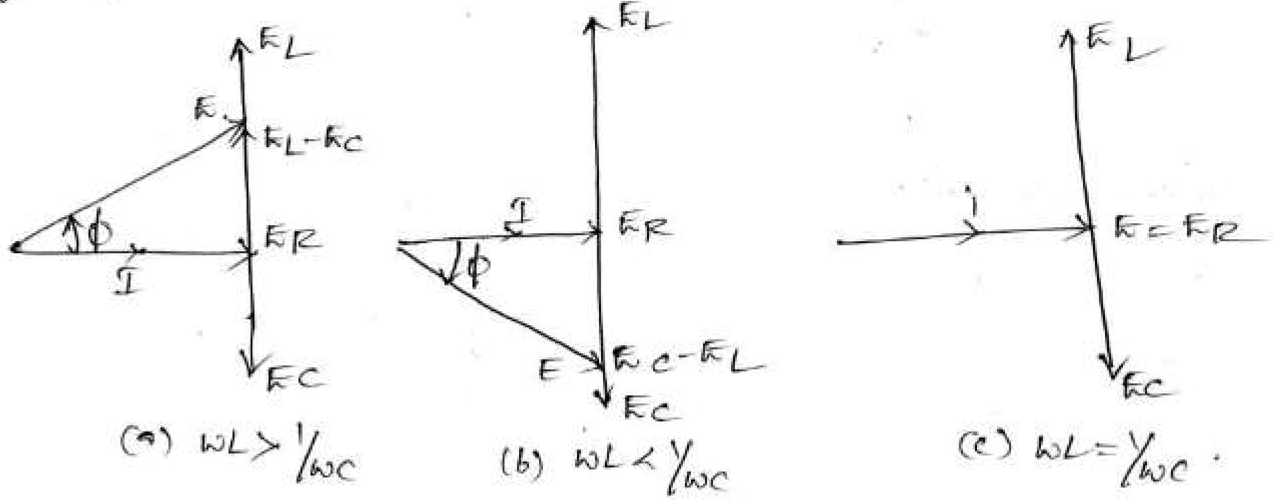
From eqn (6), the following conclusions can be drawn:

(1) if $\omega L > \frac{1}{\omega C}$, ϕ is positive, hence the voltage leads the current by an angle ϕ.

(ii) if $\omega L < \frac{1}{\omega C}$, ϕ is negative. In this case the current leads the voltage by an angle ϕ.

(iii) if $\omega L = \frac{1}{\omega C}$, $\phi = 0$. In this case, the voltage and current are in in phase.

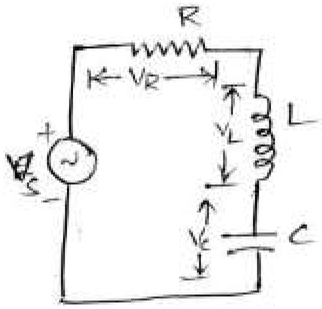
→ when $\phi = 0$, $Z = R$. The impedance is purely resistive and minimum and the current is maximum. This is the condition of resonance for a series resonance circuit.



(a) $\omega L > 1/\omega C$ (b) $\omega L < 1/\omega C$ (c) $\omega L = 1/\omega C$.

Series Resonance:—

The impedance of the series circuit is given by



$$Z = R + j(\omega L - 1/\omega C) \longrightarrow (1)$$

$$\therefore \tan\phi = \frac{\omega L - 1/\omega C}{R} \longrightarrow (2)$$

Resonance is said to occur when $\tan\phi = 0$, say at $\omega = \omega_r$

i.e., $\omega_r L - \frac{1}{\omega_r C} = 0 \Rightarrow \omega_r^2 = 1/LC$

$$\therefore \omega_r = 2\pi f_r = \frac{1}{\sqrt{LC}}$$

or $$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \longrightarrow (3)$$

At resonance, $j\omega L = j/\omega C$ [Since $X_L = X_C$]

$$\therefore \boxed{Z = R}$$

*→ When the frequency of the applied voltage is equal to the natural frequency of the circuit, the impedance is purely resistive and minimum. Therefore, the current

is maximum. Here in this case, the voltage across the inductance and capacitance are equal in magnitude and opposite in direction.

→ Series circuit is also called the 'Acceptor Circuit".

→ A series RLC circuit at resonance:

(i) The net reactance is zero because $X_L = X_C$.

(ii) The impedance of the circuit is minimum.

(iii) The admittance of the circuit is maximum.

(iv) The current in the circuit is maximum.

(v) The resonance frequency is given by

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \, (H_z)}$$

Quality factor (Q):-

The Quality factor of a circuit is defined as the ratio of potential across the inductance or capacitance at resonance to the potential drop across the resistance (or the applied voltage).

$$Q = \frac{\text{Potential drop across the inductance at resonance}}{\text{Potential drop across the resistance at resonance}}$$

$$Q = \frac{\omega_r L I}{R I} = \frac{\omega_r L}{R} = \frac{1}{\sqrt{LC}} * \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{L/C}}$$

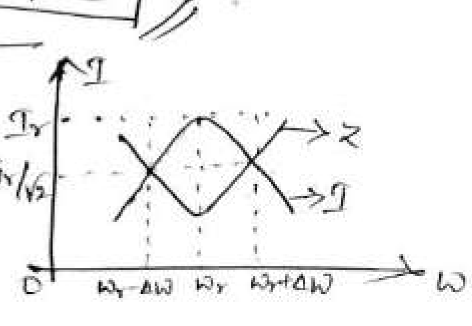
$$Q = \frac{\text{Potential drop across the capacitance at resonance}}{\text{Potential drop across the resistance at resonance}}$$

$$Q = \frac{I/\omega_r C}{R I} = \frac{1}{\omega_r C R} = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r C R} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

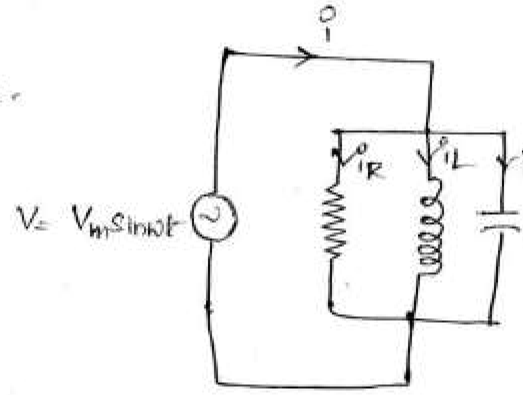
Variation of Z and I with frequency:-

*→ At resonant frequency, the current I is maximum and the impedance Z is minimum.



Unit-II (13)

2nd order Parallel RLC circuit:-



The parallel RLC circuit is shown in figure.

Here, $i = i_R + i_L + i_C$

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dV}{dt}$$

Since $V = V_m \sin\omega t$

$$\therefore i = \frac{V_m}{R}\sin\omega t - \frac{V_m}{\omega L}\cos\omega t + \omega C V_m \cos\omega t$$

Let $i = A\sin(\omega t+\theta) = A\sin\omega t\cos\theta + A\cos\omega t\sin\theta$.

By equating the cofficients of $\sin\omega t$ and $\cos\omega t$, we get

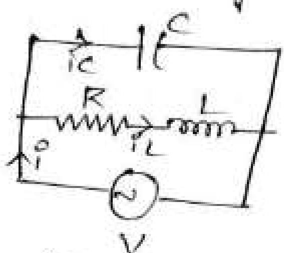
$$\frac{V_m}{R} = A\cos\theta, \quad \left[\omega C - \frac{1}{\omega L}\right]V_m = A\sin\theta, \text{ and } \tan\theta = \frac{\omega C - 1/\omega L}{1/R}$$

NOW. $A = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - 1/\omega L\right)^2}\, V_m \sin(\omega t+\theta)$

Here the ~~Stage~~ sign of the phase angle θ depends on the relative Values of ωC and $1/\omega L$

Parallel Resonance:

In a parallel circuit, where a coil (RL) is connected in parallel with a capacitor 'C'. This combination is connected across an AC voltage source.

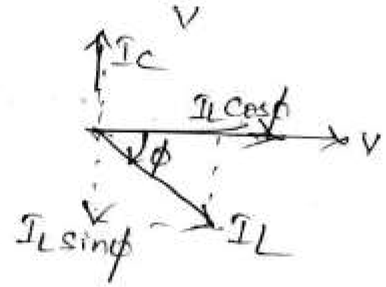


$$I_C = \frac{V}{X_C}, \quad I_L = \frac{V}{Z_L}, \quad \cos\phi = \frac{R}{Z_L}, \quad \sin\phi = \frac{X_L}{Z_L}$$

At resonance, $I_C = I_L\sin\phi$

$$\frac{V}{X_C} = \frac{V}{Z_L}\cdot\frac{X_L}{Z_L} \quad \text{or} \quad Z_L^2 = X_C X_L$$

$$= \omega_r L\left(\frac{1}{\omega_r C}\right)$$



$\phi \to$ power factor of coil

$$\therefore \boxed{Z_L = \sqrt{\frac{L}{C}}}$$

$$\sqrt{R^2 + \omega_r^2 L^2} = \sqrt{L/C}$$

$$\boxed{f_r = \frac{1}{2\pi}\left(\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\right)}$$

If 'R' is negligible, the above equation reduces to $f_r = \frac{1}{2\pi\sqrt{LC}}$. Again at resonance, the active component of current is

$$I = I_L \cos\phi$$

(or) $$\frac{V}{Z_\Omega} = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$ (where Z_Ω is the impedance of the circuit at resonance)

(or) $$Z_\Omega = \frac{Z_L^2}{R}$$

$$\therefore \boxed{Z_\Omega = \frac{L}{CR}} \qquad \left[\because Z_L = \sqrt{L/C}\right]$$

This impedance is called the "Dynamic Impedance" of the circuit.

→ If 'R' is low, Z_Ω is high and thus the current is low. This circuit is also called the "Rejector Circuit".

→ The properties of parallel resonance can therefore be listed as:

(i) Power factor is unity.

(ii) The impedance of the circuit at resonance is maximum.

(iii) The admittance of the circuit at resonance is minimum.

(iv) The value of current at resonance is minimum.

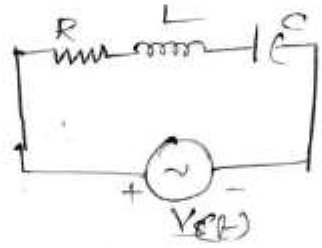
(v) The resonance frequency is given by

$$\boxed{f_r = \frac{1}{2\pi}\left(\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\right)}$$

Root Locus:—

The root locus plot is a plot of the roots of the characteristic equation to identify the stability of the system.

(i) Series RLC Circuit:—



The series RLC circuit is excited by a voltage source, $V(t)$. For this network, the KVL gives

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = V(t) \longrightarrow (1)$$

By differentiating the above eqn w.r.t 't', then

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \longrightarrow (2) \quad \left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0$$

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Unit-II (14)

The roots of the corresponding characteristic equation may be found by the quadratic formula to be

$$S_1, S_2 \text{ (or) } \boxed{D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}} \longrightarrow (3)$$

Damping (factor) ratio:—

The value of resistance at which the radical term in the above equat[ion] reduces to zero or vanishes is called the "Critical Resistance" (R_{cr})

i.e.
$$\left(\frac{R_{cr}}{2L}\right)^2 - \frac{1}{LC} = 0 \Rightarrow \left(\frac{R_{cr}}{2L}\right)^2 = \frac{1}{LC} \Rightarrow (R_{cr})^2 = \frac{4L^{\cancel{2}}}{\cancel{L}C}$$

$$\boxed{R_{cr} = 2\sqrt{\frac{L}{C}}} \longrightarrow (4)$$

→ Damping Ratio is the ratio of the actual resistance to the critical resistance, and it is the dimension less quantity.

$$\text{Damping Ratio } (\xi) = \frac{R}{R_{cr}} = R\,\frac{1}{2}\sqrt{\frac{C}{L}}$$

$$\therefore \boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}} \longrightarrow (5)$$

Natural frequency (ω_n):

$$\omega_n = \frac{1}{\sqrt{LC}}$$

The product $2\xi\omega_n$ becomes

$$2\xi\omega_n = \cancel{2}\,\frac{R}{\cancel{2}}\sqrt{\frac{C}{L}} * \frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$\Rightarrow \boxed{\xi\omega_n = \frac{R}{2L}} \longrightarrow (6)$$

Substituting these relations into equation (2) gives

$$\frac{d^2i}{dt^2} + 2\xi\omega_n\frac{di}{dt} + \omega_n^2 i = 0 \longrightarrow (7)$$

The new constants are introduced are convenient for interpreting the geometry of the root locations in the s-plane, and they have significance in understanding the response.

The quality factor (Q) of the Series RLC circuit is

$$Q = \frac{\omega_n L}{R}$$ where $\omega_n \rightarrow$ natural frequency.

Since $2\xi\omega_n = \frac{R}{L} \Rightarrow \frac{L}{R} = \frac{1}{2\xi\omega_n}$

$$\therefore Q = \omega_n\left[\frac{1}{2\xi\omega_n}\right] = \frac{1}{2\xi}.$$

$$\boxed{Q = \frac{1}{2\xi}} \rightarrow \text{Relation between Quality factor and Damping ratio}$$

$\rightarrow$ (8)

The equation (2) becomes,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \longrightarrow (9)$$

The roots of the characteristic equation are

$$\boxed{s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}} \longrightarrow (10)$$

The general solution may now be written as

$$\boxed{i = K_1 e^{\left[-\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right]t} + K_2 e^{\left[-\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right]t}} \longrightarrow (11)$$

Let us examine the behavior of the roots of the characteristic equation as the dimensionless damping ratio (ξ) varies from zero (corresponding to $R=0$) to infinity (corresponding to $R=\infty$). The three different forms of the roots are

Case(i):- $\xi > 1$. The roots are real,

case 2:- $\xi = 1$, the roots are real and repeated,

case 3:- $\xi < 1$, the roots are complex and conjugates.

As the ξ value changes from 0 to ∞, it will be recognized a locus of roots in the complex plane.

$\rightarrow$ for $\xi = 0$,

$\Rightarrow \boxed{s_1, s_2 = \pm j\omega_n}$

Here the roots are purely imaginary.

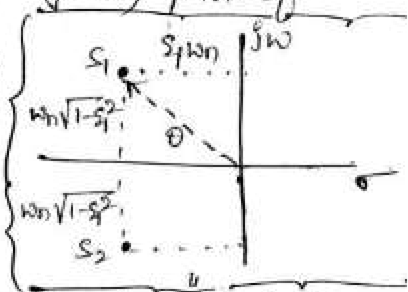
NATL unit-II (15)

→ for $\xi_1 < 1$, the roots are complex conjugates as

$$\boxed{s_1, s_2 = -\xi_f\omega_n \pm j\,\omega_n\sqrt{1-\xi_f^2}} \longrightarrow (12)$$

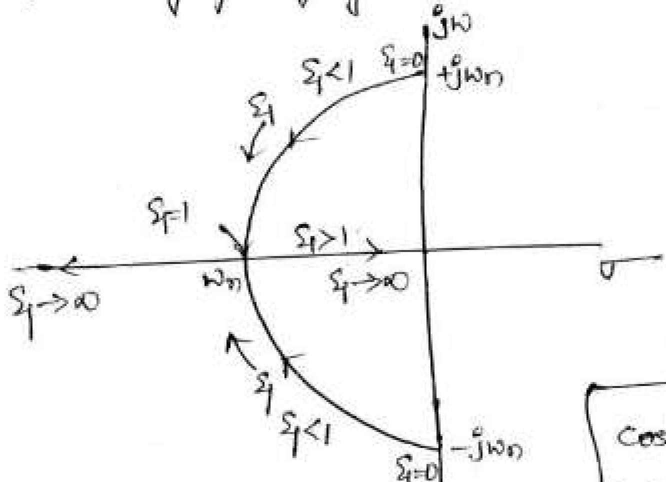
Interms of s-plane locations, we see that the real and imaginary parts of

$s = \sigma + j\omega$ are $\boxed{\sigma = -\xi_f\omega_n}$ and $\boxed{\omega = \pm\omega_n\sqrt{1-\xi_f^2}}$



and $\sigma^2 + \omega^2 = \xi_f^2\omega_n^2 + \omega_n^2\left[1-\xi_f^2\right] = \omega_n^2$.

it shows that the locus of the roots in the complex s-plane is a circle of radius (ω_n) and that this locus is formed by 'ξ_f' varying from '0' to '1'.



General Solutions interms of ξ_f, φ and ω_n:

$\cosh x = \frac{e^{x}+e^{-x}}{2}$

$\sinh x = \frac{e^{x}-e^{-x}}{2}$

$e^{x} = \cosh x + \sinh x$

$e^{-x} = \cosh x - \sinh x$

case(i):- when $\xi_f > 1$ or $\varphi < \frac{1}{2}$:

The solution for the equation (12) can be written as

$$i = e^{-\xi_f\omega_n t}\left[K_1 e^{\omega_n\sqrt{\xi_f^2-1}\,t} + K_2 e^{-\omega_n\sqrt{\xi_f^2-1}\,t}\right] \longrightarrow (13)$$

where K_1 and K_2 are arbitrary constants of integration.

This equation is sometimes more convenient to evaluate in terms of hyperbolic functions.

$$i = e^{-\xi_f\omega_n t}\left\{K_1\left[\cosh(\omega_n\sqrt{\xi_f^2-1})t + \sinh(\omega_n\sqrt{\xi_f^2-1}\,t)\right] + K_2\left\{\cosh(\omega_n\sqrt{\xi_f^2-1}\,t) - \sinh(\omega_n\sqrt{\xi_f^2-1}\,t)\right\}\right\} \longrightarrow (14)$$

(or)

$$i = e^{-\xi_f\omega_n t}\left[K_3\cosh(\omega_n\sqrt{\xi_f^2-1}\,t) + K_4\sinh(\omega_n\sqrt{\xi_f^2-1}\,t)\right] \longrightarrow (15)$$

where $K_3 = K_1 + K_2$ and $K_4 = K_1 - K_2$

The arbitrary constants are usually evaluated to find a particular solution in terms of the initial conditions.

Case ②: $\xi = 1$ or $Q = 1/2$:—

In this case, the two roots become identical. The solution with the repeated roots is

$$\boxed{i = (K_1 + K_2 t) e^{-\omega_n t}} \longrightarrow (16)$$

The limit of the quantity $t e^{-\omega_n t}$ may be investigated by L'Hospital's rule and it is equal to zero. i.e.,

$$\lim_{t \to \infty} t e^{-\omega_n t} = 0.$$

Case ③: when $\xi < 1$ or $Q > 1/2$:

In this case, the roots become complex, and the solution can be written as

$$i = e^{-\xi\omega_n t}\left[K_1 e^{j\omega_n\sqrt{1-\xi^2}\,t} + K_2 e^{-j\omega_n\sqrt{1-\xi^2}\,t}\right] \longrightarrow (17)$$

This equation may be written in terms of sine and cosine quantities i.e., $e^{\pm jx} = \cos x \pm j \sin x$

Therefore the solution reduces to

$$i = e^{-\xi\omega_n t}\left[K_5 \cos(\omega_n\sqrt{1-\xi^2}\,t) + K_6 \sin(\omega_n\sqrt{1-\xi^2}\,t)\right] \longrightarrow (18)$$

where

$$K_5 = K_1 + K_2 \quad \text{and} \quad K_6 = j(K_1 - K_2)$$

Where K_5, K_6, K_1 & K_2 are the arbitrary constants of integration. This equation may be written in different form by defining,

$$K_5 = K \sin\phi, \quad K_6 = K\cos\phi$$

using the Trigonometric identity,

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

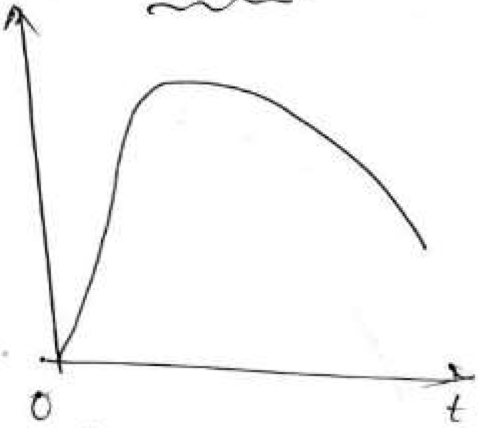
Therefore, eqn (17) reduces to

$$i = K e^{-\xi\omega_n t} \sin\left[\omega_n\sqrt{1-\xi^2}\,t + \phi\right] \longrightarrow (19)$$

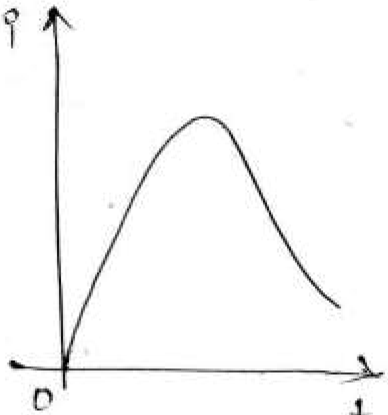
Here $K = \sqrt{K_5^2 + K_6^2}$, $\phi = \tan^{-1}(K_5/K_6)$

NATL Unit-II (16)

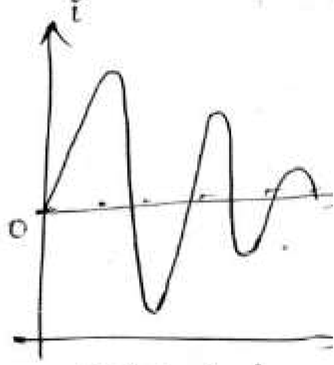
Function Solution :-



(a) overdamped ($\zeta_1 > 1$)



(b) critically damped ($\zeta_1 = 1$)



(c) underdamped ($\zeta_1 < 1$)

case(i): $\zeta_1 > 1$ or $Q < 1/2$:-

The general solution can be reduced to a particular solution for a given set of initial conditions. They are $i = e^{-\zeta_1 \omega_n t}\left[K_3 \cosh(\omega_n\sqrt{\zeta_1^2-1}\,t) + K_4 \sinh(\omega_n\sqrt{\zeta_1^2-1}\,t)\right]$

$$i(0^+) = 0$$

$$\frac{1}{C}\int_{-\infty}^{0} i\,dt = -V_0 \text{ at } \left[\text{the initial voltage on the capacitor}\right) \text{such}$$

that $\dfrac{di(0^+)}{dt} = \dfrac{V_0}{L}$.

$i(0^+) = 0$ is possible when the K_3 is zero in equation (15). which is

$$i = e^{-\zeta_1 \omega_n t}\left[K_3 \cosh\left[\omega_n\sqrt{\zeta_1^2-1}\,t\right] + K_4 \sinh\left[\omega_n\sqrt{\zeta_1^2-1}\,t\right]\right]$$

Since $K_3 = 0$ - $\boxed{i = K_4 e^{-\zeta_1 \omega_n t} \sinh\left[\omega_n\sqrt{\zeta_1^2-1}\,t\right]}$ $\longrightarrow$ (20)

The constant K_4 can be evaluated from the second initial condition:

$$\frac{di}{dt} = K_4\left[e^{-\zeta_1 \omega_n t}\cosh \omega_n\sqrt{\zeta_1^2-1}\,t\,\left(\omega_n\sqrt{\zeta_1^2-1}\right) + \sinh \omega_n\sqrt{\zeta_1^2-1}\,t\; e^{-\zeta_1\omega_n t}(-\zeta_1\omega_n)\right]$$

The hyperbolic cosine & sine terms approaches unity as $t \to 0$. Hence

$$\frac{di}{dt}(0^+) = K_4 \omega_n\sqrt{\zeta_1^2-1} = \frac{V_0}{L}$$

and $\boxed{K_4 = \dfrac{V_0}{\omega_n L\sqrt{\zeta_1^2-1}}}$

The particular solution for the overdamped case thus becomes

$$\boxed{i = \frac{V_0}{\omega_n L\sqrt{\xi_f^2-1}} e^{-\xi_f \omega_n t} \sinh\left(\omega_n \sqrt{\xi_f^2-1}\, t\right)} \longrightarrow (21)$$

Case (2):- $\xi_f = 1$

In case of critically damped, the general solution is

$$i = (K_1 + K_2 t)\, e^{-\omega_n t} \longrightarrow (22)$$

By applying initial conditions [i.e. current conditions], implies that

$K_1 = 0$. $\quad \therefore i = K_2 t\, e^{-\omega_n t}$

By differentiating the equation (22) with respect to 't', then

$$\frac{di}{dt} = K_2\left[e^{-\omega_n t} + t e^{-\omega_n t}(-\omega_n)\right] = K_2\left[1 - t\omega_n\right] e^{-\omega_n t}$$

At $t = 0^+$,

$$\frac{di}{dt}(0^+) = K_2 = \frac{V_0}{L}$$

Then the particular solution for the critically damped case is

$$\boxed{i = \frac{V_0}{L}\, t\, e^{-\omega_n t}} \longrightarrow (23)$$

Case (3):- when $\xi_f < 1$ (or) $Q > 1/2$:-

The general solution for the underdamped case is

$$i = e^{-\xi_f \omega_n t}\left[K_5 \cos\left(\omega_n \sqrt{1-\xi_f^2}\, t\right) + K_6 \sin\left(\omega_n\sqrt{1-\xi_f^2}\, t\right)\right]$$

Where $\quad K_5 = K_1 + K_2$ and $K_6 = j(K_1 - K_2)$

By applying initial conditions, $K_5 = 0$. Therefore, the solution can be written as

$$i = K_6\, e^{-\xi_f \omega_n t} \sin\left(\omega_n \sqrt{1-\xi_f^2}\, t\right) \longrightarrow (24)$$

Here the arbitrary constant 'K_6' is evaluated by using the initial condition of the derivative of the current; thus

$$\frac{di}{dt} = K_6 e^{-\xi_f \omega_n t}\left[\omega_n\sqrt{1-\xi_f^2} \cos\left(\omega_n\sqrt{1-\xi_f^2}\, t\right) - \xi_f \omega_n \sin\left(\omega_n\sqrt{1-\xi_f^2}\, t\right)\right]$$

Such that

$$\frac{di}{dt}(0^+) = K_6\, \omega_n \sqrt{1-\xi_f^2} = \frac{V_0}{L} \longrightarrow (25)$$

$$\Rightarrow K_6 = \frac{V_0}{L\omega_n\sqrt{1-\xi_1^2}}$$

Therefore the particular solution for the oscillatory case is

$$\boxed{i = \frac{V_0}{\omega_n L\sqrt{1-\xi_1^2}} e^{-\xi_1 \omega_n t} \sin(\omega_n\sqrt{1-\xi_1^2}\, t)} \longrightarrow (26)$$

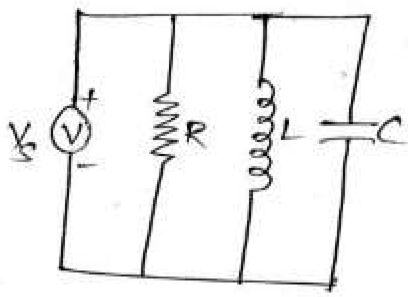
The variation of current with time for the different oscillatory cases is shown in figure. Since the current is the product of the damping factor and oscillatory term.

→ The damping factor represents an envelope (or) boundary curve for the oscillati

→ The factor '$\xi_1\omega_n$' determines how rapidly the oscillations are damped.

Parallel RLC Circuit :-

The admittance (Y) of the parallel RLC Circuit is



$$Y = \frac{1}{R} + j\omega C - \frac{1}{j\omega L}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

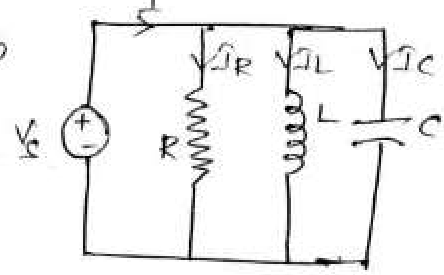
The frequency (f_r) at which resonance occurs is given by

$$\omega_r C - \frac{1}{\omega_r L} = 0 \Rightarrow \omega_r^2 = \frac{1}{LC} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

General Solutions of Parallel RLC Circuit :-

In parallel circuit, the voltage across each element is same which is V_s.



By KCL at nodes, we get: $I = I_R + I_L + I_C$

$$\frac{V_s}{R} + \frac{1}{L}\int_{-\infty}^{t} V_s\, dt + C\frac{dV_s}{dt} = I$$

Taking the derivative of the above equation w.r.t. t, we get

$$\frac{1}{R}\frac{dv}{dt} + \frac{V}{L} + C\frac{d^2V}{dt^2} = 0 \Rightarrow \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC} = 0$$

$$\Rightarrow \left[\frac{d^2}{dt^2} + \frac{1}{RC}\frac{d}{dt} + \frac{1}{LC}\right]V = 0$$

By taking the Laplace Transformation approach,

$$\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = 0$$

The roots of the characteristic equation will be

$$s_{1,2} = \frac{-1/RC \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - 1/LC}$$

$$\boxed{s_{1,2} = -\frac{1}{2RC} \pm \sqrt{(1/2RC)^2 - 1/LC}}$$

Since the roots for the generalized characteristic equation are

$$s_{1,2} = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$$

By comparing, $\omega_n^2 = 1/LC \Rightarrow \boxed{\omega_n = 1/\sqrt{LC}}$

$$\xi\omega_n = 1/2RC \Rightarrow \xi^2\omega_n^2 = \frac{1}{4R^2C^2}$$

$$\therefore \xi^2 * \frac{1}{LC} = \frac{1}{4R^2C^2} \Rightarrow \xi^2 = \frac{LC}{4R^2C^2} \Rightarrow \boxed{\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}}$$

→ Neper frequency or exponential damping coefficient (α) is a measure of how rapidly the natural response decays or damps out to its steady, final value (usually zero).

$$\alpha = \frac{1}{2RC} = \xi\omega_n$$

$$\therefore s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_n^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_n^2}$$

Since $\xi_f \omega_n = \alpha \Rightarrow \xi_f = \alpha/\omega_n$

Based on the value of ξ_f or α, the solution for the second order partial differential equation will be defined.

→ $\xi_f > 1$ or $\frac{\alpha}{\omega_n} > 1 \Rightarrow \alpha > \omega_n$ overdamped response

$\xi_f = 1$ or $\frac{\alpha}{\omega_n} = 1 \Rightarrow \alpha = \omega_n$ Critically damped response

$\xi_f < 1$ or $\frac{\alpha}{\omega_n} < 1 \Rightarrow \alpha < \omega_n$ Underdamped response

Case(i):- over damped response $[\xi_f > 1$ or $\alpha > \omega_n]$

The roots of the second order partial differential equation are.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_n^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_n^2}$$

The general solution incase of over-damped response are

$$v(t) = K_1 e^{(-\alpha + \sqrt{\alpha^2 - \omega_n^2})t} + K_2 e^{(-\alpha - \sqrt{\alpha^2 - \omega_n^2})t}$$

$$= K_1 e^{-\alpha t} e^{\sqrt{\alpha^2 - \omega_n^2}\, t} + K_2 e^{-\alpha t} e^{-\sqrt{\alpha^2 - \omega_n^2}\, t}$$

$$= e^{-\alpha t}\left[K_1 e^{\sqrt{\alpha^2 - \omega_n^2}\, t} + K_2 e^{-\sqrt{\alpha^2 - \omega_n^2}\, t}\right]$$

$$= e^{-\alpha t}\left[K_1\left(\cosh\sqrt{\alpha^2 - \omega_n^2}\, t + \sinh\sqrt{\alpha^2 - \omega_n^2}\, t\right) + K_2\left(\cosh\sqrt{\alpha^2 - \omega_n^2}\, t - \sinh\sqrt{\alpha^2 - \omega_n^2}\, t\right)\right]$$

$$= e^{-\alpha t}\left[\cosh\sqrt{\alpha^2 - \omega_n^2}\, t\,(K_1 + K_2) + \sinh\sqrt{\alpha^2 - \omega_n^2}\, t\,(K_1 - K_2)\right]$$

$$= e^{-\alpha t}\left[K_3 \cosh\sqrt{\alpha^2 - \omega_n^2}\, t + \sinh\sqrt{\alpha^2 - \omega_n^2}\, t \cdot K_4\right]$$

where $K_3 = K_1 + K_2$, $K_4 = K_1 - K_2$

To find the final solution, apply initial conditions.

$$V(0^+) = 0, \quad \frac{dV(0^+)}{dt} = i/C$$

By applying 1st initial condition,

at $t=0$,

$V(0) = 0 = K_3(1) + K_4(0) \Rightarrow K_3 = 0$

$\therefore V(t) = K_4 e^{-\alpha t} \sinh\sqrt{\alpha^2 - \omega_n^2}\, t$

By differentiating the above equation w.r.t. 't', we get

$$\frac{dv(t)}{dt} = K_4\left[e^{-\alpha t}\cosh\sqrt{\alpha^2-\omega_n^2}\,t\,(\sqrt{\alpha^2-\omega_n^2}) + e^{-\alpha t}(-\alpha)\sinh\sqrt{\alpha^2-\omega_n^2}\,t\right]$$

$$= K_4 e^{-\alpha t}\left[\sqrt{\alpha^2-\omega_n^2}\cosh\sqrt{\alpha^2-\omega_n^2}\,t - \alpha\sinh\sqrt{\alpha^2-\omega_n^2}\,t\right]$$

By applying the 2nd initial condition at $t=0$,

$$\frac{dv(0)}{dt} = \frac{i}{C} = K_4\left[\sqrt{\alpha^2-\omega_n^2}\right] \Rightarrow \boxed{K_4 = \frac{i}{C\sqrt{\alpha^2-\omega_n^2}}}$$

Therefore the particular solution becomes

$$\boxed{v(t) = \frac{i}{C\sqrt{\alpha^2-\omega_n^2}}\,e^{-\alpha t}\sinh\sqrt{\alpha^2-\omega_n^2}\,t.}$$

Case ②:— Critically damped Response $[\xi=1 \text{ or } \alpha=\omega_n]$

The roots of the 2nd order partial differential equation are

$S_1=-\alpha, \quad S_2=-\alpha$. The roots are real and equal. Hence the solution is

$$v(t) = (K_5+K_6 t)e^{-\alpha t}$$

For particular solution, apply initial conditions

$$v(0^+)=0, \quad \frac{dv(0^+)}{dt} = i/C$$

By applying 1st initial condition at $t=0$,

$$v(0)=0=[K_5+K_6(0)]e^0 \Rightarrow 0=K_5+0 \Rightarrow \boxed{K_5=0}$$

$$\therefore v(t) = K_6 t\, e^{-\alpha t}$$

By differentiating the above equation w.r.t. 't', we get

$$\frac{dv(t)}{dt} = K_6\left[t e^{-\alpha t}(-\alpha) + e^{-\alpha t}\right] = K_6 e^{-\alpha t}[1-\alpha t]$$

By applying the 2nd initial condition, at $t=0$,

$$\frac{dv(0)}{dt} = i/C = K_6 e^0[1-0] \Rightarrow i/C = K_6[1] \Rightarrow \boxed{K_6 = i/C}$$

Therefore, the particular solution becomes

$$\boxed{v(t) = \frac{i}{C}e^{-\alpha t}(t)}$$

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case (3) :- Under-damped Response [$\xi < 1$ or $\alpha < \omega_n$]

Since $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2} = -\alpha \pm \sqrt{-1(\omega_n^2 - \alpha^2)} = -\alpha \pm \sqrt{j^2(\omega_n^2 -}$

$$S_{1,2} = -\alpha \pm j\sqrt{\omega_n^2 - \alpha^2}$$

$S_1 = -\alpha + j\sqrt{\omega_n^2 - \alpha^2}$, $S_2 = -\alpha - j\sqrt{\omega_n^2 - \alpha^2}$. two roots are comp Conjuga

The general solution is

$$v(t) = e^{-\alpha t}\left[K_5 \cos\sqrt{\omega_n^2 - \alpha^2}\, t + jK_6 \sin\sqrt{\omega_n^2 - \alpha^2}\, t\right]$$

By applying the 1st Boundary Condition i.e, $v(0) = 0$, we get at $t = 0$,

$$v(0) = 0 = (1)\left[K_5(1) + jK_6(0)\right] \Rightarrow K_5 = 0.$$

$$\therefore \boxed{v(t) = jK_6 e^{-\alpha t} \sin\sqrt{\omega_n^2 - \alpha^2}\, t}$$

By differentiating the above equation w.r.t. 't', we can get

$$\frac{dv(t)}{dt} = jK_6\left[e^{-\alpha t} \cos\sqrt{\omega_n^2 - \alpha^2}\, t \left(\sqrt{\omega_n^2 - \alpha^2}\right) + e^{-\alpha t}(-\alpha) \sin\sqrt{\omega_n^2 - \alpha^2}\, t\right]$$

$$\frac{dv(t)}{dt} = jK_6 e^{-\alpha t}\left[\sqrt{\omega_n^2 - \alpha^2} \cos\sqrt{\omega_n^2 - \alpha^2}\, t - \alpha \sin\sqrt{\omega_n^2 - \alpha^2}\, t\right]$$

By applying the 2nd boundary condition, i.e, $\frac{dv(0^+)}{dt} = i/c$, we can get at $t = 0$,

$$\frac{dv(0)}{dt} = i/c = jK_6\left[\sqrt{\omega_n^2 - \alpha^2}\,(1) - \alpha(0)\right] = jK_6\sqrt{\omega_n^2 - \alpha^2}$$

$$\therefore \boxed{K_6 = \frac{i}{jC\sqrt{\omega_n^2 - \alpha^2}}}$$

Therefore, the particular solution becomes,

$$v(t) = \cancel{j} * \frac{i}{\cancel{j}C\sqrt{\omega_n^2 - \alpha^2}} e^{-\alpha t} \sin\sqrt{\omega_n^2 - \alpha^2}\, t$$

$$\therefore \boxed{v(t) = \frac{i}{C\sqrt{\omega_n^2 - \alpha^2}} e^{-\alpha t} \sin\sqrt{\omega_n^2 - \alpha^2}\, t}$$

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Two-port Network parameters

→ A general network having two pairs of terminals, one often labeled the 'input terminals' and the other the "output terminals". These terminals are very important in an electronic system.

→ In this pair of terminals, an electrical signal enters the network through the input terminals and leaves via the output terminals.

One-port network:—

→ A pair of terminals at which a signal may enter or leave a network is called a "port".

→ A network having only one such pair of terminals is called a "one-port network" or simply a "one-port".

Multi-port network:—

→ When more than one port of terminals is present then that network is known as "Multiport network".

→ If a circuit [or network] is connected to a pair of ports, it is called a 'two port circuit' [or two port network].

→ A pair of ports containing no sources in their branches are called "Passive ports".

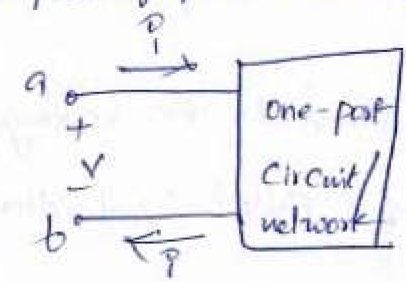

figure: one-port circuit

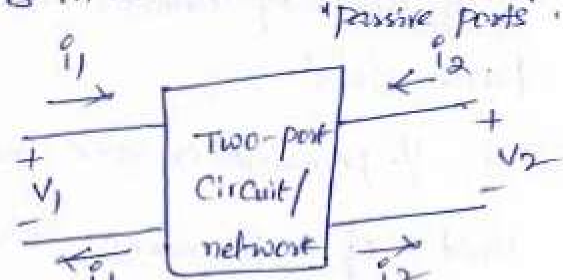

figure: Two port circuit

→ A pair of ports containing sources in their branches are called "Active ports".

Two port network:

→ The port on the left side is called 'port 1' and the port on the right side is called 'port 2'. An input signal is applied to the circuit through port 1 and and an output signal from the circuit is taken from port 2.

→ Two port circuit does not contain independent sources. There are ~~the~~ four variables in the two port circuit. These are voltage V_1 and current I_1 at port 1 and voltage V_2 and current I_2 at port 2. Two of the four variables are independent and the other two variables are dependent.

→ Depending on which two of the four ~~parameters~~ variables are selected as independent variables, there are six different representations of the circuit.

→ The ~~coefficiencients~~ coefficients of the representation are called 'parameters'. The six different parameters are z-parameters, y-parameters, h-parameters, g-parameters, ABCD parameters and b-parameters.

→ The z-parameters are ratios of voltage to current with units of ohms, and they are called 'Impedance parameters'.

→ The y-parameters are ratios of current to voltage with units of siemens, and they are called 'Admittance parameters'.

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→ The h-parameters represent input impedance, reverse voltage gain, forward current gain and output admittance and hence these are called 'hybrid parameters'.

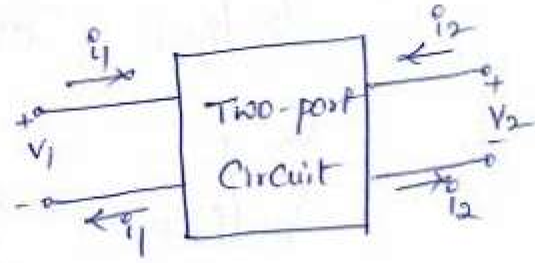
→ The g-parameters represent output impedance, forward voltage gain, reverse current gain and input impedance and hence these are called 'inverse hybrid parameters'.

→ The ABCD parameters represent voltage ratio, transfer impedance, transfer admittance and current ratio and hence these are called "Transmission parameters".

→ The two-port circuits can be interconnected. The interconnection can be in cascade, series, parallel and a combination of series and parallel circuits.

Z-parameters: (Impedance parameters)

→ from the two-port circuit of Z-parameters, V_1 and V_2 are observed to be dependent variables and i_1 & i_2 are observed to be independent variables.



→ The dependent variables are written as linear combinations of independent variables as

$$\boxed{\begin{aligned} V_1 &= z_{11} i_1 + z_{12} i_2 \\ V_2 &= z_{21} i_1 + z_{22} i_2 \end{aligned}}$$

$\longrightarrow$ ①
$\longrightarrow$ ②

In a matrix notation, we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \longrightarrow ③$$

This equation can be written as

$$V = Z i$$

where

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}, \quad i = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where Z_{11}, Z_{12}, Z_{21} & Z_{22} are the network functions, and are called impedance (Z) parameters.

→ Coefficient Z_{11} can be found by taking the ratio of 'V_1' over 'i_1' when port 2 is open circuited ($I_2 = 0$)

$$\therefore Z_{11} = \left.\frac{V_1}{I_1}\right|_{i_2=0} \longrightarrow (4)$$

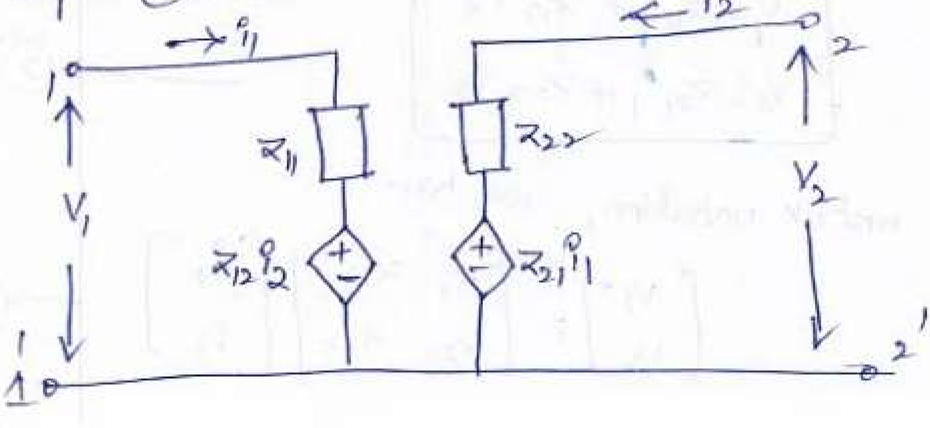
Z_{11} → input impedance with port 2 is open circuited.

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{i_1=0}$ → Transfer impedance with port 1 open circuited → (5)

$Z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0}$ → Transfer impedance with port 2 open circuited → (6)

$Z_{22} = \left.\frac{V_2}{i_2}\right|_{i_1=0}$ → output impedance with port 1 open circuited. → (7)

from eqns (1) & (2).



Equivalent Circuit

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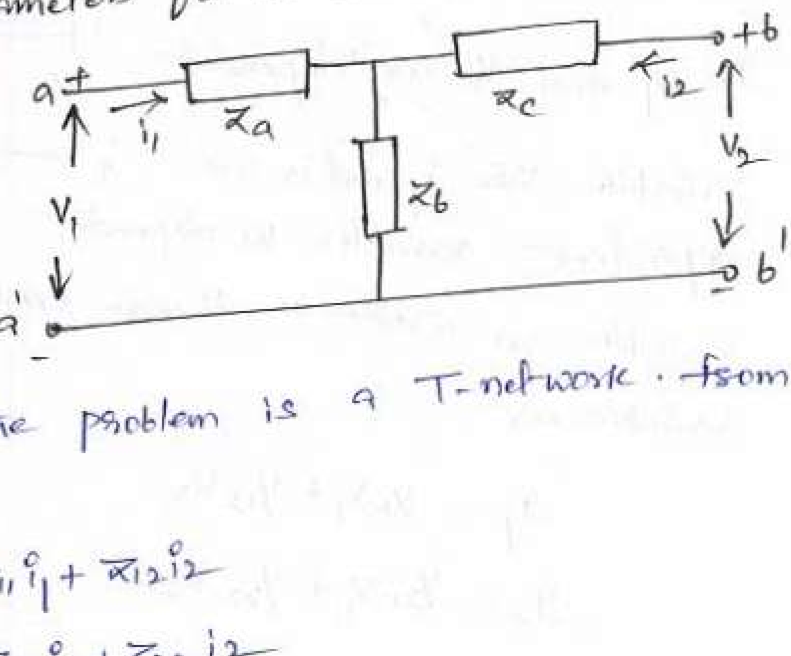
If the network under study is reciprocal or bilateral, then according to reciprocity principle.

$$\frac{V_2}{i_1}\Big|_{i_2=0} = \frac{V_1}{i_2}\Big|_{i_1=0}$$

or $\boxed{Z_{21} = Z_{12}}$

All the parameters have the dimensions of impedance. Moreover individual parameters are specified only when the current in one of the ports is zero. This corresponds to one of the ports being open-circuited from which the Z-parameters also derive the name "open-circuit impedance parameters".

Prob:- find the z-parameters for the circuit shown below.



Sol:- The circuit in the problem is a T-network. from eqns ① & ②, we have

$$V_1 = Z_{11} i_1 + Z_{12} i_2$$

$$V_2 = Z_{21} i_1 + Z_{22} i_2$$

→ when the port bb' is open circuited, $Z_{11} = V_1/i_1$

where $V_1 = i_1(Z_a + Z_b) \Rightarrow \boxed{Z_{11} = Z_a + Z_b}$

$$Z_{21} = \frac{V_2}{i_1}\Big|_{i_2=0} \quad \text{where } V_2 = I_1 Z_b$$

$$\therefore \boxed{Z_{21} = Z_b}$$

→ when the port a-a' is open circuited, $i_1 = 0$

$$Z_{22} = \frac{V_2}{i_2}\Big|_{i_1=0}$$

where $V_2 = i_2 (Z_b + Z_c)$

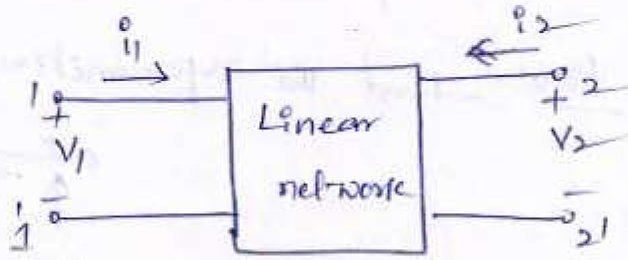
$$\therefore \; Z_{22} = Z_b + Z_c$$

$$Z_{12} = \frac{V_1}{i_2}\Big|_{i_1=0}$$

where $V_1 = i_2 Z_b \Rightarrow \boxed{Z_{12} = Z_b}$

It can be observed that $Z_{12} = Z_{21}$, so the network is a bilateral network which satisfies the principle of reciprocity.

Y (Admittance) - parameters:



If V_1 and V_2 are independent variables, then i_1 and i_2 are dependent variables. The dependent variables are written as linear combinations of independent variables as

$$I_1 = y_{11} V_1 + y_{12} V_2 \longrightarrow (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \longrightarrow (2)$$

In matrix notation, we have

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Coefficient y_{11} can be measured by taking the ratio of I_1 over V_1 when port-2 is short-circuited ($V_2 = 0$).

$$y_{11} = I_1 / V_1 \big|_{V_2 = 0}$$

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Coefficient Y_{11} → input admittance with port 2 short circuited.

Similarly,

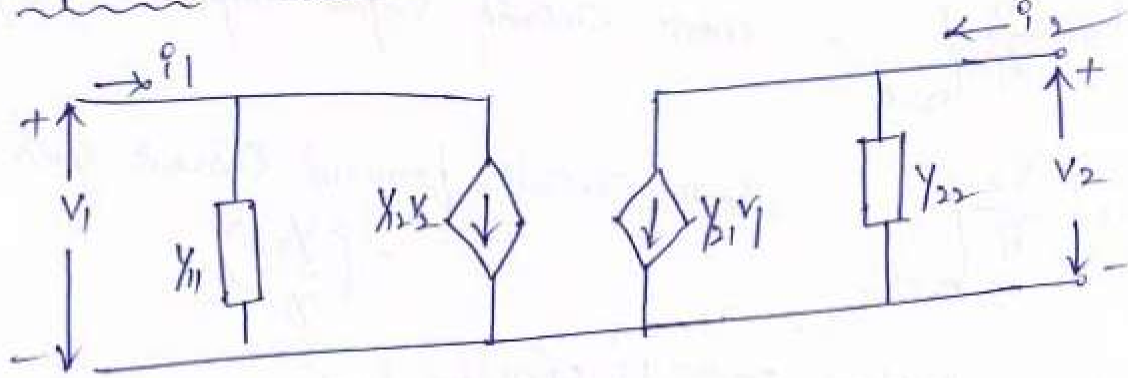
$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \rightarrow \text{Transfer admittance with port 1 short circuited}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} \rightarrow \text{Transfer admittance with port 2 short circuited}$$

$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} \rightarrow \text{output admittance with port 1 short circuit}$$

→ coefficients Y_{11}, Y_{12}, Y_{21} & Y_{22} have units of Siemens (A/V)

Equivalent Circuit:



*→ If the circuit is reciprocal, we have

$$\boxed{Y_{12} = Y_{21}}$$

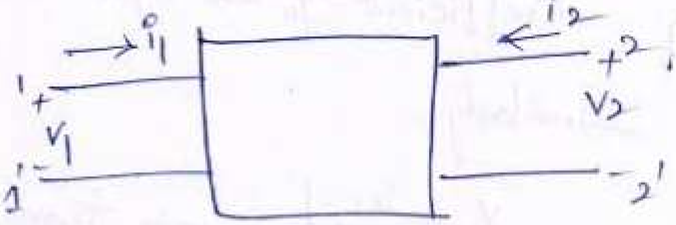
h [Hybrid] parameters:—

→ Hybrid parameters find extensive use in transistor circuits.

→ The hybrid matrices describes a two-port network, when the voltage of one port and the current of other port are taken as the independent variables.

$$V_1 = h_{11} i_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} i_1 + h_{22} V_2 \longrightarrow (2)$$



The Coefficients in the above equations are called "hybrid parameters". In Matrix notation,

$$\begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix}$$

The individual h-parameters may be deffined by Letting $i_1 = 0$ & $V_2 = 0$.

→ when $V_2 = 0$, the port $22'$ is short circuited.

then $h_{11} = \left.\frac{V_1}{i_1}\right|_{V_2=0}$ = short circuit input impedance $= \left[\frac{1}{Y_{11}}\right]$

$h_{21} = \left.\frac{i_2}{i_1}\right|_{V_2=0}$ = short circuit forward current gain $= \left(\frac{Y_{21}}{Y_{11}}\right)$

→ Similarly, by letting port $11'$ open, i.e. $i_1 = 0$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{i_1=0}$ ⟹ open circuit reverse voltage gain $= \left[\frac{Z_{12}}{Z_{22}}\right]$

$h_{22} = \left.\frac{I_2}{V_2}\right|_{i_1=0}$ ⟹ open circuit output admittance $= \frac{1}{Z_{22}}$

Since the h-parameters represent dimensionally an impedance, an admittance, a voltage gain and current gain, these are called "hybrid parameters".

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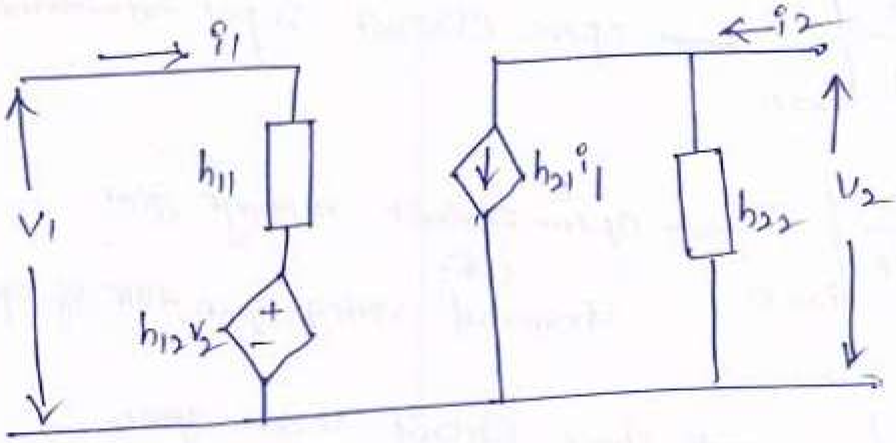
Equivalent Circuit:

An equivalent circuit of a two-port network interms of hybrid parameters is shown in the following figure.



* The two-port circuit is reciprocal when

$$\boxed{h_{12} = -h_{21}}$$ **

g [Inverse Hybrid] - parameters:

→ The input current I_1 and output voltage can be expressed interms of i_2 and V_1. The equations are as follows:

$$i_1 = g_{11}V_1 + g_{12}i_2 \longrightarrow (1)$$

$$V_2 = g_{21}V_1 + g_{22}i_2 \longrightarrow (2)$$

The coefficients in the above equation are called the "inverse hybrid parameters".

In Matrix notation.

$$\begin{bmatrix} i_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ i_2 \end{bmatrix}$$

It can be verified that

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

The individual g-parameters may be defined by Letting $i_2=0$ and $v_1=0$ in the above equations ① & ②. Thus,

When $i_2=0$

$$g_{11} = \frac{i_1}{v_1}\bigg|_{i_2=0} \rightarrow \text{open-circuit input admittance} = \frac{1}{z_{11}}$$

$$g_{21} = \frac{v_2}{v_1}\bigg|_{i_2=0} \rightarrow \text{open-circuit voltage gain}$$

(or) forward voltage gain with o/p open

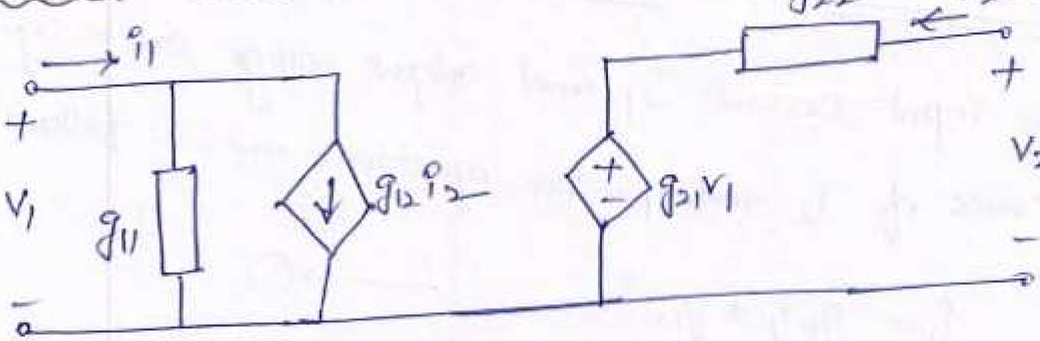
When $v_1=0$:

$$g_{12} = \frac{i_1}{i_2}\bigg|_{v_1=0} \rightarrow \text{short circuit reverse gain}$$

(or) reverse current gain with i/p short

$$g_{22} = \frac{v_2}{i_2}\bigg|_{v_1=0} \rightarrow \text{short circuit output impedance} = \frac{1}{y_{22}}.$$

Equivalent Circuit



ABCD [Transmission] parameters:—

If v_2 and i_2 are independent variables, then v_1 and i_1 are dependent variables. The dependent variables are written as linear combinations of independent variables as

$$\boxed{\begin{aligned} V_1 &= AV_2 - Bi_2 \\ i_1 &= CV_2 - Di_2 \end{aligned}}$$

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In Matrix notation, we have

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

→ Transmission parameters provide a direct relationship between input & o/p.

Here,

Coefficient 'A' can be found by taking the ratio of V_1 over V_2 when port '2' is open circuited ($i_2=0$):

$$A = \left.\frac{V_1}{V_2}\right|_{i_2=0}$$ = Reverse voltage gain with o/p is open circuited.

$$B = \left.\frac{V_1}{-i_2}\right|_{V_2=0}$$ → Negative transfer impedance with o/p short circuited.

$$C = \left.\frac{i_1}{V_2}\right|_{i_2=0}$$ → Transfer admittance with output open circuited

$$D = \left.\frac{i_1}{-i_2}\right|_{V_2=0}$$ → Negative current ratio (or) Negative reverse current gain with output is short circuited

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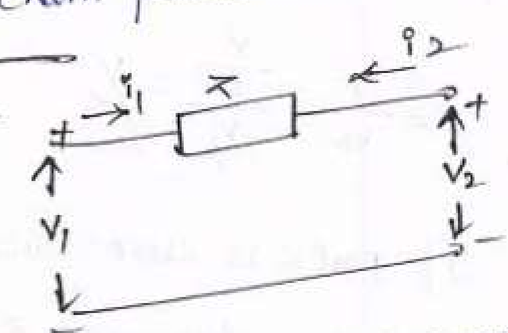
→ Transmission parameters also called 'General circuit parameters' or 'chain parameters'.

Prob:- 

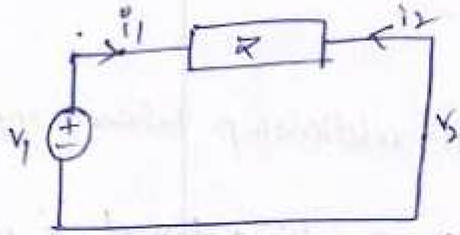
Find the ABCD parameters of the circuit → ?

Sol:- when port-2 or output is open circuited

i_1 & i_2 are zero. $\therefore V_2 = V_1 \Rightarrow A = \frac{V_2}{V_1} = 1$

$$C = \left.\frac{I_1}{V_2}\right|_{i_2=0} = \frac{0}{V_1} = 0$$

when port 2 or output port is shorted



No two currents are exist in a single path. $\therefore I_2 = -I_1$.

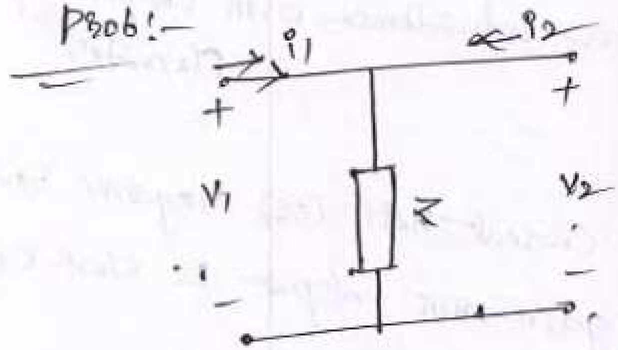
$$I_1 = \frac{V_1}{Z} \implies I_2 = -V_1/Z$$

$$\therefore B = \frac{-V_1}{i_2} = \frac{-V_1}{\frac{-V_1}{Z}} = Z$$

$$D = \left.\frac{-I_1}{i_2}\right|_{V_2=0} = -\frac{I_1}{-I_1} = 1$$

$$\therefore T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \longrightarrow (1)$$

Prob:- find ABCD parameters for the circuit shown.



Sol:-

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$I_2 = 0 \implies$ open circuit at an o/p port. The voltage drop across Z is V_1 which is equal to V_2.

$$C = \left.\frac{i_1}{V_2}\right|_{i_2=0}$$

$$\therefore A = \frac{V_1}{V_2} = \frac{V_1}{V_1} = 1, \quad C = \frac{i_1}{V_2} = \frac{\frac{V_1}{Z}}{V_1} = 1/Z$$

$$B = \left.\frac{V_1}{-i_2}\right|_{V_2=0}$$

If port 2 is short circuited ($V_2=0$), the voltage drop across Z is 0. Thus $V_1 = V_2 = 0$. Also, since there is no current through Z, we have $I_1 = -i_2$.

$$D = \left.\frac{i_1}{-i_2}\right|_{V_2=0} = \frac{i_1}{i_1} = 1$$

$$\therefore T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix} \longrightarrow (2)$$

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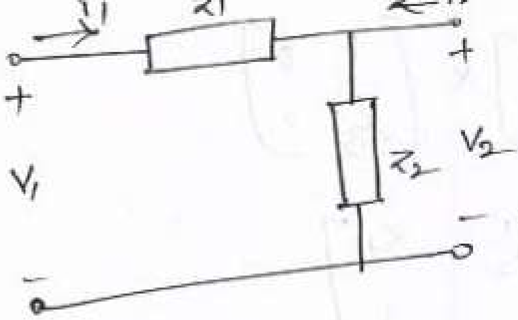


find the ABCD parameters for the Circuit shown.

Sol:-

A= To find A & C, make port-2 is open circuited.

when port-2 is open i.e. $i_2=0$, from the voltage divider rule, voltage V_2 is

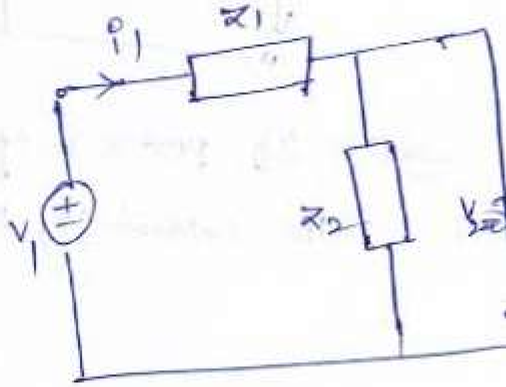
$$V_2 = \frac{z_2}{z_1+z_2} V_1 \quad \text{and} \quad i_1 = \frac{V_1}{z_1+z_2}$$

Since $A = \frac{V_1}{V_2} = \frac{z_1+z_2}{z_2} = 1+\frac{z_1}{z_2}$ //

$$C = \frac{i_1}{V_2} = \frac{V_1/z_1+z_2}{\left(z_2/z_1+z_2\right)V_1} = \frac{1}{z_1+z_2} \times \frac{z_1+z_2}{z_2} = 1/z_2 \text{ //}$$

To find B & D, make output is short circuit [i.e. $V_2=0$].

when port-2 is shorted i.e. $V_2=0$.



$$i_1 = \frac{V_1}{z_1} = -i_2$$

$$B = \frac{V_1}{-i_2} = \frac{V_1}{+V_1/z_1} = z_1$$

$$D = \frac{i_1}{-i_2} = \frac{i_1}{i_1} = 1$$

$$\therefore\ T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{z_1}{z_2} & z_1 \\ \frac{1}{z_2} & 1 \end{bmatrix} \longrightarrow (3)$$

The given circuit can be interpreted as a cascade of series impedance z_1 and parallel impedance 'z_2'. Matrix T can be obtained by multiplying the matrices of each section

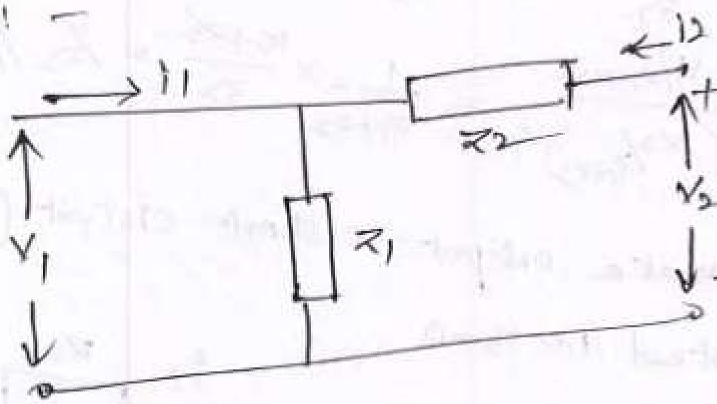
$$T=\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\times\begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1+\frac{Z_1}{Z_2} & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}.$$

*→ The determinant of the product of the two square matrices of equal size is the product of the determinants of each matrix, that is

$$\det(EF)=\det(E)\det(F).$$

Prob: –



Find the ABCD Parameters for the Circuits shown in figure.

Sol: – If port 2 is open-circuited ($i_2=0$), then $V_2=V_1$.

The Current I_1 is given by

$$i_1=\frac{V_1}{Z_1}$$

Coefficients A & C are given by

$$A=\frac{V_1}{V_2}\bigg|_{i_2=0}=1 \quad , \quad C=\frac{i_1}{V_2}\bigg|_{i_2=0}=\frac{V_1/Z_1}{V_1}=1/Z_1$$

If port 2 is short Circuited ($V_2=0$), the Current I_1 is given by

$$i_1=\frac{V_1}{Z_1}+\frac{V_1}{Z_2}=\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)V_1$$

The Current $-I_2$ is given by

$$-I_2=V_1/Z_2$$

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coefficients B and D are given by

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z_2} = Z_2$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)V_1}{V_1/Z_2} = 1 + \frac{Z_2}{Z_1}$$

$$\therefore T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_2 \\ \frac{1}{Z_1} & 1+\frac{Z_2}{Z_1} \end{bmatrix}$$

The above circuit can be interpreted as a cascade of parallel impedance Z_1 and series impedance Z_2. Matrix T can be obtained by multiplying the matrices of each section:

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z_2 \\ \frac{1}{Z_1} & 1+\frac{Z_2}{Z_1} \end{bmatrix}$$

Conversion of parameters:

(i) Conversion of Z-parameters to all other parameters:

Z to Y:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \qquad I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Since $V_1 = Z_{11}i_1 + Z_{12}i_2 \qquad V_2 = Z_{21}i_1 + Z_{22}i_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

Inverse of Z-matrix is given by

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \frac{Adj(Z)}{Det(Z)} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

where $\Delta Z = det(Z) = Z_{11}Z_{22} - Z_{12}Z_{21}$

By comparison,

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{12}}{\Delta Z} \quad Y_{21} = \frac{-Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

(ii) Conversion of Z-parameters to ABCD parameters

The equations of ABCD parameters are

$$V_1 = AV_2 - Bi_2 \rightarrow (1) \qquad i_1 = CV_2 - Di_2 \rightarrow (2)$$

Since $V_2 = Z_{21}i_1 + Z_{22}i_2 \Rightarrow \boxed{i_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}i_2} \rightarrow (3)$

from (2) & (3).

$$C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$$

Since $V_1 = Z_{11}i_1 + Z_{12}i_2 \rightarrow (4)$

Substituting eqn (3) in (4),

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}i_2\right] + Z_{12}i_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]i_2 \rightarrow (5)$$

from (1) & (5) $A = \frac{Z_{11}}{Z_{21}}$ // $B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$ //

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Conversion of z-parameters to h-parameters

The equations of h-parameters are

$$v_1 = h_{11} i_1 + h_{12} v_2 \longrightarrow (1) \quad i_2 = h_{21} i_1 + h_{22} v_2 \longrightarrow (2)$$

Since $v_1 = z_{11} i_1 + z_{12} i_2 \longrightarrow (3) \quad v_2 = z_{21} i_1 + z_{22} i_2 \longrightarrow (4)$

from eqn (4), we have

$$i_2 = -\frac{z_{21}}{z_{22}} i_1 + \frac{1}{z_{22}} v_2 \longrightarrow (5)$$

By comparing (2) & (5).

$$h_{21} = \frac{-z_{21}}{z_{22}} \text{ //} \quad h_{22} = \frac{1}{z_{22}} \text{ //} \longrightarrow (6)$$

Substituting eqn (5) in eqn (3).

$$v_1 = z_{11} i_1 + z_{12}\left[-\frac{z_{21}}{z_{22}} i_1 + \frac{1}{z_{22}} v_2\right] = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} i_1 + \frac{z_{12}}{z_{22}} v_2 \longrightarrow (7)$$

By comparing eqns (1) & (7),

$$h_{11} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} = \frac{\Delta z}{z_{22}} \text{ //} \quad h_{12} = \frac{z_{12}}{z_{22}} \text{ //}$$

Conversion of z-parameters to g-parameters:

Equations for g-parameters are

$$i_1 = g_{11} v_1 + g_{12} i_2 \longrightarrow (1)$$

$$v_2 = g_{21} v_1 + g_{22} i_2 \longrightarrow (2)$$

Since $v_1 = z_{11} i_1 + z_{12} i_2 \longrightarrow (3)$, $v_2 = z_{21} i_1 + z_{22} i_2 \longrightarrow (4)$

from eqn (3). $i_1 = \frac{1}{z_{11}} v_1 - \frac{z_{12}}{z_{11}} i_2 \longrightarrow (5)$

Comparing eqns (1) & (5),

$$g_{11} = \frac{1}{z_{11}}, \quad g_{12} = -\frac{z_{12}}{z_{11}} \text{ //}$$

Substituting eqn (5) into eqn (4) results in

$$V_2 = z_{21}\left(\frac{1}{z_{11}} V_1 - \frac{z_{12}}{z_{11}} i_2\right) + z_{22} i_2$$

$$= \frac{z_{21}}{z_{11}} V_1 + \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}} i_2 \longrightarrow (6)$$

By Comparing (2) & (6),

$$g_{21} = \frac{z_{21}}{z_{11}}, \quad g_{22} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}} = \frac{\Delta z}{z_{11}}$$

$$\Delta z = \det(z) = \underline{z_{11}z_{22} - z_{12}z_{21}}$$

Inter-relation between Two-port parameters

	$[z]$	$[y]$	$[ABCD]$	$[h]$	$[g]$
$[z]$	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta g}{g_{11}} \end{bmatrix}$
$[y]$	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$
$[ABCD]$	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ -\frac{\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{21}} & -\frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta g}{g_{21}} \end{bmatrix}$
$[h]$	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{g_{22}}{\Delta g} & \frac{-g_{12}}{\Delta g} \\ \frac{-g_{21}}{\Delta g} & \frac{g_{11}}{\Delta g} \end{bmatrix}$
$[g]$	$\begin{bmatrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta z}{z_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & -\frac{1}{y_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{C}{A} & -\frac{\Delta T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{bmatrix}$	$\begin{bmatrix} \frac{h_{22}}{\Delta h} & \frac{-h_{12}}{\Delta h} \\ \frac{-h_{21}}{\Delta h} & \frac{h_{11}}{\Delta h} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

where

$\Delta z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta h = h_{11}h_{22} - h_{12}h_{21}$ // $\Delta T = AD - BC$, $\Delta g = g_{11}g_{22} - g_{12}g_{21}$

$\Delta y = y_{11}y_{22} - y_{12}y_{21}$

Image Impedance :-

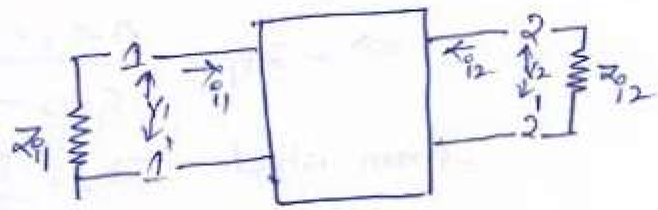

figure 1

The image impedance z_{i1} and z_{i2} of a two-port network shown in figure ① are two values of impedance such that, if the port 1-1' of the network is terminated in z_{i1}, the input impedance of port 2-2' is z_{i2}; and if the port 2-2' is terminated in z_{i2}, the input impedance at the port 1-1' is z_{i1}. Then, z_{i1} and z_{i2} are called "Image impedances" of the two-port network. These parameters can be obtained interms of two-port parameters. Since,

$$V_1 = AV_2 - Bi_2$$
$$i_1 = CV_2 - Di_2$$

If the network is terminated in z_{i2} at 2-2' as shown in figure 2.

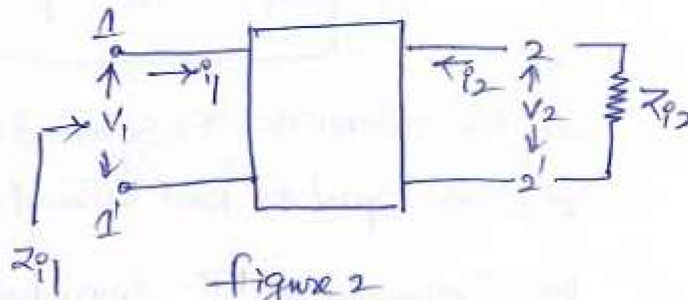

figure 2

$$V_2 = -i_2 z_{i2}$$

$$\frac{V_1}{i_1} = \frac{AV_2 - Bi_2}{CV_2 - Di_2} = z_{i1} \Rightarrow z_{i1} = \frac{-Ai_2 z_{i2} - Bi_2}{-Ci_2 z_{i2} - Di_2} = \frac{-Az_{i2} - B}{-Cz_{i2} - D}$$

$$\text{or } z_{i1} = \frac{Az_{i2} + B}{Cz_{i2} + D} //$$

Similarly,

If the network is terminated in z_{i1} at the port 1-1' as shown in figure ③, then

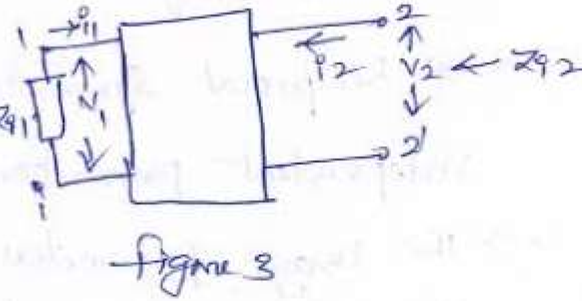

figure 3

$$V_1 = -i_1 Z_{i1}$$

$$\frac{V_2}{i_2} = Z_{i2}$$

$$\therefore -Z_{i1} = \frac{V_1}{i_1} = \frac{AV_2 - Bi_2}{CV_2 - Di_2} \Rightarrow -Z_{i1} = \frac{Ai_2 Z_{i2} - Bi_2}{Ci_2 Z_{i2} - Di_2}$$

$$\Rightarrow -Z_{i1} = \frac{AZ_{i2} - B}{CZ_{i2} - D}$$

$$-Z_{i1} C Z_{i2} + D Z_{i1} = A Z_{i2} - B$$

$$\Rightarrow B + DZ_{i1} = AZ_{i2} + CZ_{i1}Z_{i2}$$

$$\Rightarrow B + DZ_{i1} = Z_{i2}\left[A + CZ_{i1}\right]$$

$$\Rightarrow Z_{i2} = \frac{B + DZ_{i1}}{A + CZ_{i1}}$$

from which, $Z_{i2} = \dfrac{DZ_{i1} + B}{CZ_{i1} + A}$ //

Substituting the value of Z_1 in the above equation,

$$Z_{i2}\left[C\frac{(-AZ_{i2}+B)}{(CZ_{i2}-D)} + A\right] = D\left[\frac{-AZ_{i2}+B}{CZ_{i2}-D}\right] + B$$

from which, $Z_{i2} = \sqrt{\dfrac{BD}{AC}}$

Similarly, we can find $Z_{i1} = \sqrt{\dfrac{AB}{CD}}$

If the network is symmetrical, then $A = D$

$$\therefore \boxed{Z_{i1} = Z_{i2} = \sqrt{B/C}}$$

If the network is symmetrical, the image impedances Z_{i1} and Z_{i2} are equal to each other; the image impedance is then called the "characteristic impedance" or "the iterative impedance".

→ If a symmetrical network is terminated in Z_L, its input impedance will also be Z_L or its impedance transformation ratio is unity.

→ A Reciprocal symmetrical network can be described by two independent parameters.

→ The image parameters Z_{i1} and Z_{i2} are sufficient to characterize reciprocal symmetrical networks.

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→ for a generalized network, the two image parameters z_{i1} and z_{i2} are not completely defines a network. A third parameter called 'image transfer constant ϕ' is also used to describe ~~symm~~ reciprocal networks.

The image transfer constant may be obtained from the voltage and current ratios.

If the image impedance z_{i2} is connected across the port 2-2' then

$$V_1 = AV_2 - Bi_2 \longrightarrow (1)$$

$$V_2 = -i_2 z_{i2} \Rightarrow i_2 = -\frac{V_2}{z_{i2}} \longrightarrow (2)$$

$$\therefore V_1 = AV_2 - B\left[\frac{-V_2}{z_{i2}}\right] = \left[A + \frac{B}{z_{i2}}\right]V_2 \longrightarrow (3)$$

Since $I_1 = CV_2 - Di_2 \longrightarrow (4)$

$$I_1 = CV_2 - D\left[\frac{-V_2}{z_{i2}}\right] = CV_2 + \frac{D}{z_{i2}}V_2 = \left(C + \frac{D}{z_{i2}}\right)V_2.$$

or

$$I_1 = C\left[-i_2 z_{i2}\right] - Di_2 = -\left[Cz_{i2} + D\right]i_2 \longrightarrow (5)$$

from equation (3),

$$V_1 = \left[A + \frac{B}{z_{i2}}\right]V_2 \Rightarrow \frac{V_1}{V_2} = \left(A + \frac{B}{z_{i2}}\right) \longrightarrow (6)$$

Since $z_{i2} = \sqrt{\frac{BD}{AC}} \longrightarrow (7)$

$$\therefore \frac{V_1}{V_2} = A + B\sqrt{\frac{AC}{BD}} \Rightarrow \frac{V_1}{V_2} = A + \sqrt{\frac{ABC}{D}} = A + \frac{\sqrt{ABCD}}{D} \longrightarrow (8)$$

from equation (5),

$$\frac{-I_1}{i_2} = \left[Cz_{i2} + D\right] = D + C\sqrt{\frac{BD}{AC}}$$

$$\Rightarrow -I_1/i_2 = D + \sqrt{\frac{BCD}{A}} = D + \frac{\sqrt{ABCD}}{A} \longrightarrow (9)$$

Multiplying equations (8) and (9), we have

$$\frac{+V_1}{V_2} \times \frac{i_1}{-i_2} = \left[\frac{AD+\sqrt{ABCD}}{D}\right]\left[\frac{AD+\sqrt{ABCD}}{A}\right]$$

$$= \frac{\left[AD+\sqrt{ABCD}\right]^2}{AD} = \frac{(AD)^2+(ABCD)+2AD\sqrt{ABCD}}{AD}$$

$$= AD+BC+2\sqrt{(AD)BC} = \left[\sqrt{AD}+\sqrt{BC}\right]^2$$

or $\sqrt{AD}+\sqrt{BC} = \sqrt{\frac{-V_1}{V_2} * \frac{i_1}{i_2}}$ Since $AD-BC=1$

$\therefore BC = AD-1$

$$\therefore \sqrt{AD}+\sqrt{AD-1} = \sqrt{\frac{-V_1}{V_2} \times \frac{i_1}{i_2}}$$

Let $\cosh\phi = \sqrt{AD}$, $\sinh\phi = \sqrt{AD-1}$.

$$\therefore \tanh\phi = \frac{\sqrt{AD-1}}{\sqrt{AD}} = \sqrt{\frac{BC}{AD}}$$

$$\therefore \boxed{\phi = \tanh^{-1}\sqrt{\frac{BC}{AD}}}$$

Also, $e^{\phi} = \cosh\phi + \sinh\phi = \sqrt{\frac{-V_1 i_1}{V_2 i_2}}$

$$\phi = \log_e \sqrt{\left(-\frac{V_1 i_1}{V_2 i_2}\right)} = \frac{1}{2}\log_e\left[\frac{-V_1}{V_2}\frac{i_1}{i_2}\right]$$

Since $V_1 = Z_{i_1} i_1$, $V_2 = -Z_{i_2} i_2$

$$\therefore \phi = \frac{1}{2}\log_e\left[\frac{Z_{i_1}}{Z_{i_2}}\right] + \log\left[i_1/i_2\right]$$

For symmetrical reciprocal networks,

$$Z_{i_1} = Z_{i_2}$$

$$\boxed{\phi = \log_e\left(\frac{i_1}{i_2}\right) = \gamma}$$

where γ is the propagation constant.

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Network function:— [Driving Point and Transfer functions]

A network function gives the relation between Currents or voltages at different parts of the network. It is broadly classified as "Driving point and Transfer function". It is associated with terminals and ports.

(i) Driving-point functions:—

If excitation and response are measured at the same ports, the network function is known as the 'Driving-point function'.

→ for a one-port network, only one voltage and current are specified and hence only one network function (and its reciprocal) can be defined.

(a) Driving-point Impedance function: It is defined as the ratio of the voltage transform at one-port to the Current transform at the same port. It is denoted by $Z(s)$.

$$\boxed{Z(s) = \frac{V(s)}{I(s)}}$$

(b) Driving-point Admittance function:—

It is defined as the ratio of the Current transform at one port to the voltage transform at the same port. It is denoted by

$$\boxed{Y(s) = \frac{I(s)}{V(s)}}$$

for a two-port network, the driving-point impedance function and driving-point admittance function at port-1 are

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad , \quad Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

Similarly at port 2,

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)} \quad , \quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

(ii) Transfer functions:—

The transfer function is used to describe networks which have atleast two-ports.

→ Transfer function relates a voltage or current at one-port to the voltage or current at another port. These functions are also defined as the ratio of a response transform to an excitation transform. Thus, there are four possible forms of transfer functions.

(a) Voltage transfer function:—

It is defined as the ratio of the voltage transform at one port to the voltage transform at another port. It is denoted by $G(s)$.

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)}, \quad G_{21}(s) = \frac{V_1(s)}{V_2(s)}$$

(b) Current transfer function:—

It is defined as the ratio of the current transform at one port to the current transform at another port. It is denoted by $\alpha(s)$.

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)}, \quad \alpha_{21}(s) = \frac{I_1(s)}{I_2(s)} //$$

(c) Transfer impedance function:—

It is defined as the ratio of the voltage transform at one port to the current transform at another port. It is denoted by $Z(s)$.

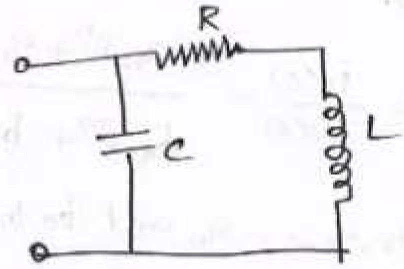
$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}, \quad Z_{21}(s) = \frac{V_1(s)}{I_2(s)} //$$

(d) Transfer admittance function:—

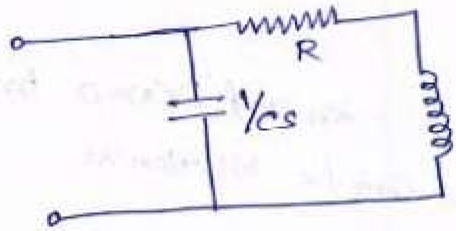
It is defined as the ratio of the current transform at one port to the voltage transform at another port. It is denoted by $Y(s)$.

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)}, \quad Y_{21}(s) = \frac{I_1(s)}{V_2(s)} \,//$$

Prob:- Determine the driving-point impedance function of a one-port network.



Sol:- The transformed network is

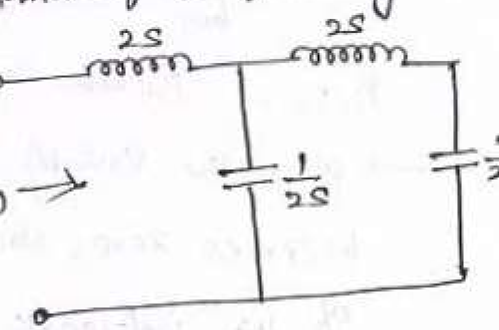


$$Z(s) = \frac{\frac{1}{Cs}(R+Ls)}{\frac{1}{Cs} + R + Ls}$$

$$= \frac{R+Ls}{LCs^2 + RCs + 1}$$

$$Z(s) = \frac{1}{C} * \frac{s + R/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Prob:- Determine the driving-point impedance of the following network



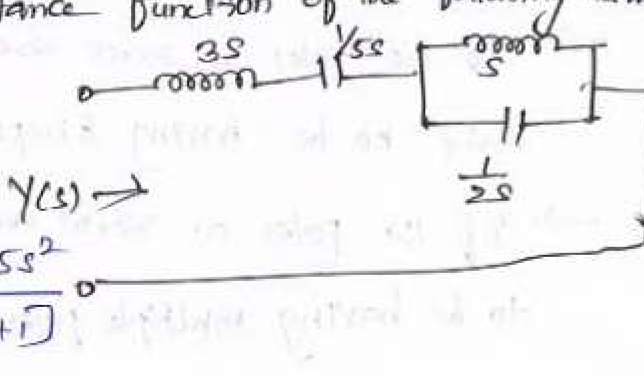
Sol:-

$$Z(s) = 2s + \frac{\frac{1}{2s}\left[2s + \frac{1}{2s}\right]}{\frac{1}{2s} + 2s + \frac{1}{2s}} = 2s + \frac{\frac{1}{2s}\left[2s + \frac{1}{2s}\right]}{\frac{2+4s^2}{2s}}$$

$$= 2s + \frac{2s + \frac{1}{2s}}{2+4s^2} = \frac{4s + 8s^3 + 2s + 1/2s}{2+4s^2}$$

$$Z(s) = \frac{16s^4 + 12s^2 + 1}{8s^3 + 4s}$$

Prob:- Find the driving-point admittance function of the following network



Sol: $Z(s) = 3s + \frac{1}{5s} + \frac{s\left[\frac{1}{2s}\right]}{s + 1/2s}$

$$Z(s) = 3s + \frac{1}{5s} + \frac{s}{2s^2+1} = \frac{30s^4 + 15s^2 + 2s^2 + 1 + 5s^2}{5s[2s^2+1]}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{5s(2s^2+1)}{30s^4 + 22s^2 + 1} \,//$$

Poles and Zeros of Network functions:

The network function F(s) can be written as ratio of two polynomials.

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

where $a_0, a_1, \dots a_n$ and $b_0, b_1, \dots b_m$ are the coefficients of the polynomials N(s) and D(s). These are real and positive for networks of passive elements.

Let N(s)=0 have 'n' roots as $z_1, z_2, \dots z_n$ and D(s)=0 have 'm' roots as $P_1, P_2, \dots P_m$. Then F(s) can be written as

$$F(s) = H \frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-P_1)(s-P_2)\dots(s-P_m)}$$

where $\frac{a_n}{b_m}$ is a constant called "scale factor" and $z_1, z_2, \dots z_n$.

$P_1, P_2, \dots P_m$ are complex frequencies.

→ When the variable 's' has the values $z_1, z_2, \dots z_n$, the network function becomes zero; such complex frequencies are known as the "zeros of the network function".

→ when the variable 's' has values $P_1, P_2, \dots P_m$, the network function becomes infinite; such complex frequencies are known as the poles of the network function.

→ A network function is completely ~~sp~~ specified by its poles, zeros and the scale factor.

→ If the poles or zeros are not repeated, then the function is said to be having simple poles or simple zeros. ~~If~~

→ If the poles or zeros are repeated, then the function is said to be having multiple poles or multiple zeros.

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→ When $n > m$, then $(n-m)$ zeros are at $S=\infty$, and for $m > n$, $(m-n)$ poles are at $S=\infty$.

→ For any network function, poles and zeros at zero and infinity are taken into account in addition to finite poles and zeros.

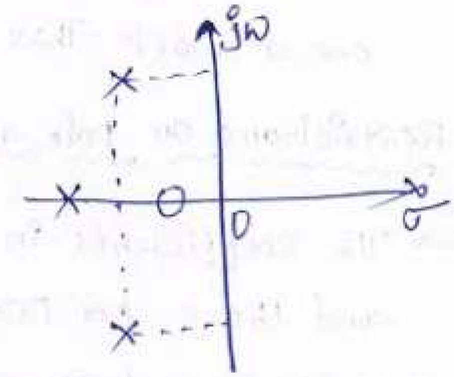

fig. pole-zero plot

→ Poles and zeros are critical frequencies.

→ The network function becomes infinite at poles, while the network function becomes zero at zeros.

→ The network function has a finite, non-zero value at other frequencies.

→ Poles and zeros provide a representation of a network function as shown in figure. The zeros are shown by circles and the poles by crosses.

Restrictions on pole and zero locations for driving-point functions:

→ The coefficients in the polynomials N(s) and D(s) must be real and positive.

→ The poles and zeros, if complex or imaginary, must occur in conjugate pairs.

→ The real part of all poles and zeros, must be negative or zero, i.e. the poles and zeros must lie in left half of S-plane.

→ If the real part of pole or zero is zero, then that pole or zero must be simple.

→ The polynomials N(s) and D(s) may not have missing terms between those of highest and lowest degree, unless all even or all odd terms are missing.

→ The degree of N(s) and D(s) may differ by either zero or one only. This condition prevents multiple poles and zeros at $S=\infty$.

→ The terms of lowest degree in N(s) and D(s) may differ in degree by one at most. This condition prevents multiple poles and zeros at s=0.

Restrictions on pole and zero Locations for Transfer functions:—

→ The coefficients in the polynomials N(s) and D(s) must be real, and those for D(s) must be positive.

→ The poles and zeros, if complex or imaginary, must occur in conjugate pairs.

→ The real part of poles must be negative or zero. If the real part is zero, then that pole must be simple.

→ The polynomial D(s) may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.

→ The polynomial N(s) may have terms missing between the terms of lowest and highest degree, and some of the coefficients may be negative.

→ The degree of N(s) may be as small as zero, independent of the degree of D(s).

→ For voltage and current transfer functions, the maximum degree of N(s) is the degree of D(s).

→ For transfer impedance and admittance functions, the maximum degree of N(s) is the degree of D(s) plus one.

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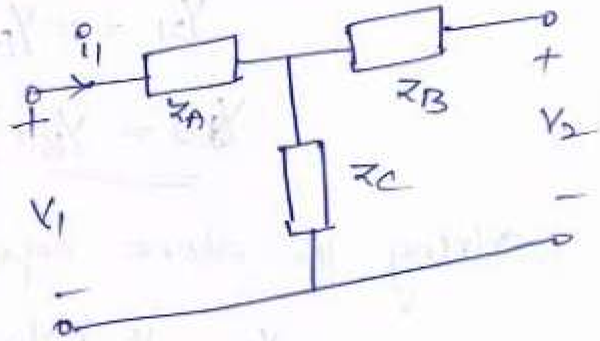
T-network:

Any two-port network can be represented by an equivalent T network as shown in the figure. The elements of the equivalent T network may be expressed interms of z-parameters.



Applying KVL to mesh 1:

$$V_1 = Z_A i_1 + Z_C(i_1+i_2)$$
$$= (Z_A+Z_C)i_1 + Z_C i_2 \longrightarrow (1)$$

Applying KVL to mesh 2:

$$V_2 = Z_B i_2 + Z_C(i_1+i_2) = Z_C i_1 + (Z_B+Z_C) i_2 \longrightarrow (2)$$

Comparing equations (1) & (2) with z-parameters equations.

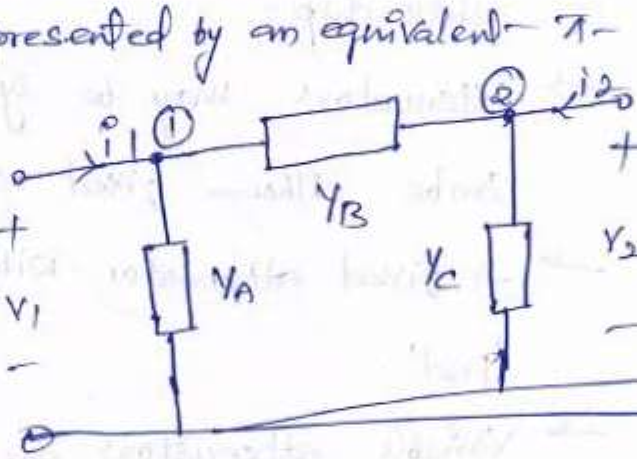
$$Z_{11} = Z_A + Z_C, \quad Z_{12} = Z_C, \quad Z_{21} = Z_C, \quad Z_{22} = Z_B + Z_C$$

Solving the above equations.

$$Z_A = Z_{11} - Z_{12} = Z_{11} - Z_{21}$$
$$Z_B = Z_{22} - Z_{21} = Z_{22} - Z_{12}$$
$$Z_C = Z_{12} = Z_{21}$$

π-network:

Any two-port network can be represented by an equivalent π-network as shown in figure.



Applying KCL at node (1).

$$i_1 = Y_A V_1 + Y_B(V_1 - V_2)$$
$$= (Y_A+Y_B)V_1 - Y_B V_2 \longrightarrow (1)$$

Applying KCL at node 2,

$$i_2 = Y_C V_2 + Y_B(V_2 - V_1) = -Y_B V_1 + (Y_B+Y_C)V_2 \longrightarrow (2)$$

By comparing equations ① & ② with Y-parameter equations,

$$Y_{11} = Y_A + Y_B$$

$$Y_{12} = -Y_B$$

$$Y_{21} = -Y_B$$

$$Y_{22} = Y_B + Y_C$$

solving the above equations,

$$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$$

$$Y_B = -Y_{12} = -Y_{21}$$

$$Y_C = Y_{22} + Y_{12} = Y_{22} + Y_{21}$$

Attenuators :-

→ An attenuator is a two-port resistive network and is used to reduce the signal level by a given amount

→ It is necessary to introduce a specified loss between source and a matched load without altering the impedance relationship.

→ Attenuators may be symmetrical or Asymmetrical, and can be either fixed or variable.

→ A fixed attenuator with constant attenuation is called a 'pad'.

→ Variable attenuators are used as volume controls in radio broadcasting sections.

→ Attenuators are also used in laboratories to obtain small values

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of voltage or current for testing circuits.

→ Attenuation is expressed either in decibles or nepers.

→ Attenuation offered by a network in decibels is

$$\text{Attenuation (dB)} = 10 \log_{10} \left[P_1/P_2 \right]$$

where $P_1 \rightarrow$ input power, $P_2 \rightarrow$ output power

→ for a properly matched network,

$$P_1 = i_1^2 R_0 = \frac{V_1^2}{R_0}$$

$$P_2 = i_2^2 R_0 = \frac{V_2^2}{R_0}$$

where $R_0 \rightarrow$ characteristic resistance of the network.

Hence

$$\text{Attenuation (dB)} = 20 \log_{10} \frac{V_1}{V_2} = 20 \log_{10} \frac{i_1}{i_2}$$

→ Attenuation is also expressed in nepers as the natural algorithm of the voltage or current ratio

$$\text{Attenuation in nepers} = \ln \frac{V_1}{V_2} = \ln \frac{i_1}{i_2} = \frac{1}{2} \ln \frac{P_1}{P_2}$$

$$\frac{V_1}{V_2} = \frac{i_1}{i_2} = N \text{ then } \frac{P_1}{P_2} = N^2$$

$$\text{Attenuation in dB} = 20 \log_{10} N$$

$$N = \text{antilog} \left(\frac{dB}{20} \right)$$

where 'N' is the attenuation in nepers.

Relation between Decibel and Neper. —

$$\text{Attenuation [Nepers]} = \ln \frac{V_1}{V_2}$$

$$\text{Attenuation (dB)} = 20 \log_{10} V_1/V_2$$

changing the base of the algorithm,

$$\text{Attenuation (dB)} = 20\,\frac{\ln V_1/V_2}{\ln 10} = \frac{20}{2.303}\ln\frac{V_1}{V_2}$$

$$= 8.686 \text{ (attenuation in Neper)}$$

$$\text{Attenuation in nepers} = \frac{1}{8.686}\text{ (Attenuation in dB)}$$

$$= 0.115 \text{ (Attenuation in dB)}$$

Design of Attenuators:—

There are four types of attenuators, T, π, lattic and Bridged T-type.

→ An attenuator is to be designed for desired values of characteristic resistance 'R_0' and attenuation.

(i) T-type Attenuator:—

→ The values of the arms of the network can be specified in terms of characteristic impedance Z_0, and propagation constant, γ of the network.

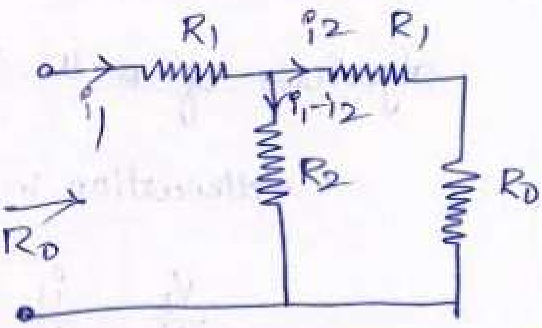

fig: symmetrical T-attenuator

Since the network is symmetrical resistive circuit hence $Z_0 = R_0$ and $\gamma = \alpha$.

→ The design equations can be obtained by applying Kirchhoff's law to the network.

from KVL,

$$R_2(i_1 - i_2) = i_2(R_1 + R_0)$$

$$\Rightarrow i_2(R_2 + R_1 + R_0) = i_1 R_2$$

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$$\frac{i_1}{i_2} = \frac{R_1+R_0+R_2}{R_2} = N \longrightarrow (1)$$

The characteristic impedance of the attenuator is R_0 when it is terminated in a load of R_0.

Hence, $$R_0 = R_1 + \frac{R_2(R_1+R_0)}{R_1+R_0+R_2} = R_1 + \frac{R_1+R_0}{N}$$

$$\Rightarrow NR_0 = NR_1 + R_1 + R_0 \Rightarrow \boxed{R_1 = \frac{R_0(N-1)}{N+1}} \longrightarrow (2)$$

from eqn (1), we have

$$NR_2 = R_1+R_0+R_2 \Rightarrow (N-1)R_2 = (R_1+R_0)$$

Substituting the value of R_1 from (2) we have

$$(N-1)R_2 = \frac{R_0(N-1)}{N+1} + R_0 \Rightarrow (N-1)R_2 = \frac{2NR_0}{N+1}$$

$$\Rightarrow \boxed{R_2 = \frac{2NR_0}{N^2-1}} \longrightarrow (3)$$

Equations (2) & (3) are the design equations for the symmetrical T-attenuator.

(ii) π-type Attenuator:

→ The series and shunt arms of the attenuator can be specified in terms of z_0 and propagation constant 'γ'.

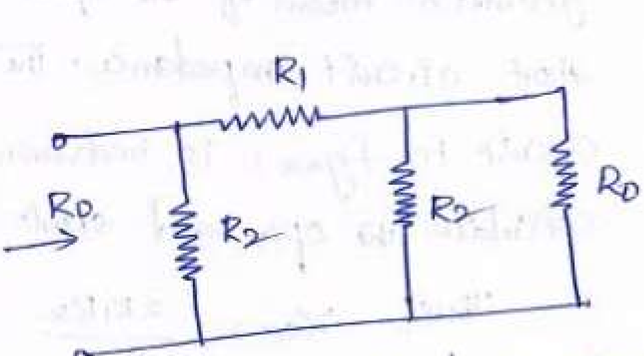


Fig: Symmetrical attenuator

→ The network is formed by resistors and is symmetrical and hence $z_0 = R_0$ and $\gamma = \alpha$.

from fundamental equations,

$$R_1 = R_0 \cdot \frac{N - \frac{1}{N}}{2} = R_0 \frac{N^2-1}{2N}$$

$$R_2 = R_0 \frac{N-1}{N+1}$$

where $N = \frac{i_1}{i_2} = e^{\gamma}$, $\gamma = \alpha$.

(iii) Lattice Attenuator: —

→ A symmetrical resistance lattice is shown in figure 1. The series and diagonal arm of the network can be specified in terms of the characteristic impedance Z_0 and propagation constant 'γ'.

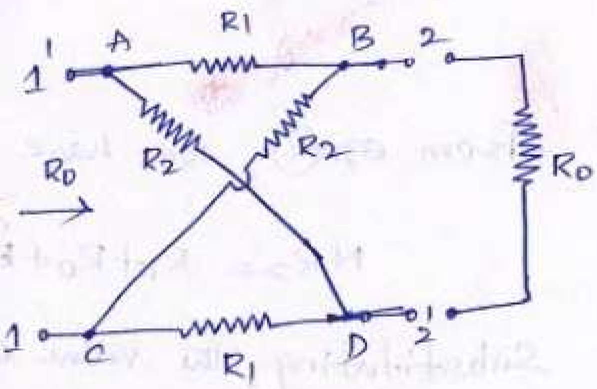


fig: 1

Since the characteristic impedance of symmetric network is the geometric mean of the open and short-circuit impedance. The circuit in figure 1 is redrawn to calculate the open and short circuit impedances.

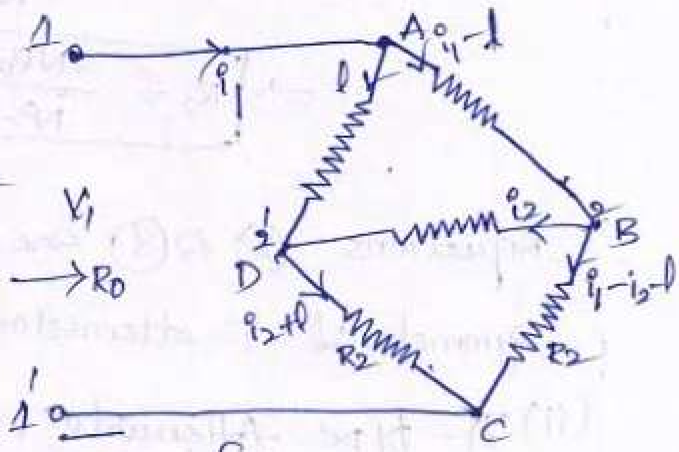


figure 2

Thus; $Z_{SC} = \frac{2R_1R_2}{R_1+R_2}$, $Z_{OC} = \frac{R_1+R_2}{2}$ ⟶ (1)

Hence, $Z_0 = R_0 = \sqrt{Z_{OC} Z_{SC}}$, $R_0 = \sqrt{R_1R_2}$ ⟶ (2)

In figure 2, the input impedance at 1-1' is R_0. When the network is terminated in 'R_0' at 2-2'.

By KVL, we get—

$$V_1 = i_1 R_0 = (i_1 - l)R_1 + i_2 R_0 + (l + i_2)R_1$$

$$i_1 R_0 = R_1 (i_1 + i_2) + i_2 R_0$$

$$\Rightarrow i_1 (R_0 - R_1) = i_2 (R_1 + R_0)$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{R_1 + R_0}{R_0 - R_1} = \frac{1 + R_1/R_0}{1 - R_1/R_0} \longrightarrow (3)$$

$$N = e^{\alpha} = \frac{i_1}{i_2} = \frac{1 + R_1/R_0}{1 - R_1/R_0} = \frac{1 + \sqrt{R_1/R_2}}{1 - \sqrt{R_1/R_2}} \longrightarrow (4)$$

∴ the propagation constant becomes

$$\alpha = \log \left[\frac{1 + \sqrt{R_1/R_2}}{1 - \sqrt{R_1/R_2}} \right]$$

Since from eqn (4).

$$N \left[1 - R_1/R_0 \right] = \left[1 + \frac{R_1}{R_0} \right] \Rightarrow \boxed{R_1 = R_0 \frac{(N-1)}{(N+1)}} \longrightarrow (5)$$

Similarly, we can express $\boxed{R_2 = R_0 \frac{(N+1)}{(N-1)}} \longrightarrow (6)$

Eqns (5) & (6) are the design equations for lattice attenuator.

(iv) Bridged-T Attenuator :—

A bridge-T attenuator is shown in figure 1. Since the attenuator is formed by resistors only hence.

$Z_0 = R_0$ and $\gamma = \alpha$.

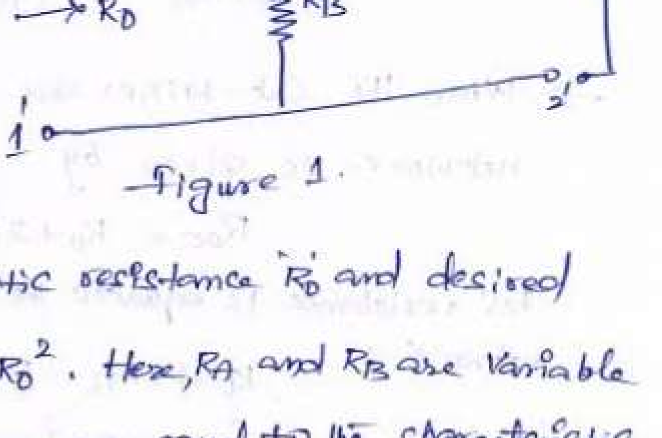


Figure 1.

→ The bridged-T network may be designed to have any characteristic resistance R_0 and desired attenuation by making $R_A R_B = R_0^2$. Here, R_A and R_B are variable resistances and all other resistances are equal to the characteristic resistance R_0 of the network.

→ The design equations of the bridged-T attenuator are obtained by bisection theorem.

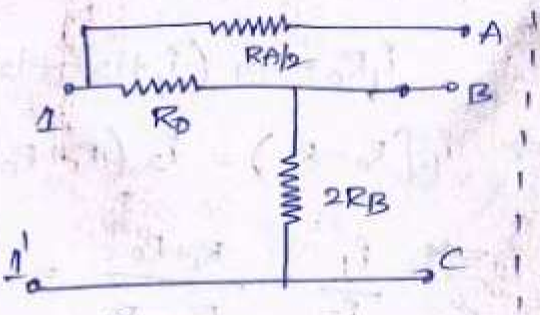


figure 2

According to bisection theorem, a network having mirror image symmetry can be reduced to an equivalent lattice structure.

The series arm of the equivalent lattice is found by bisecting the given network into two parts, short circuiting all the cut-wires and equating the series impedance of the lattice to the input impedance of the bisected network; the diagonal arm is equal to the input impedance of the bisected network when cut wires are open circuited.

from figure 2, when the cut wires A, B and C are shorted, the input resistance of the network is given by

$$R_{SC} = \frac{R_D * R_{A/2}}{R_D + R_{A/2}} = \frac{R_D R_A}{2R_D + R_A} \longrightarrow (1)$$

This resistance is equal to the series arm-resistance of the lattice network.

$$\therefore \frac{R_D R_A}{2R_D + R_A} = R_1 \longrightarrow (2)$$

Since $R_1 = R_D \frac{(N-1)}{N+1}$, [Lattice Attenuator]

Hence, $\frac{R_D R_A}{2R_D + R_A} = R_D \frac{(N-1)}{(N+1)} \Rightarrow \boxed{R_A = R_D (N-1)} \longrightarrow (3)$

→ When the cut-wires are open, the input resistance of the network is given by

$$R_{OC} = R_D + 2R_B \longrightarrow (4)$$

This resistance is equal to the diagonal arm resistance of the lattice network:

$$R_D + 2R_B = R_2 \Rightarrow R_2 = R_D \frac{N+1}{N-1}$$

Hence, $(R_D + 2R_B) = R_D \frac{N+1}{N-1} \Rightarrow \boxed{R_B = \frac{R_D}{N-1}}$

NAS

①

Unit-IV

Filters

→ Filters are frequency selective networks that attenuate signals at some frequency and allow others to pass with or without attenuation.

→ A filter is constructed from purely reactive elements.

→ An ideal filter, produces no attenuation in the desired band. Called 'pass Band' and provides attenuation at all other frequencies called "Attenuation Band" or 'Stop Band'.

→ The frequency which separates the pass band and the stop band is called the "Cut-off frequency".

→ Filters are mostly used in communication systems to separate various channels in carrier-frequency telephone circuits to avoid the interference.

→ Applications could also find in instrumentation, telemetering equipment etc.

Classification of filters:

Filters are classified into four categories based on their frequency characteristics:

(i) Low-pass filter

(ii) High-pass filter

(iii) Band-pass filter

(iv) Band-stop filter

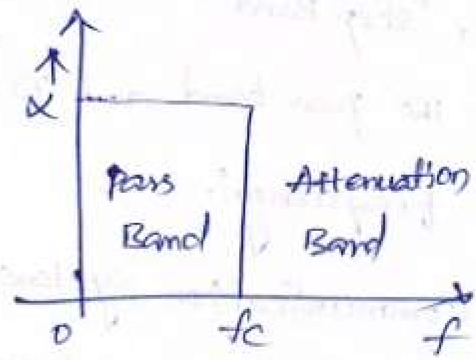
** A low pass filter allows all frequencies up to a certain cut-off frequency to pass through it and attenuates all the other frequencies above the cut-off frequency.

→ A high pass filter attenuates all the frequencies below the cutoff frequency and allows all other frequencies above the cut-off frequency to pass through it.

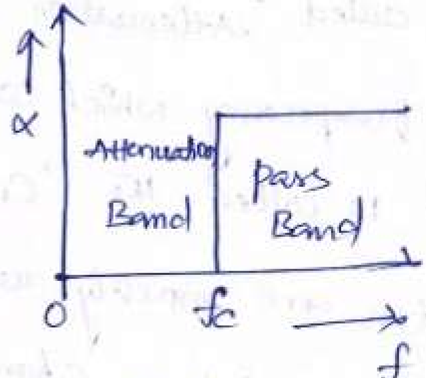
→ A band-pass filter allows a limited band of frequencies to pass through it and attenuates all other frequencies below or above the frequency band.

→ A band-stop filter attenuates a limited band of frequencies but allows all other frequencies to pass through it.

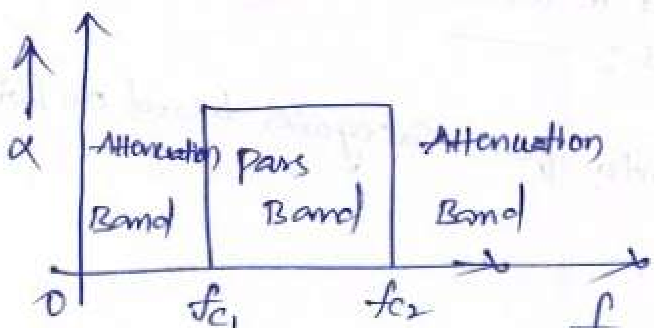
Filter Networks.



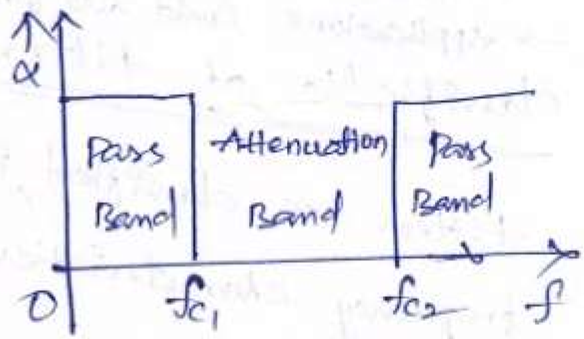
Low pass filter



High pass filter



Band-pass filter



Band-stop filter

→ Band pass filter has two cut-off frequencies and will have the pass band $f_{c_2} - f_{c_1}$. Where 'f_{c_1}' is the lower cut-off frequency and 'f_{c_2}' is the upper cut-off frequency.

Unit-2 NAS (2)

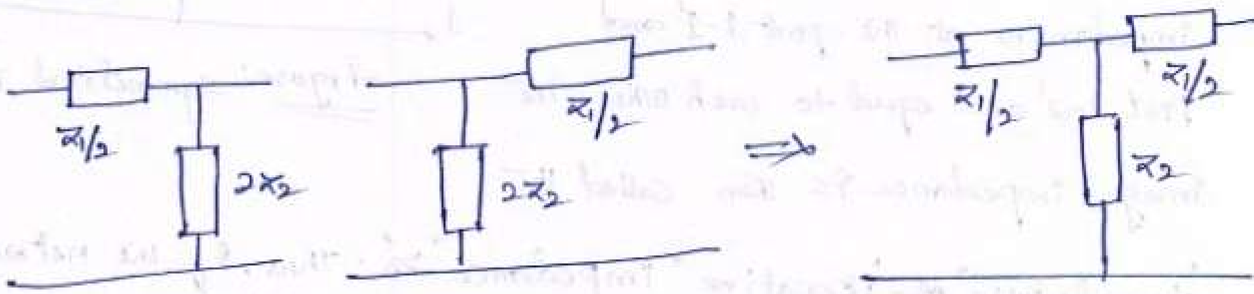
filter Networks:

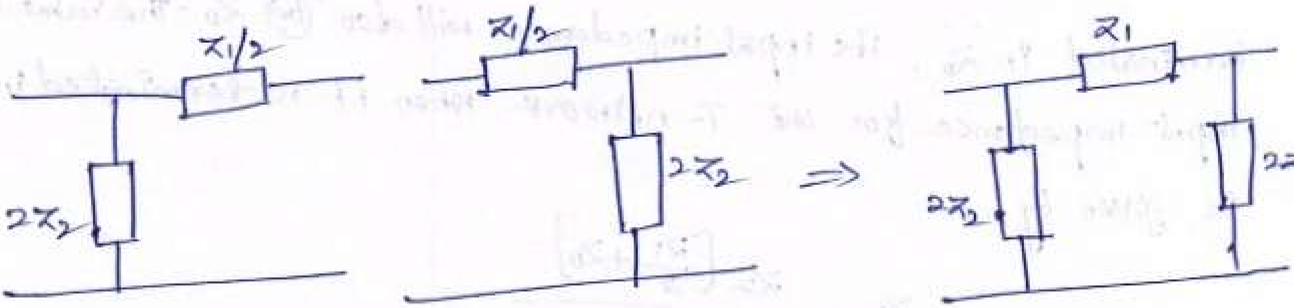
→ Ideal filters should have zero attenuation in the pass band.

→ filters are designed with an assumption that the elements of the filters are purely reactive.

→ filters are made of symmetrical T or π sections.

→ T and π sections can be considered as combinations of unsymmetrical L-sections.





→ The ladder structure is one of the commonest forms of filter network.

→ A cascade connection of several T and π sections constitutes a 'Ladder Network'.

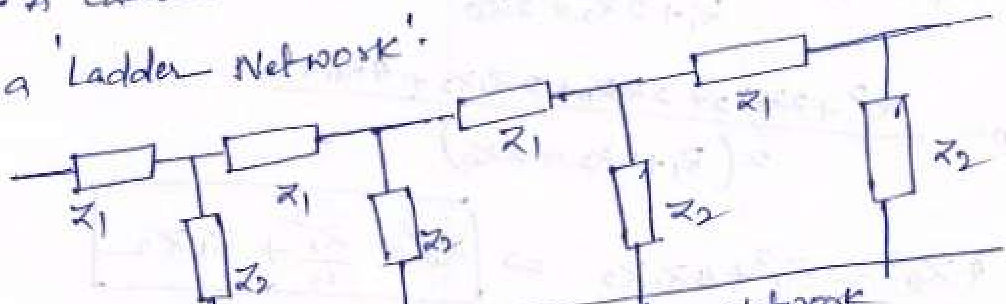


fig: T-section ladder Network

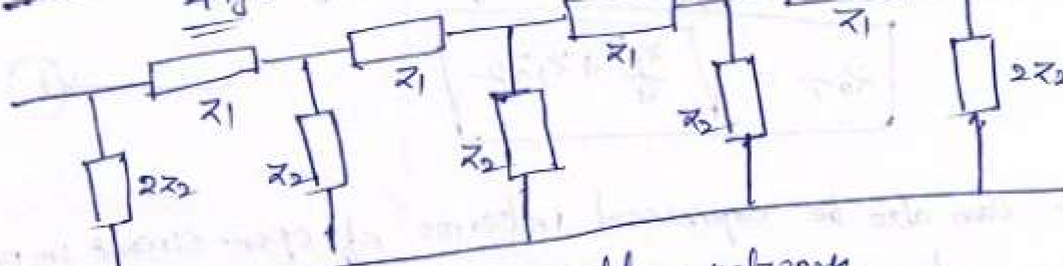


fig: π-section ladder network

To study the behaviour of any filter requires the calculation of its propagation constant (γ), attenuation constant (α), phase constant (β), and its characteristic impedance Z_0.

T-Network:

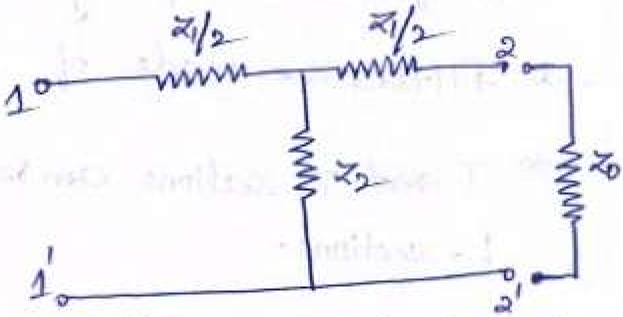


Figure: symmetrical T-network

Consider a symmetrical T-network.

As we know that if the image impedances at the port 1-1' and port 2-2' are equal to each other, the image impedance is then called the "characteristic" or "iterative" impedance "Z_0". Thus, if the network is terminated in Z_0, its input impedance will also become Z_0. The value of input impedance for the T-network when it is terminated in "Z_0" is given by.

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left[\frac{Z_1}{2}+Z_0\right]}{\frac{Z_1}{2}+Z_2+Z_0}$$

Also. $Z_{in} = Z_0$.

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left[\frac{Z_1}{2}+Z_0\right]}{Z_1+2Z_2+2Z_0} = \frac{Z_1}{2} + \frac{Z_1Z_2+2Z_2Z_0}{Z_1+2Z_2+2Z_0}$$

$$Z_0 = \frac{Z_1^2+2Z_1Z_2+2Z_1Z_0+2Z_1Z_2+4Z_0Z_2}{2(Z_1+2Z_2+2Z_0)}$$

$$\therefore \quad 4Z_0^2 = Z_1^2+4Z_1Z_2 \Rightarrow \boxed{Z_0^2 = \frac{Z_1^2}{4}+Z_1Z_2}$$

The characteristic impedance of a symmetrical T-section is

$$\boxed{Z_{0T} = \sqrt{\frac{Z_1^2}{4}+Z_1Z_2}} \longrightarrow (1)$$

→ Z_{0T} can also be expressed interms of open-circuit impedance Z_{OC} and short-circuit impedance Z_{SC} of the T-network.

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(3)

From figure, the open-circuit impedance $Z_{OC} = \frac{Z_1}{2} + Z_2$ and

$$Z_{SC} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2}$$

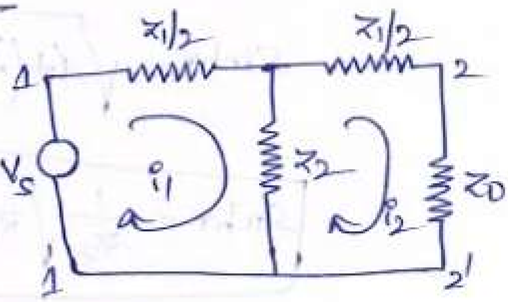
$$Z_{SC} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{OC} * Z_{SC} = Z_1Z_2 + \frac{Z_1^2}{4} = Z_{OT}^2$$

$$\text{Since } Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

$$\therefore \boxed{Z_{OT} = \sqrt{Z_{SC} * Z_{OC}}} \longrightarrow (2)$$

Propagation Constant of T-network :—



figure

The propagation constant (γ) of the network is given by

$$\boxed{\gamma = \log_e i_1/i_2}$$

Writing mesh equations for the second mesh, we get

$$i_1 Z_2 = i_2\left[\frac{Z_1}{2} + Z_2 + Z_0\right] \Rightarrow \frac{i_1}{i_2} = \frac{Z_1/2 + Z_2 + Z_0}{Z_2} = e^{\gamma}$$

$$\therefore \frac{Z_1}{2} + Z_2 + Z_0 = Z_2 e^{\gamma} \Rightarrow \boxed{Z_0 = Z_2\left[e^{\gamma} - 1\right] - Z_1/2} \longrightarrow (3)$$

The characteristic impedance of a T-network is given by

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2} \longrightarrow (4)$$

from equations (3) & (4), $\left(Z_2\left[e^{\gamma} - 1\right] - Z_1/2\right) = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$

$$Z_2^2\left[e^{\gamma} - 1\right]^2 + \frac{Z_1^2}{4} - Z_1Z_2\left[e^{\gamma} - 1\right] - \frac{Z_1^2}{4} - Z_1Z_2 = 0$$

$$\Rightarrow Z_2^2\left[e^{\gamma} - 1\right]^2 - Z_1Z_2\left[1 + e^{\gamma} - 1\right] = 0$$

$$Z_2^2\left[e^{\gamma}-1\right]^2 - Z_1 Z_2 e^{\gamma} = 0$$

$$\Rightarrow Z_2\left[e^{\gamma}-1\right]^2 - Z_1 e^{\gamma} = 0$$

$$\Rightarrow \left[e^{\gamma}-1\right]^2 = \frac{Z_1 e^{\gamma}}{Z_2} \Rightarrow e^{2\gamma}+1-2e^{\gamma} = \frac{Z_1}{Z_2 e^{-\gamma}}$$

Rearranging the above equation, we have

$$e^{-\gamma}\left[e^{2\gamma}+1-2e^{\gamma}\right] = \frac{Z_1}{Z_2} \Rightarrow \left[e^{\gamma}+e^{-\gamma}-2\right] = \frac{Z_1}{Z_2}$$

$$\frac{e^{\gamma}+e^{-\gamma}}{2} - 1 = \frac{Z_1}{2Z_2} \Rightarrow \frac{e^{\gamma}+e^{-\gamma}}{2} = 1+\frac{Z_1}{2Z_2}$$

$$\boxed{\cosh\gamma = 1+\frac{Z_1}{2Z_2}} \longrightarrow (5)$$

Since $\sinh^2\gamma + \cosh^2\gamma = 1$

$$\Rightarrow \sinh\gamma = \sqrt{\cosh^2\gamma - 1} = \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1}$$

$$\sinh\gamma = \sqrt{\cancel{1}+\left(\frac{Z_1}{2Z_2}\right)^2 + \frac{\cancel{2}Z_1}{\cancel{2}Z_2} - \cancel{1}} = \sqrt{\frac{Z_1}{Z_2}+\left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\boxed{\sinh\gamma = \frac{1}{Z_2}\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_{0T}}{Z_2}} \longrightarrow (6)$$

from equations (5) & (6), we get

$$\boxed{\tanh\gamma = \frac{Z_{0T}}{Z_2+\frac{Z_1}{2}}}$$

But $Z_2+\frac{Z_1}{2} = Z_{0C}$

Also, from equation (2), $Z_{0T} = \sqrt{Z_{0C}Z_{SC}}$

$$\tanh\gamma = \sqrt{\frac{Z_{SC}}{Z_{0C}}}$$

Also, $\sinh\frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cosh\gamma - 1)}$

where $\cosh\gamma = 1+\frac{Z_1}{2Z_2}$ //

$$\boxed{\sinh\gamma/2 = \sqrt{\frac{Z_1}{4Z_2}}} \;// \longrightarrow (7)$$

$$\left\{\begin{aligned} &\cosh 2hx = 1+2\sin^2 hx \\ &\cosh x = 1+2\sin^2 hx/2 \\ &2\sin^2\frac{hx}{2} = \cosh x - 1 \\ &\sin^2\frac{hx}{2} = \frac{\cosh x - 1}{2} \\ &\sinh\frac{x}{2} = \sqrt{\frac{1}{2}(\cosh x - 1)} \end{aligned}\right.$$

unit-I
NAS
(H)

π-network filter:

When the network is terminated in Z_0 at the port 2-2', its input impedance is given by

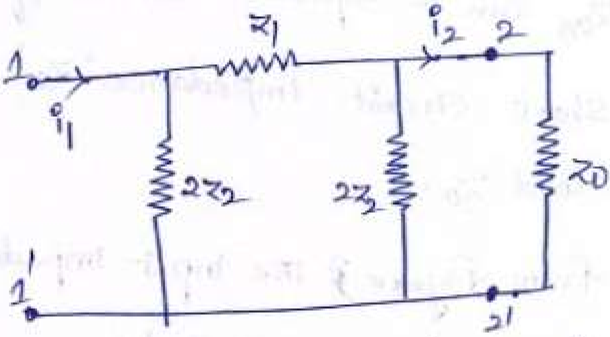


figure: Asymmetrical π-section

$$Z_{in} = \frac{2Z_2\left[Z_1 + \dfrac{2Z_2 Z_0}{2Z_2+Z_0}\right]}{Z_1 + \dfrac{2Z_2 Z_0}{2Z_2+Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

$$Z_0 = \frac{2Z_2\left[Z_1 + \dfrac{2Z_2 Z_0}{2Z_2+Z_0}\right]}{Z_1 + \dfrac{2Z_2 Z_0}{2Z_2+Z_0} + 2Z_2}$$

$$\Rightarrow Z_0 Z_1 + \frac{2Z_2 Z_0^2}{2Z_2+Z_0} + 2Z_0 Z_2 = \frac{2Z_2(2Z_1Z_2 + Z_0Z_1 + 2Z_0Z_2)}{2Z_2+Z_0}$$

$$\Rightarrow 2Z_0Z_1Z_2 + Z_1Z_0^2 + 2Z_0^2 Z_2 + 4Z_2^2 Z_0 + 2Z_2 Z_0^2 = 4Z_1Z_2^2 + 2Z_0Z_1Z_2 + 4Z_0$$

$$\Rightarrow Z_1Z_0^2 + 4Z_2Z_0^2 = 4Z_1Z_2^2 \Rightarrow Z_0^2\left[Z_1 + 4Z_2\right] = 4Z_1Z_2^2$$

$$\therefore Z_0^2 = \frac{4Z_1Z_2^2}{Z_1+4Z_2}$$

Rearranging the above equation leads to

$$\boxed{Z_0 = \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}}} \longrightarrow (8)$$

Which is the characteristic impedance of a symmetrical π-networ

$$Z_{0\pi} = \frac{Z_1Z_2}{\sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}}$$

from equation (1), $Z_{0T} = \sqrt{\dfrac{Z_1^2}{4} + Z_1Z_2}$

$$\therefore \boxed{Z_{0\pi} = \frac{Z_1Z_2}{Z_{0T}}} \longrightarrow (9)$$

$Z_{0\pi}$ can be expressed interms of the open-circuit impedance "Z_{oc}" and short circuit impedance "Z_{sc}" of the π-network exclusive of the load Z_0.

from figure of the input impedance at the port 1-1' when the port 2-2' is open is given by

$$Z_{oc} = \frac{2Z_2(Z_1+2Z_2)}{Z_1+4Z_2}$$

Similarly, the input impedance at the port 1-1' when the port 2-2' is short circuited is given by

$$Z_{sc} = \frac{2Z_1Z_2}{2Z_2+Z_1}$$

Hence, $Z_{oc} * Z_{sc} = \frac{4Z_1Z_2^2}{Z_1+4Z_2} = \frac{Z_1Z_2}{1+Z_1/4Z_2}$

Thus, from equation (8),

$$\boxed{Z_{0\pi} = \sqrt{Z_{oc}Z_{sc}}} \longrightarrow (10)$$

** The propagation constant of a symmetrical π-section is the same as that for a symmetrical - T-section,

i.e., $\boxed{\cosh\gamma = 1+\frac{Z_1}{2Z_2}}$

→ for symmetrical T (or) π-sections, the expression for propagation constant 'γ' interms of the hyperbolic functions is given by equations (5) and (7).

from eqn (7), $\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

** If Z_1 and Z_2 are both pure imaginary values, their ratio, and hence $\frac{Z_1}{4Z_2}$, will be a pure real number between the infinite limits. Then $\sinh\frac{\gamma}{2} = \frac{\sqrt{Z_1}}{\sqrt{4Z_2}}$ will also have infinite limits.

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but may be either real or imaginary depending upon whether $\frac{Z_1}{4Z_2}$ is positive or negative.

Since $\gamma = \alpha + j\beta$.

$\therefore \sinh\frac{\gamma}{2} = \sinh\left[\frac{\alpha}{2} + \frac{j\beta}{2}\right] = \sinh\frac{\alpha}{2}\cos\beta/2 + j\cosh\alpha/2 \sin\beta/2$

$$\boxed{\sinh\gamma/2 = \sqrt{\frac{Z_1}{4Z_2}}} \longrightarrow (11)$$

Case(i): —

If Z_1 and Z_2 are the same type of reactances, then $\left|\frac{Z_1}{4Z_2}\right|$ is real and equal to say $+x$.

The imaginary part of the equation (11) must be zero.

$$\cosh\frac{\alpha}{2}\sin\beta/2 = 0 \longrightarrow (12) \qquad \sinh\alpha/2 \cos\beta/2 = x \longrightarrow (13)$$

'α' and 'β' must satisfy both the equations.

Equation (12) can be satisfied if $\beta/2 = 0$ or $n\pi$ where $n = 0, 1, 2 \ldots$ then

$\cos\beta/2 = 1$ and $\sinh\alpha/2 = x = \sqrt{Z_1/4Z_2}$ //.

$\therefore$ 'x' should be always positive implies that

$$\boxed{\left|\frac{Z_1}{4Z_2}\right| > 0 \text{ and } \alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \longrightarrow (14)$$

Since $\alpha \neq 0$, it indicates that the attenuation exists.

Case(ii): —

Consider the case of Z_1 and Z_2 being opposite type of reactance

i.e. $\frac{Z_1}{4Z_2}$ is negative, making $\sqrt{\frac{Z_1}{4Z_2}}$ imaginary and equal to say jx

$\therefore$ The real part of eqn (11) must be zero.

$$\sinh\alpha/2 \cos\beta/2 = 0 \longrightarrow (15)$$

$$\cosh\alpha/2 \sin\beta/2 = x \longrightarrow (16)$$

Both the equations must be satisfied simultaneously by 'α' and 'β'.

Equation (15) may be satisfied when $\alpha = 0$ or when $\beta = \pi$.

a) **When $\alpha = 0$:**

$$\sinh\alpha/2 = 0, \quad \sin\beta/2 = x = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\boxed{\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \longrightarrow (17)$$

b) **When $\beta = \pi$:**

$$\cos\beta/2 = 0, \quad \sin\beta/2 = \pm 1, \quad \cosh\alpha/2 = x = \sqrt{\frac{Z_1}{4Z_2}}$$

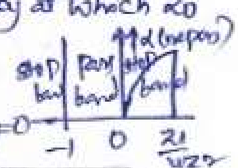
Since $\cosh\alpha/2 \geq 1$, this is valid for negative $Z_1/4Z_2$ and having magnitude greater than or equal to unity. It indicates the condition of stop band since $\alpha \neq 0$.

$$\boxed{\alpha = 2\cosh^{-1}\sqrt{Z_1/4Z_2}}$$

The frequency which separates the attenuation band from pass band or vice versa is called 'Cut-off frequency'. The cut-off frequency is denoted by f_c and it is also termed "Nominal frequency" since Z_0 is real in pass band and imaginary in attenuation band. f_c is the frequency at which Z_0 changes from being real to being imaginary. These frequencies occur at

$$\frac{Z_1}{4Z_2} = 0 \text{ or } Z_1 = 0$$

$$\frac{Z_1}{4Z_2} = -1 \text{ or } Z_1 + 4Z_2 = 0$$



Constant-K Low Pass Filter:

A network, either T or π is said to be of the constant-K type if Z_1 and Z_2 of the network satisfy the relation

$$\boxed{Z_1 Z_2 = K^2} \longrightarrow (1)$$

where Z_1 and Z_2 are impedances in the T and π-sections. Equation (1) states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. K is a real constant, that is the resistance. K is often termed as design impedance or nominal impedance of the Constant K-filter.

The constant K, T or π-type filter is also known as the "prototype" because other more complex networks can be derived from it.

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(6)

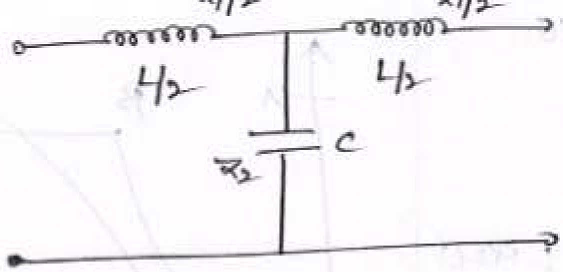

fig: prototype T-section

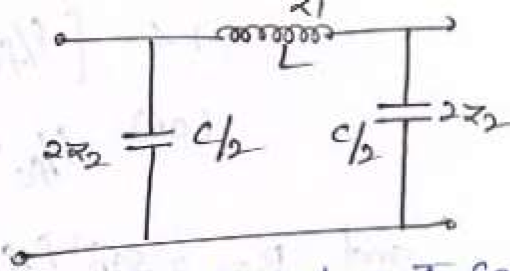

fig: prototype π-section

from figures, $Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$. Hence, $Z_1 Z_2 = \frac{L}{C} = K^2$

which is independent of frequency.

$$\boxed{Z_1 Z_2 = K^2 = \frac{L}{C} \quad \text{or} \quad K = \sqrt{\frac{L}{C}}} \longrightarrow (2)$$

Since the product Z_1 and Z_2 is constant, the filter is a constant-K type.

The cut-off frequency can be calculated by $\frac{Z_1}{4Z_2} = 0$.

$$\left[\frac{Z_1}{4Z_2} = \frac{j\omega L}{4 \times \frac{1}{j\omega C}} = \frac{+j^2\omega^2 LC}{4}\right]$$

i.e., $\frac{-\omega^2 LC}{4} = 0 \Rightarrow f = 0$.

and $\therefore \frac{Z_1}{4Z_2} = -1 \Rightarrow \frac{-\omega^2 LC}{4} = -1$

$$\text{(or)} \quad \boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \longrightarrow (3)$$

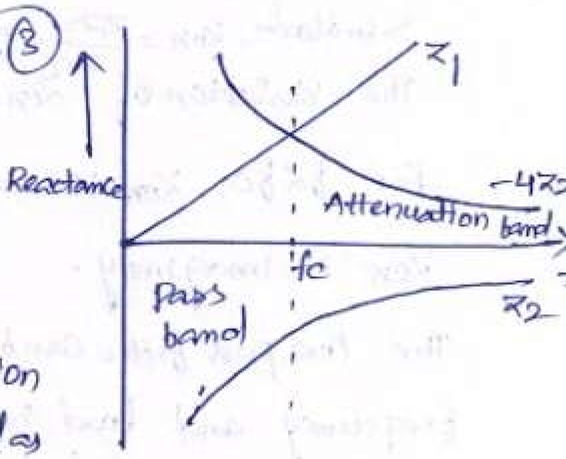

Figure 1

→ The reactance of Z_1 and $4Z_2$ will vary with frequency as shown in Figure (1).

→ The cut-off frequency at the intersection of the curves Z_1 and $-4Z_2$ is indicated as "f_c".

We also have that,

$$\sinh\left[\gamma/2\right] = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{j\omega\sqrt{LC}}{2}$$

from eqn (3), $\sqrt{LC} = \frac{1}{\pi f_c}$

$$\therefore \sinh\gamma/2 = \frac{j\cancel{2}\pi f}{\cancel{2}} * \frac{1}{\pi f_c} = jf/f_c$$

We also know that in the pass band,

$$-1 < Z_1/4Z_2 < 0 \Rightarrow -1 < \frac{-\omega^2 LC}{4} < 0$$

$$-1 < -\left[f/f_c\right]^2 < 0$$

(or) $f/f_c < 1$

and $\beta = 2\sin^{-1}\left[f/f_c\right]$; $\alpha = 0$

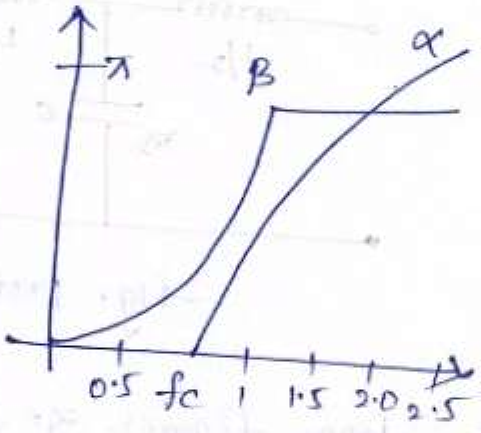

figure ②

In the attenuation band,

$\frac{Z_1}{4Z_2} < -1$ i.e. $f/f_c > 1$

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right] = 2\cosh^{-1}\left[f/f_c\right];\ \beta = \pi$$

Thus, from figure ②,

$$\alpha = 0,\ \beta = 2\sinh^{-1}\left[f/f_c\right] \text{ for } f < f_c$$

$$\alpha = 2\cosh^{-1}\left[f/f_c\right];\ \beta = \pi \text{ for } f > f_c.$$

The characteristic impedance can be calculated as follows:

$$Z_{0T} = \sqrt{Z_1 Z_2\left[1 + \frac{Z_1}{4Z_2}\right]} = \sqrt{\frac{L}{C}\left[1 - \frac{\omega^2 LC}{4}\right]} = K\sqrt{1 - \left(f/f_c\right)^2}$$

Similarly, $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{K}{\sqrt{1-(f/f_c)^2}}$ — Z_{0T} is real when $f<f_c$ and at $f=f_c \Rightarrow Z_{0T}=0$, for $f>f_c \rightarrow Z_{0T}$ imaginary. ④

The variation of $Z_{0\pi}$ with frequency is shown in figure ②.

for $f<f_c$, $Z_{0\pi}$ is real; at $f=f_c$, $Z_{0\pi}$ is infinite and for $f>f_c$, $Z_{0\pi}$ is imaginary.

The low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut-off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C} \Rightarrow \pi^2 f_c^2 LC = 1$$

$\left\{\right.$ since at $f=f_c \Rightarrow Z_{0T}=0$ $\therefore \sqrt{Z_1Z_2\left[1+\frac{Z_1}{4Z_2}\right]} = 0 \Rightarrow 1+\frac{Z_1}{4Z_2}=0$

Also, we know that $K = \sqrt{\frac{L}{C}}$ is called the 'Design Impedance' or 'Load resistance'.

$$\therefore K^2 = L/C \Rightarrow \pi^2 f_c^2 K^2 C^2 = 1 \Rightarrow \boxed{C = \frac{1}{\pi f_c K}} \text{ shunt capacitance.}$$

and $L = K^2 C = \frac{K}{\pi f_c} \rightarrow$ Series inductance.

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Constant-K High pass filter:

→ A constant-K high pass filter prototype are shown in the following figure. where $Z_1 = -j/\omega C$ and $Z_2 = j\omega L$.

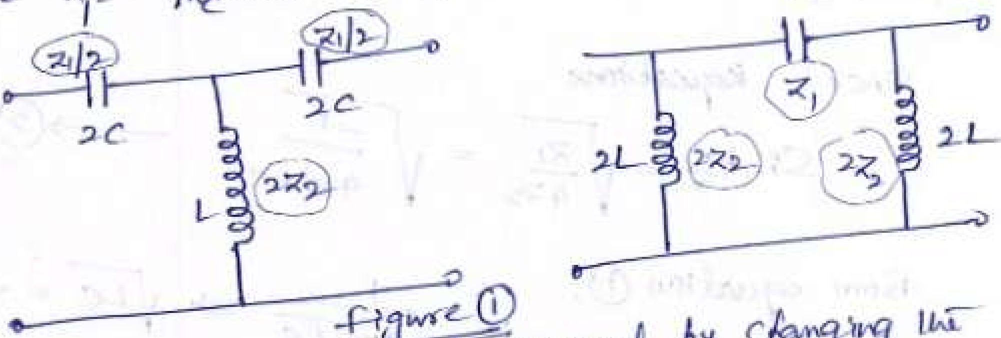

figure ①

→ A constant-K high pass filter can be obtained by changing the positions of series and shunt arms of the constant-K low pass filter.

→ It can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant-K-type. Thus, $Z_1 Z_2$ are given by

$$Z_1 Z_2 = \frac{-j}{\omega C} * j\omega L = \frac{L}{C} = K^2$$

$$\therefore \boxed{K = \sqrt{L/C}}$$

→ The cut-off frequencies are given by $Z_1 = 0$ and $Z_1 = -4Z_2$.

$Z_1 = 0$ indicates $\frac{j}{\omega_c C} = 0$. as $\omega_c \to \infty$ $\left[\text{since } C \neq \infty\right]$

from $Z_1 = -4Z_2$, $\frac{-j}{\omega_c C} = -4j\omega L \Rightarrow \omega_c^2 LC = \frac{1}{4} \Rightarrow 4f_c^2\pi^2 LC = \frac{1}{4}$

$f_c^2 = \frac{1}{16\pi^2 LC}$ (or) $\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}} \longrightarrow ①$

The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in figure ②.

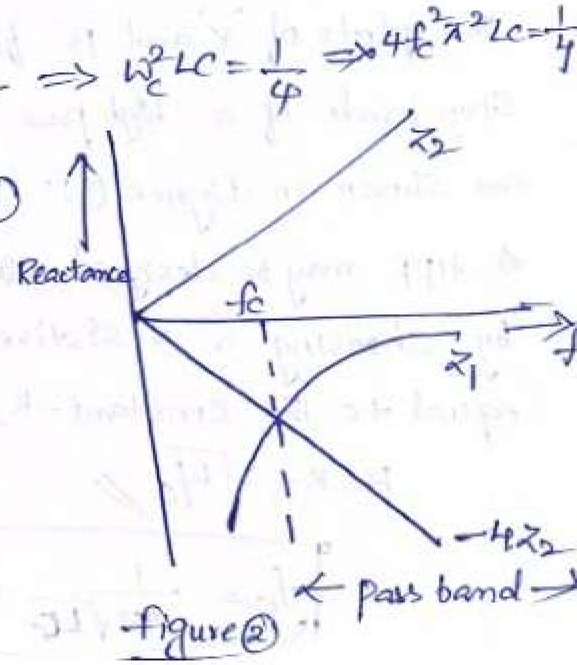

figure ②

As seen from figure ②, the filter transmits all frequencies between $f = f_c$ and $f = \infty$. The point f_c from the graph is a point at which $Z_1 = -4Z_2$.

Since, ~~Equations~~

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}} \longrightarrow ②$$

$$\frac{Z_1}{4Z_2} = \frac{-1}{4\omega^2 LC}$$

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$-1 < \frac{-f_c^2}{f^2} < 0$$

$$\Rightarrow \left(\frac{f_c}{f}\right)^2 < 1$$

$$\therefore \frac{f_c}{f} < 1$$

from equation ①: $f_c = \frac{1}{4\pi\sqrt{LC}} \Rightarrow \sqrt{LC} = \frac{1}{4\pi f_c}$

therefore from equation ②.

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2(f_c)^2}{4\omega^2 \cancel{LC}}} = \sqrt{\frac{-(4\pi)^2 f_c^2}{4 * 4\pi^2 f^2}}$$

$$\Rightarrow \sinh\frac{\gamma}{2} = \sqrt{\frac{-f_c^2}{f^2}} = \sqrt{\frac{j^2 f_c^2}{f^2}} = \frac{jf_c}{f}$$

In the pass band, $-1 < \frac{Z_1}{4Z_2} < 0$, $\alpha = 0$ or the region in which $\frac{f_c}{f} < 1$ is a pass band. $\beta = 2\sin^{-1}(f_c/f)$

→ In the attenuation band, $\frac{Z_1}{4Z_2} < -1$ i.e. $\frac{f_c}{f} > 1$

$$\frac{Z_1}{4Z_2} < -1 \Rightarrow -\left(\frac{f_c}{f}\right)^2 < -1 \Rightarrow f_c > f$$

$$\therefore \alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right] = 2\cosh^{-1}\left[\frac{f_c}{f}\right];\ \beta = \pi$$

The plots of 'α' and 'β' for pass and stop bands of a high pass filter network are shown in figure ③.

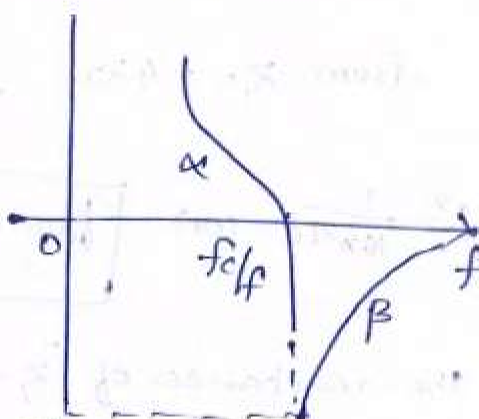

figure ③

A HPF may be designed ~~similar to the~~ by choosing a resistive load 'r' equal to the constant-K, such that

$$R = K = \sqrt{L/C}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{K}{4\pi L} = \frac{1}{4\pi CK}} \longrightarrow ③$$

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Since $\sqrt{C} = \frac{\sqrt{L}}{K} \Rightarrow \frac{L}{C} = K^2$

$$\therefore \boxed{L = \frac{K}{4\pi f_c} \quad \text{and} \quad C = \frac{1}{4\pi f_c K}} \longrightarrow (4)$$

The characteristic impedance can be calculated using the relation

$$Z_{0T} = \sqrt{Z_1 Z_2 \left[1 + \frac{Z_1}{4Z_2}\right]} = \sqrt{\frac{L}{C}\left[1 - \frac{1}{4\omega^2 LC}\right]}$$

$$\therefore \boxed{Z_{0T} = K\sqrt{1 - \left[f_c/f\right]^2}} \longrightarrow (5)$$

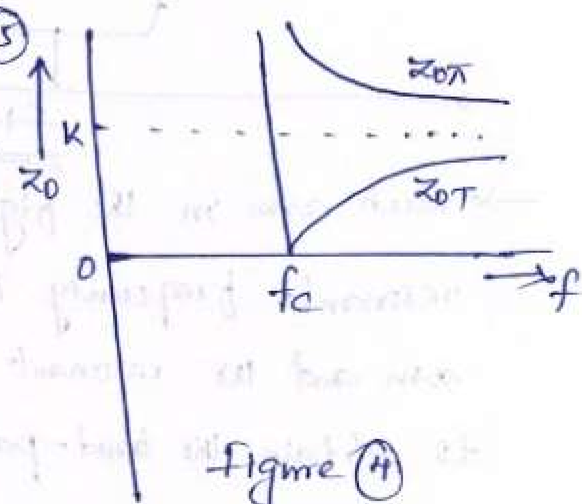


Figure (4)

Similarly, the characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{K^2}{Z_{0T}}$$

$$\boxed{Z_{0\pi} = \frac{K}{\sqrt{1 - (f_c/f)^2}}} \longrightarrow (6)$$

The plot of characteristic impedances with respect to frequency is shown in figure (4).

Constant-K Band Pass filter :

→ A band pass filter is one which attenuates all frequencies below a lower cutoff frequency 'f_1' and above an upper cut-off frequency 'f_2'.

→ frequencies lying between f_1 and f_2 comprise the pass band, and are transmitted with zero attenuation.

→ A band pass filter may be obtained by using a low-pass filter followed by a high-pass filter in which the cut-off frequency of the Low pass filter is above the cut-off frequency of the hi

Pass filter, the overlap thus allowing only a band of frequencies to pass.

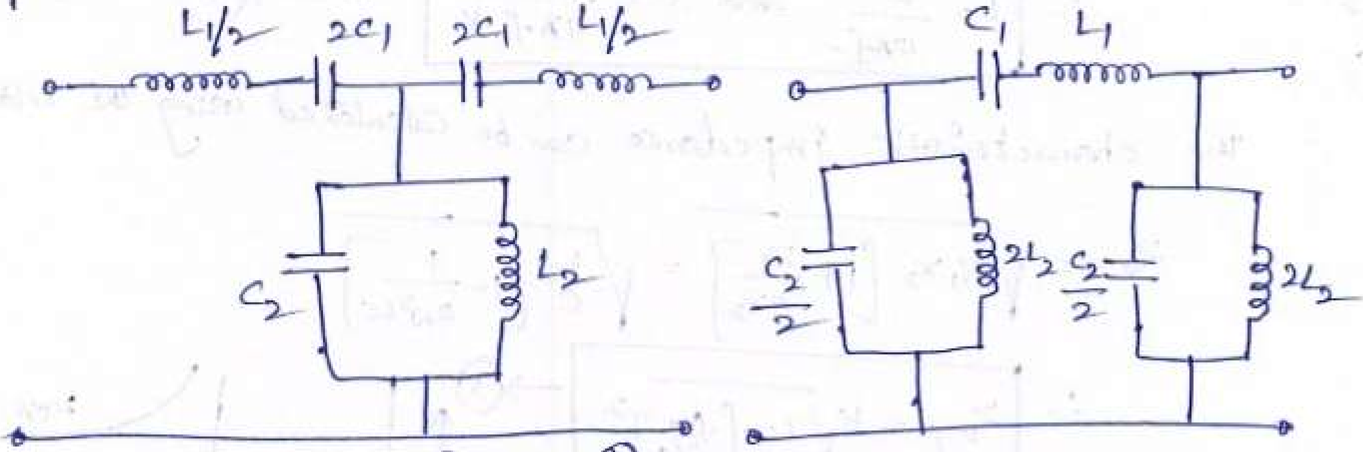


figure ①

→ Each arm in the figure ① has a resonant circuit with same resonant frequency i.e, the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band-pass characteristic.

For the equal resonant frequency condition,

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

from which, $\omega_0^2 L_1 C_1 = 1 \longrightarrow ①$

and $\frac{1}{\omega_0 C_2} = \omega_0 L_2$ for the shunt arm

from which, $\omega_0^2 L_2 C_2 = 1 \longrightarrow ②$

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2 \Rightarrow \boxed{L_1 C_1 = L_2 C_2} \longrightarrow ③$$

The impedance of the series arm, Z_1 is given by

$$Z_1 = \left[j\omega L_1 - \frac{j}{\omega C_1}\right] = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right]$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 * \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right]\left[\frac{j\omega L_2}{1-\omega^2 L_2 C_2}\right]$$

$$= \frac{-L_2}{C_1}\left[\frac{\omega^2 L_1 C_1 - 1}{1-\omega^2 L_2 C_2}\right]$$

from eqn (3), $L_1 C_1 = L_2 C_2$

$$\therefore \boxed{Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2}$$

where $K \rightarrow$ Constant. Thus the filter is a Constant K-type

Therefore, for a Constant K-type in the pass band,

$$-1 < \frac{Z_1}{4Z_2} < 0 \quad \text{and at Cut-off frequency}$$

$$Z_1 = -4Z_2 \Rightarrow Z_1^2 = -4Z_1 Z_2 = -4K^2$$

$$\therefore \boxed{Z_1 = \pm j2K}$$

i.e. the value of Z_1 at lower cut-off frequency is equal to the negative of the value of Z_1 at the upper cut-off frequency.

$$\therefore \left[\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right] = -\left[\frac{1}{j\omega_2 C_1} + j\omega_2 L_1\right]$$

$$\text{or} \quad \left[\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right] = \left[\frac{1}{\omega_2 C_1} - \omega_2 L_1\right]$$

$$\Rightarrow \left[1 - \omega_1^2 L_1 C_1\right] = \frac{\omega_1}{\omega_2}\left(\omega_2^2 L_1 C_1 - 1\right) \longrightarrow (4)$$

Since $L_1 C_1 = \frac{1}{\omega_0^2}$

therefore from equation (4),

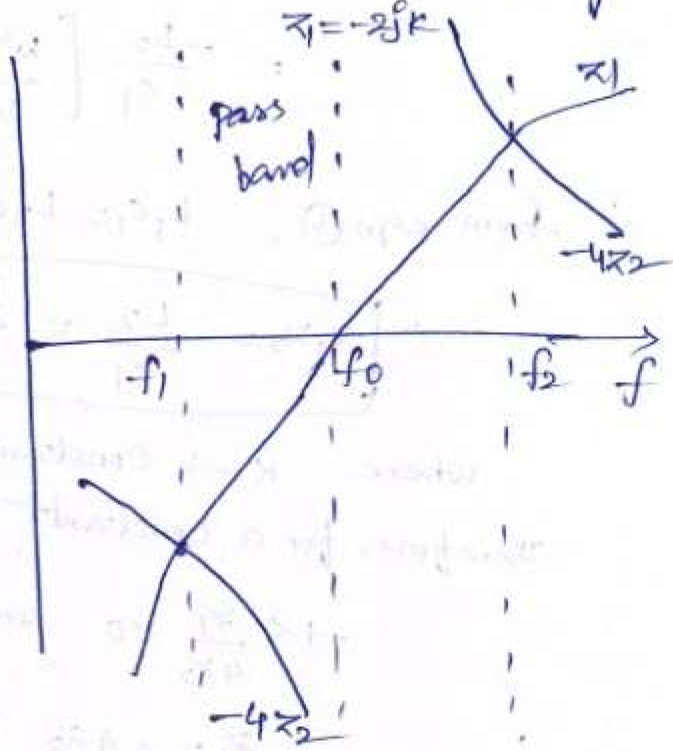
$$\left[1 - \frac{\omega_1^2}{\omega_0^2}\right] = \frac{\omega_1}{\omega_2}\left[\frac{\omega_2^2}{\omega_0^2} - 1\right] \Rightarrow \left(\omega_0^2 - \omega_1^2\right) = \frac{\omega_1}{\omega_2}\left\{\omega_2^2 - \omega_0^2\right\}$$

$$\Rightarrow \omega_0^2 \omega_2 - \omega_1^2 \omega_2 = \omega_2^2 \omega_1 - \omega_0^2 \omega_1 \Rightarrow \omega_0^2 \omega_2 + \omega_0^2 \omega_1 = \omega_2^2 \omega_1 + \omega_1^2 \omega_2$$

$$\Rightarrow \omega_0^2\left[\omega_1 + \omega_2\right] = \omega_1 \omega_2\left[\omega_1 + \omega_2\right] \Rightarrow \boxed{\omega_1 \omega_2 = \omega_0^2}$$

$$\therefore \boxed{f_0 = f_1 f_2} \longrightarrow (5)$$

Thus, the resonant frequency is the geometric mean of the cut-off frequencies. The variation of reactances with respect to frequency is shown in figure ②.



Design: If the filter is terminated in a load resistance $R = K$, then at the lower cut-off frequency,

$$\left[\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right] = -2jK$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2K$$

$$\Rightarrow 1 - \omega_1^2 C_1 L_1 = 2K\omega_1 C_1$$

Since, $L_1 C_1 = \frac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2K\omega_1 C_1$$

or $1 - \left(\frac{f_1}{f_0}\right)^2 = 4\pi K f_1 C_1 \Rightarrow 1 - \frac{f_1^2}{f_1 f_2} = 4\pi K f_1 C_1$ $\left[\because f_0 = \sqrt{f_1 f_2}\right]$

$$\therefore f_2 - f_1 = 4\pi K f_1 f_2 C_1$$

$$\Rightarrow \boxed{C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2}} \longrightarrow (6)$$

Since $L_1 C_1 = \frac{1}{\omega_0^2} \Rightarrow L_1 = \frac{1}{\omega_0^2 C_1} = \frac{4\pi K f_1 f_2}{\omega_0^2 (f_2 - f_1)}$

$$\therefore \boxed{L_1 = \frac{K}{\pi(f_2 - f_1)}} \longrightarrow (7)$$

To evaluate the values of the shunt-arm, consider the equation

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2$$

$$L_2 = C_1 K^2 = \frac{(f_2 - f_1)K}{4\pi f_1 f_2} \longrightarrow (8)$$

and $\boxed{C_2 = \frac{L_1}{K^2} = \frac{1}{\pi(f_2 - f_1)K}} \longrightarrow (9)$

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Equations (6) through (9) are the design equations of a prototype band pass filter. The variation of α, β with respect to frequency is shown in figure (3).

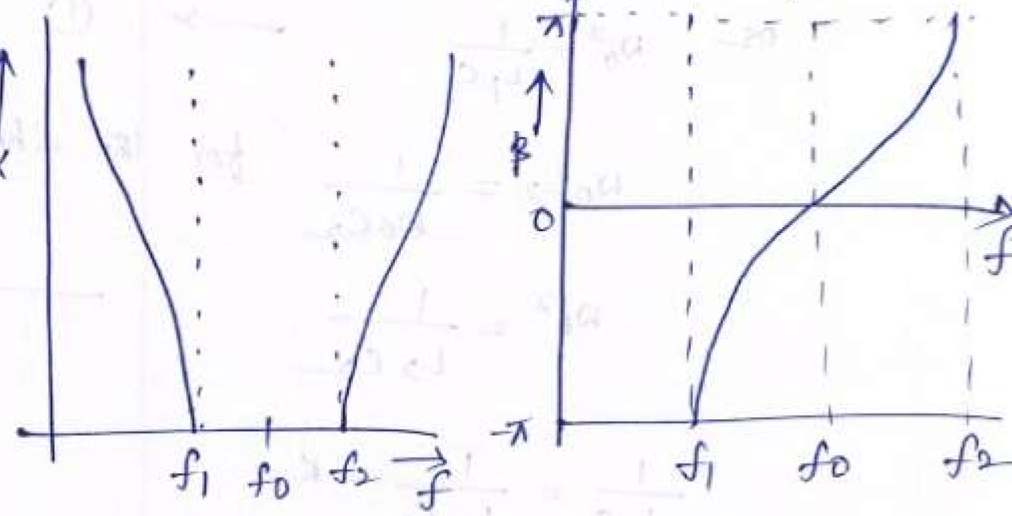

figure (3)

Band-stop filter (or) Band-Elimination filter

→ A band elimination filter is one which passes without attenuation at all frequencies less than lower cut-off frequency 'f_1' and greater than the upper cut-off frequency 'f_2'.

→ frequencies lying between 'f_1' and 'f_2' are attenuated. Therefore a band-stop filter can be realized by connecting a low-pass filter in parallel with a high pass section, in which the cut-off frequency of low-pass filter is below that of a high pass filter.

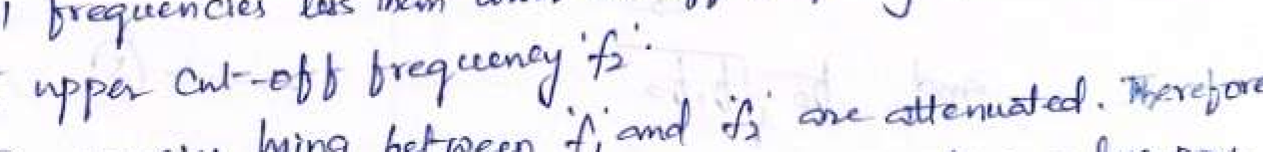

figure (1)

As for the band-pass filter, the series and shunt-arms are chosen to resonate at the same frequency 'ω_0'. Therefore, from figure (1), for the condition of equal resonant frequencies,

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the Series arm.}$$

or $$\omega_0^2 = \frac{1}{L_1 C_1} \longrightarrow (1)$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \quad \text{for the Shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \longrightarrow (2)$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = K$$

Thus, $$\boxed{L_1 C_1 = L_2 C_2} \longrightarrow (3)$$

It can also be verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = K^2 \longrightarrow (4)$$

and $$f_0 = \sqrt{f_1 f_2} \longrightarrow (5)$$

At Cut-off frequency, $Z_1 = -4Z_2$

Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = K^2$$

$$\boxed{Z_2 = \pm jK/2} \longrightarrow (6)$$

If the load is terminated in load resistance, $R = K$, then at lower Cut-off frequency. $Z_2 = -jK/2$

$$Z_2 = j\left[\frac{1}{\omega_1 C_2} - \omega_1 L_2\right] = j\frac{K}{2} \Rightarrow \frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{K}{2}$$

$$\Rightarrow 1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{K}{2}$$

from eqn (2). $L_2 C_2 = \frac{1}{\omega_0^2}$

$$\therefore 1 - \frac{\omega_1^2}{\omega_0^2} = \frac{K}{2}\omega_1 C_2 \Rightarrow 1 - (f_1/f_0)^2 = K\pi f_1 C_2$$

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$$\Rightarrow \boxed{C_2 = \frac{1}{K\pi f_1}\left[1-\left(f_1/f_0\right)^2\right]}$$

Since $f_0 = \sqrt{f_1 f_2}$

$$\therefore \boxed{C_2 = \frac{1}{K\pi}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]} = \frac{1}{K\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] \rightarrow (7)$$

and from eqn (2). $\omega_0^2 = \frac{1}{L_2 C_2}$

$$\therefore L_2 = \frac{1}{\omega_0^2 C_2} = \frac{\pi K f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

Since $f_0 = \sqrt{f_1 f_2}$

$$\therefore \boxed{L_2 = \frac{K}{4\pi(f_2 - f_1)}} \rightarrow (8)$$

Also from (4). $K^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$

$$\boxed{L_1 = K^2 C_2 = \frac{K}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right)} \rightarrow (9)$$

and $$\boxed{C_1 = \frac{L_2}{K^2} = \frac{1}{4\pi K(f_2 - f_1)}} \rightarrow (10)$$

The variation of the reactances with respect to frequency is shown in fig (2).

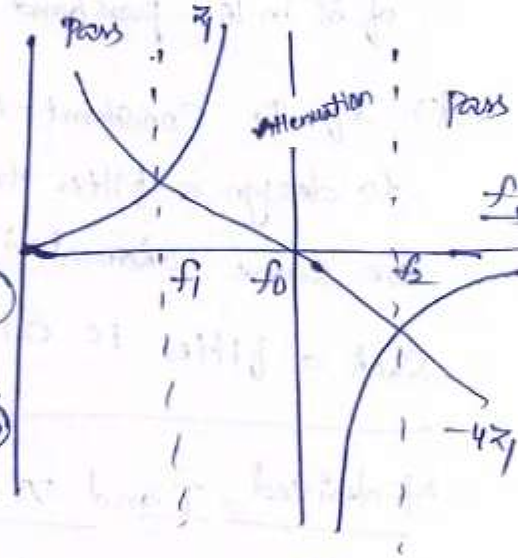

Figure (2)

Equations (7) through (10) are the design equations of Band stop filter.

The variation of α, β with respect to frequency is shown in figure (3).

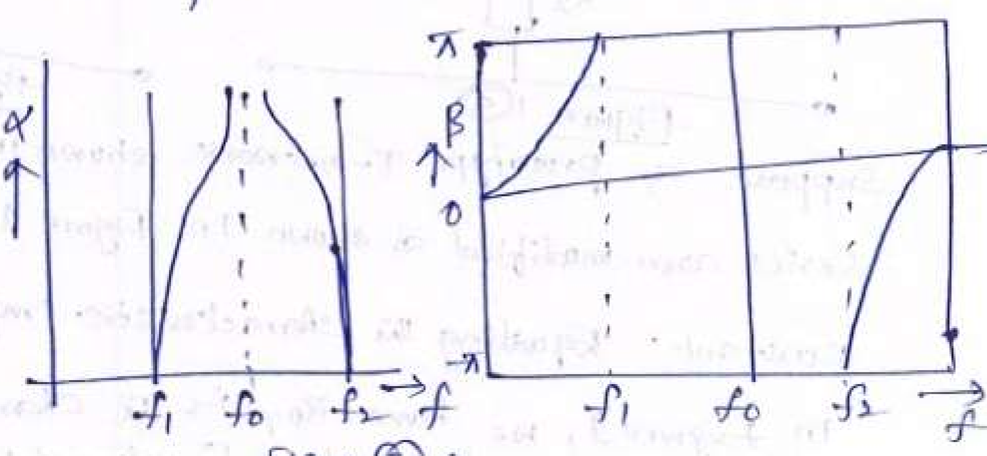

Figure (3)

M-derived filters:—

→ The Attenuation is not sharp in the stop band for K-type filters.

→ The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band.

→ Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. But, their characteristic impedances be equal to each other at all frequencies.

→ If their characteristic impedances match at all frequencies, they would also have the same pass band.

→ Since practical elements have a certain resistance which gives rise to attenuation in the pass band too. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'α' in the pass band.

*) If the Constant-K section regarded as the prototype it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called "m-derived filter".

M-derived T and π- filters:—

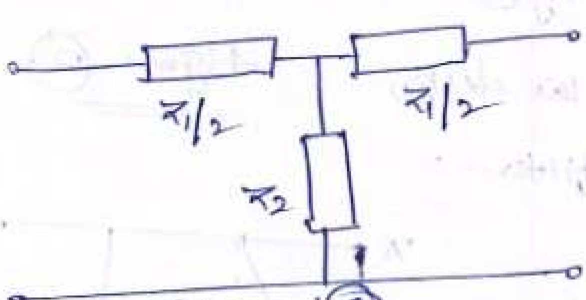

figure 1(a)

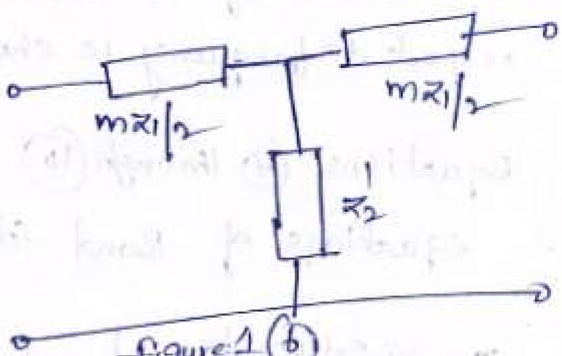

figure 1(b)

Suppose a prototype T-network shown in figure 1(a) has the series arm modified as shown in figure 1(b), where 'm' is a constant. Equating the characteristic impedance of the networks in figure 1, we have Z_{OT}' is the characteristic impedance of the modified (m-derived) network.

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(12)

$$Z_{0T} = Z_{0T}'$$

$$\sqrt{\frac{Z_1^2}{4}+Z_1Z_2} = \sqrt{\frac{m^2Z_1^2}{4}+mZ_1Z_2'}$$

$$\Rightarrow \frac{Z_1^2}{4}+Z_1Z_2 = \frac{m^2Z_1^2}{4}+mZ_1Z_2'$$

$$\Rightarrow mZ_1Z_2' = \frac{Z_1^2}{4}\left(1-m^2\right)+Z_1Z_2$$

$$\therefore \boxed{Z_2' = \frac{Z_1}{4m}\left(1-m^2\right)+\frac{Z_2}{m}} \longrightarrow (1)$$

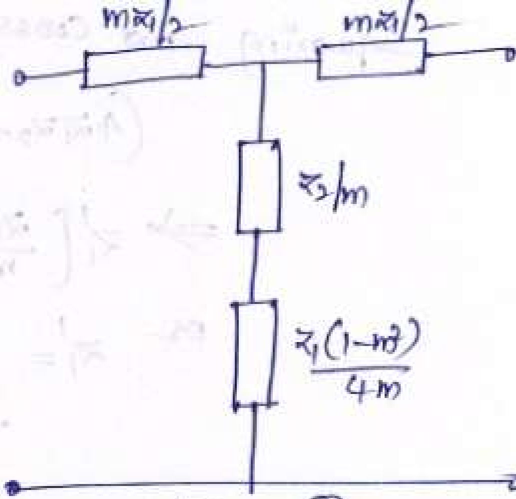

Figure (2)

It appears that the shunt-arm Z_2' consists of two impedances in series as shown in figure (2).

from figure (2). $\frac{1-m^2}{4m}$ should be positive to realise the impedance Z_2' physically i.e. $0<m<1$. Thus, the m-derived section can be obtained from the prototype by modifying its series and shunt arms.

A similar type of technique can be applied to a π-section network.

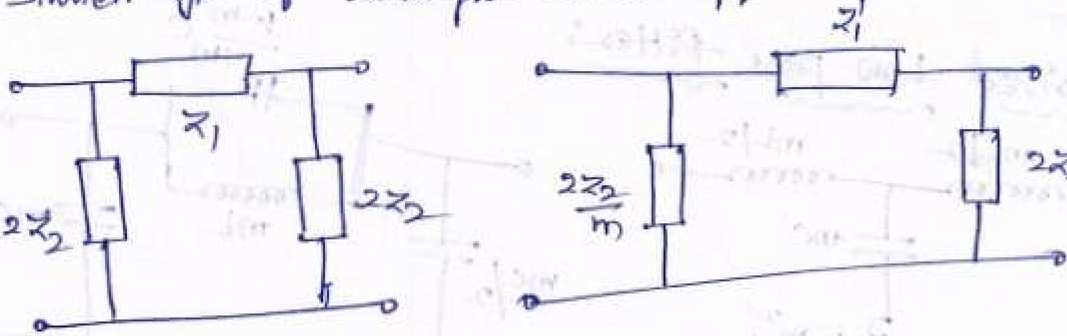

figure (3)

The characteristic impedances of the prototype and its modified sections have to be equal for matching.

$$Z_{0\pi} = Z_{0\pi}'$$

$$\therefore \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1'Z_2/m}{1+\frac{Z_1'}{4Z_2/m}}}$$

Squaring and cross multiplying the above equation results as under.

$$(4Z_1Z_2 + mZ_1'Z_1) = \frac{4Z_1'Z_2 + Z_1Z_1'}{m}$$

$$\Rightarrow Z_1'\left[\frac{Z_1}{m} + \frac{4Z_2}{m} - mZ_1\right] = 4Z_1Z_2$$

or $$Z_1' = \frac{Z_1Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}} = \frac{Z_1Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}[1-m^2]}$$

$$Z_1' = \frac{Z_1Z_2\frac{4m^2}{(1-m^2)}}{\frac{Z_2\,4m^2}{m(1-m^2)} + Z_1m} = \frac{mZ_1\frac{Z_2 4m}{(1-m^2)}}{mZ_1 + \frac{Z_2 4m}{(1-m^2)}} \longrightarrow (2)$$

It appears, that the series arm of the m-derived π-section is a parallel combination of mZ_1 and $\frac{4mZ_2}{1-m^2}$.

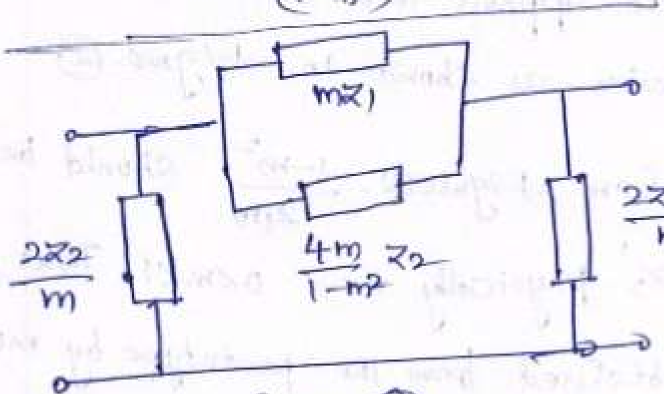

figure (4)

M-derived Low pass filter:

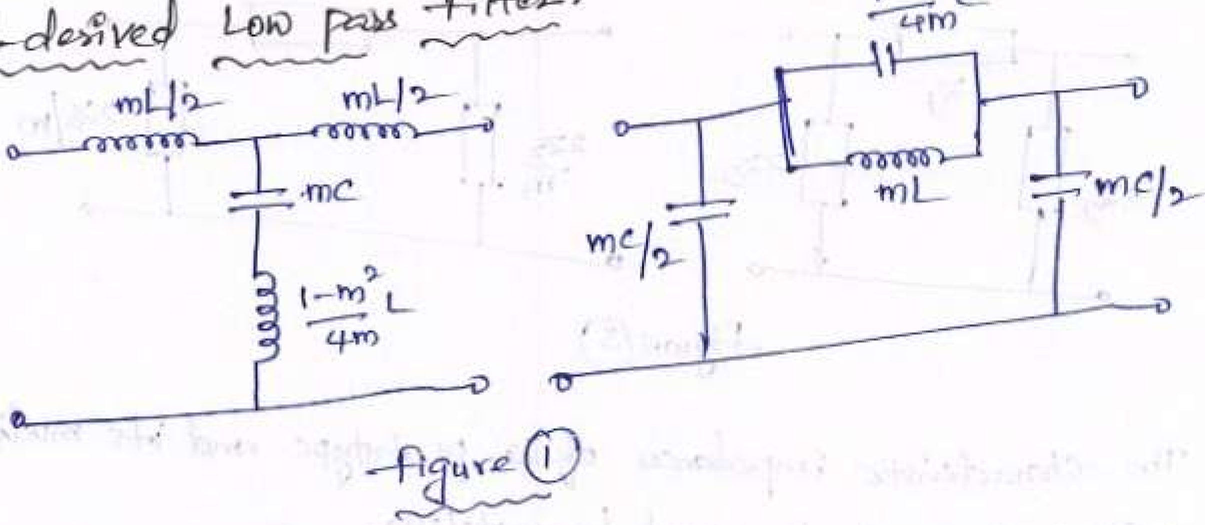

figure (1)

In figure (1), both m-derived low pass T and π-filter sections are shown. for the T-section shown in figure (1)a, the shunt arm is to be chosen so that it is resonant at some frequency 'f_∞' above cut-off frequency 'f_c'.

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$$\frac{1}{m\omega_r C} = \frac{1-m^2}{4m}\omega_r L \qquad \text{where } \omega_r \to \text{Resonant frequency}$$

$$\Rightarrow \omega_r^2 = \frac{4}{(1-m^2)LC} \Rightarrow f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since the Cut-off frequency for the low-pass filter is $f_c = \frac{1}{\pi\sqrt{LC}}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \longrightarrow (1)$$

(or)

$$m = \sqrt{1-(f_c/f_\infty)^2} \longrightarrow (2)$$

If a sharp Cut-off is desired, f_∞ should be near to f_c. It is clear that the smaller the value of m, f_∞ comes close to f_c.

→ Similarly, for m-derived π-section, the inductance and capacitance in the series arm constitute a resonant circuit. Thus, at f_∞ a frequency corresponds to infinite attenuation i.e., at f_r.

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4m}\right)\omega_r C} \Rightarrow \omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$\therefore \boxed{f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}}$$

Since $f_c = \frac{1}{\pi\sqrt{LC}}$

$$\therefore \boxed{f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\infty} \longrightarrow (3)$$

Thus, for both m-derived low-pass networks for a positive value of m $(0<m<1)$, $f_\infty > f_c$. Equations (2) or (3) can be used to choose the value of m, knowing f_c and f_r. After the value of m is evaluated, the elements of the T or π-networks can be found. The variation of attenuation for a low-pass m-derived section can be verified from $\alpha = 2\cosh^{-1}\sqrt{Z_1/4Z_2}$ for $f_c < f < f_\infty$.

for $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the prototype.

$$\therefore \alpha = 2\cosh^{-1}\left[\frac{mf/f_c}{\sqrt{1-(f/f_\infty)^2}}\right]$$

and $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\left(\frac{mf/f_c}{\sqrt{1-(f/f_c)^2(1-m^2)}}\right)$$

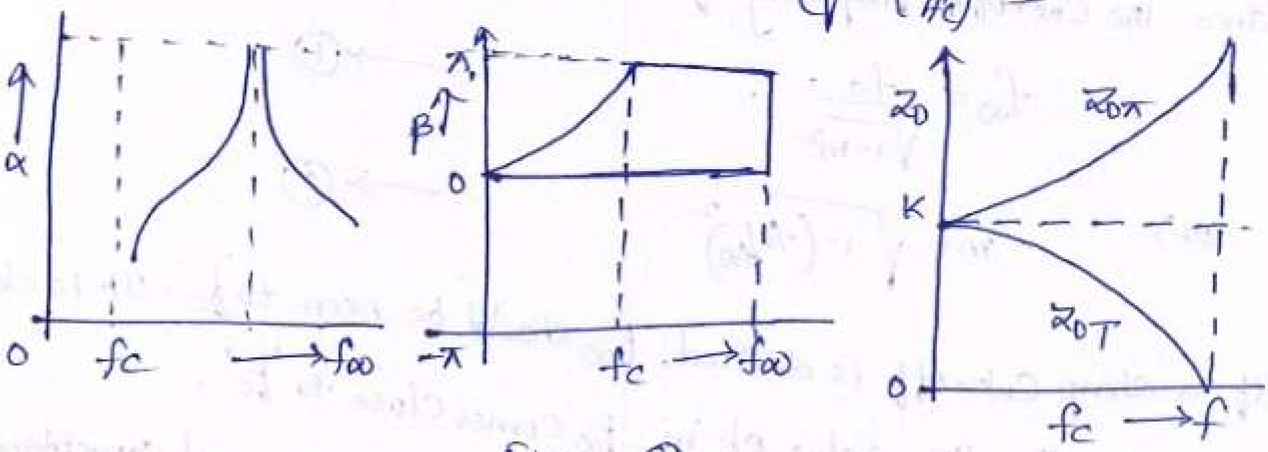

figure ②

figure ② shows the variation of α, β, and Z_0 with respect to frequency for an m-derived low-pass filter.

m-derived high-pass filter:—

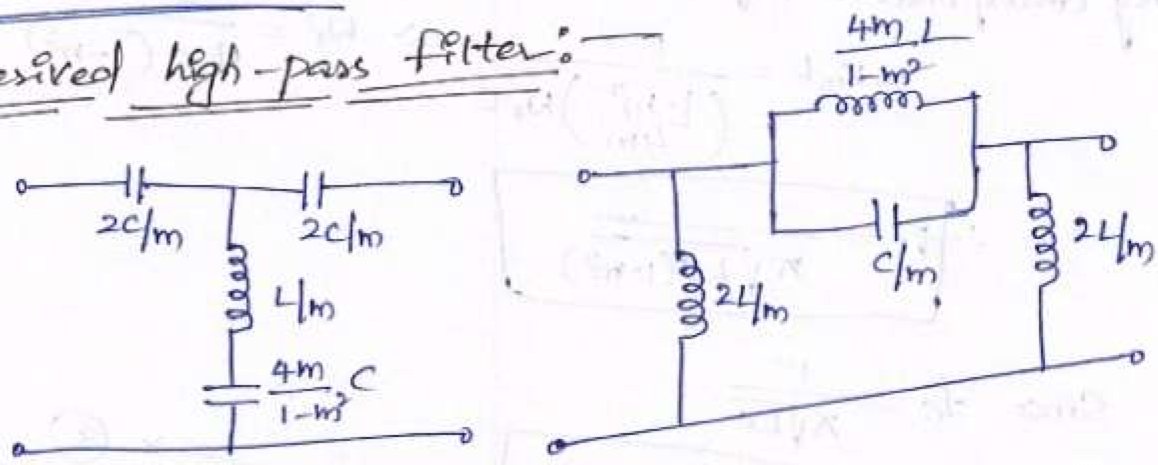

figure ①

In figure ①, both m-derived high pass T and π sections are shown.

If the shunt-arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

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$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

$$\Rightarrow \omega_r^2 = \omega_\infty^2 = \frac{1}{\frac{L}{m} \times \frac{4m}{1-m^2} C} = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } \boxed{f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}}$$

The Cut-off frequency f_c of a high pass prototype filter is given by → (1)

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = f_c\sqrt{1-m^2} \Rightarrow m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \longrightarrow (2)$$

Similarly, for the m-derived π-section, the resonant circuit is constituted by the series arm inductance and capacitance. Thus, at f_r,

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m} C} \Rightarrow \omega_r^2 = \omega_\infty^2 = \frac{1-m^2}{4LC}$$

$$\Rightarrow \omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } \boxed{f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}}$$

Thus, the frequency corresponding to infinite attenuation is the same for both sections.

The variation of α, β and Z_0 with frequency is shown in the following figure (2).

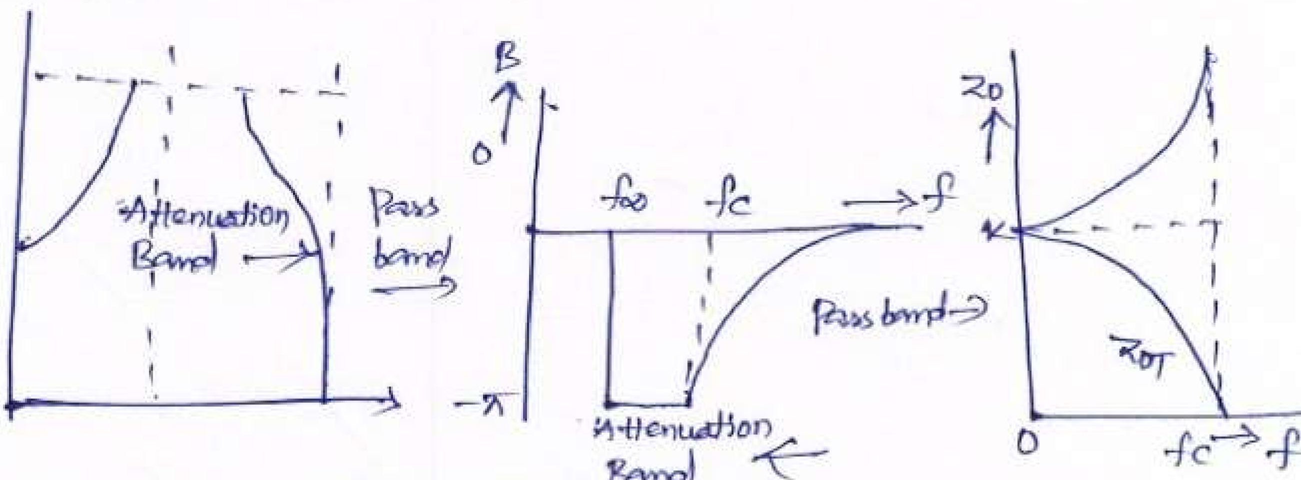

Figure (2)

unit-II

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(15)

Attenuators

→ An attenuator is a two-port resistive network and is used to reduce the signal level by a given amount.

→ Attenuation sometimes it is necessary to introduce a specified loss between source and a matched load without altering the impedance relationship.

→ Attenuation can be measured in decibels or nepers.

$$\text{Attenuation (dB)} = 10\log_{10}\left[\frac{P_1}{P_2}\right]$$

$P_1 \to$ input power, $P_2 \to$ output power

$$\text{Attenuation (dB)} = 20\log_{10}\left[\frac{V_1}{V_2}\right]$$

$$= 20\log_{10}\left[\frac{i_1}{i_2}\right]$$

If $\frac{V_1}{V_2} = \frac{i_1}{i_2} = N$ then $\frac{P_1}{P_2} = N^2$.

and $\boxed{dB = 20\log_{10} N}$ or $\boxed{N = \text{antilog}\left[\frac{dB}{20}\right]}$

Types of Attenuators:

(i) T-type Attenuator:-

A symmetrical T-attenuator is presented in figure 1. An attenuator is to be designed for desired values of characteristic resistance, R_0 and attenuation.

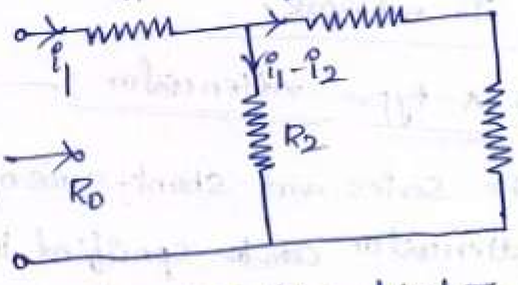

figure 1: Symmetrical T-attenuator

The values of the arms of the network can be specified in terms of characteristic impedance (Z_0), propagation constant (γ) of the network.

The network in the figure is a symmetrical resistive circuit: hence $Z_0 = R_0$ and $\gamma = \alpha$.

The design equations can be obtained by applying KVL to the network.

$$R_2[i_1 - i_2] = i_2[R_1 + R_0] \Rightarrow \frac{i_1}{i_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \longrightarrow (1)$$

The characteristic impedance of the attenuator is R_0 when it is terminated in a load of R_0. Hence,

$$R_0 = R_1 + \frac{R_2[R_1 + R_0]}{R_1 + R_0 + R_2} \longrightarrow (2)$$

from (1) & (2),

$$R_0 = R_1 + \frac{R_1 + R_0}{N} \Rightarrow NR_0 = NR_1 + R_1 + R_0$$

$$R_0[N-1] = R_1[N+1] \Rightarrow \boxed{R_1 = \frac{R_0[N-1]}{N+1}} \longrightarrow (3)$$

from eqn (1), $NR_2 = R_1 + R_0 + R_2$

$$\Rightarrow (N-1)R_2 = R_1 + R_0$$

Substituting eqn (3) in the above eqn,

$$(N-1)R_2 = R_0\left[\frac{N-1}{N+1}\right] + R_0 \Rightarrow (N-1)R_2 = \frac{2NR_0}{N+1}$$

$$\therefore \boxed{R_2 = \frac{2NR_0}{N^2-1}} \longrightarrow (4)$$

Equations (3) and (4) are the design equations for the symmetrical T-network.

(ii) π-type Attenuator :-

The series and shunt arms of the attenuator can be specified in terms of Z_0 and propagation constant γ.

→ The network is formed by resistors and is symmetrical, hence $Z_0 = R_0$ and $\gamma = \alpha$.

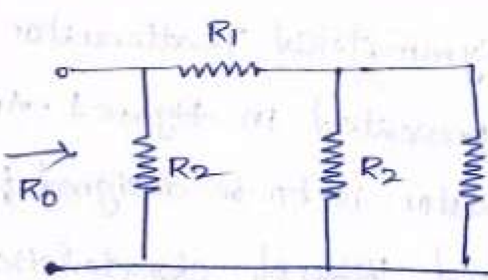

figure (1)

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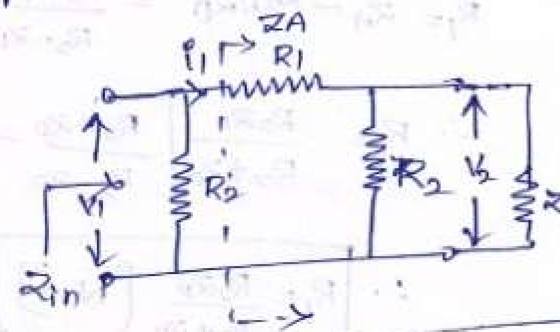
In an attenuator, input and output should be matched with the characteristic impedance.



Since,

$$\frac{1}{Z_{in}} = \frac{1}{Z_A} + \frac{1}{R_2} \Rightarrow \frac{1}{Z_A} = \frac{1}{Z_{in}} - \frac{1}{R_2}$$

$$\boxed{Z_A = R_1 + R_2 \| Z_0}$$

$$\Rightarrow \frac{1}{Z_A} = \frac{R_2 - Z_{in}}{R_2 Z_{in}}$$

$$\therefore Z_A = \frac{R_2 Z_{in}}{R_2 - Z_{in}}$$

Since $Z_{in} = Z_0 \Rightarrow \boxed{Z_A = \frac{R_2 Z_0}{R_2 - Z_0}} \longrightarrow (1)$

$V_1 = i_1 Z_A$ and $V_2 = i_1 [R_2 \| Z_0]$

Therefore, Attenuation ratio $N = \frac{V_1}{V_2} = \frac{i_1 Z_A}{i_1 [R_2 \| Z_0]} = \frac{Z_A}{R_2 \| Z_0}$

$$\therefore \boxed{N = \frac{Z_A}{\frac{R_2 * Z_0}{R_2 + Z_0}}} \longrightarrow (2)$$

from (1) & (2),

$$N = \frac{R_2 + Z_0}{R_2 Z_0} * \frac{R_2 Z_0}{R_2 - Z_0} = \frac{R_2 + Z_0}{R_2 - Z_0}$$

$$\Rightarrow N[R_2 - Z_0] = R_2 + Z_0 \Rightarrow NR_2 - NZ_0 = R_2 + Z_0 \Rightarrow NR_2 - R_2 = NZ_0 + Z_0$$

$$\Rightarrow R_2[N-1] = [N+1] Z_0$$

$$\therefore \boxed{R_2 = \left(\frac{N+1}{N-1}\right) Z_0}$$

Since $Z_0 = R_0 \Rightarrow \boxed{R_2 = \left(\frac{N+1}{N-1}\right) R_0} \longrightarrow (3)$

Also, $R_1 = Z_A - (R_2 \| Z_0) \Rightarrow R_1 = \frac{R_2 Z_0}{R_2 - Z_0} - \frac{R_2 Z_0}{R_2 + Z_0}$

$$\Rightarrow R_1 = R_2 Z_0 \left[\frac{1}{R_2 - Z_0} - \frac{1}{R_2 + Z_0}\right] = R_2 Z_0 \left[\frac{\cancel{R_2} + Z_0 - \cancel{R_2} + Z_0}{R_2^2 - Z_0^2}\right]$$

(or)

$$R_1 = Z_A - R_2 \| Z_0 = \frac{R_2 Z_0}{R_2 - Z_0} - \frac{R_2 Z_0}{R_2 + Z_0}$$

$$\therefore R_1 = \frac{R_2 Z_0}{R_2 + Z_0}\left[\frac{R_2 + Z_0}{R_2 - Z_0} - 1\right]$$

$$\therefore \boxed{R_1 = \frac{R_2 Z_0}{R_2 + Z_0}\left[N-1\right]} \longrightarrow (4) \qquad \left[\text{Since } N = \frac{R_2 + Z_0}{R_2 - Z_0}\right]$$

Since $Z_0 = R_0 \Rightarrow R_1 = \frac{R_2 R_0}{R_2 + R_0}\left[N-1\right]$

from equations (3) & (4), since $R_2 = R_0\left(\frac{N+1}{N-1}\right)$

$$\therefore R_1 = R_0 \frac{[N+1]}{\cancel{N-1}} * R_0 * \frac{1}{R_0\left[\frac{N+1}{N-1}\right] + R_0}\left[\cancel{N-1}\right]$$

$$R_1 = \frac{R_0^2 [N+1] * (N-1)}{R_0[N+1] + R_0[N-1]}$$

$$\therefore R_1 = \frac{R_0^2 [N^2-1]}{R_0 N + \cancel{R_0} + R_0 N - \cancel{R_0}} = \frac{R_0^2 [N^2-1]}{2 R_0 N}$$

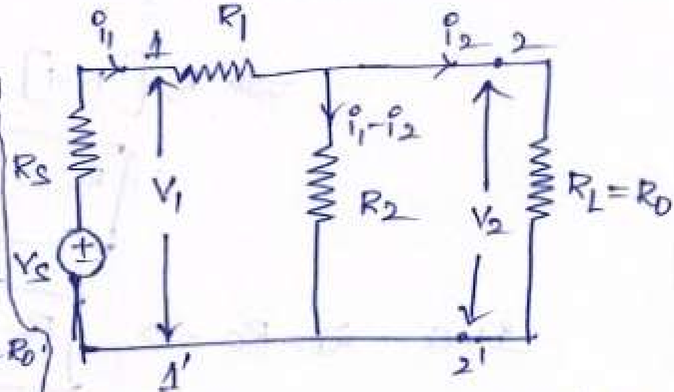
$$\therefore \boxed{R_1 = \frac{R_0 [N^2-1]}{2N}} \longrightarrow (5)$$

Substitute and $\boxed{R_2 = R_0\left(\frac{N+1}{N-1}\right)} \longrightarrow (6)$

Equations (5) and (6) are the design equations of π-Attenuator.

(iii) <u>L-type Attenuator</u> :—

The L-type attenuator shown in figure(1) is connected between a source with source resistance $R_S = R_0$ and load resistance $R_L = R_0$



figure(1) L-type Asymmetrical Attenuator

By mesh equations in the loop(2),

$$V_2 = (i_1 - i_2) R_2 = i_2 R_L$$

(or) $i_1 R_2 = i_2 [R_2 + R_L] \Rightarrow \frac{i_1}{i_2} = \frac{R_2 + R_L}{R_2} = N$

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$$1+\frac{R_L}{R_2} = N \Rightarrow \boxed{R_2 = \frac{R_L}{(N-1)}} \longrightarrow (1)$$

Since, $R_L = R_0$,

$$\therefore \boxed{R_2 = \frac{R_0}{N-1}} \longrightarrow (2)$$

The resistance of the network as viewed from $1-1'$ into the network is

$$R_0 = R_1 + \frac{R_2 R_0}{R_2+R_0} \Rightarrow R_1 = R_0 - \frac{R_2 R_0}{R_2+R_0} = \frac{\cancel{R_2 R_0} + R_0^2 - \cancel{R_2 R_0}}{R_2+R_0}$$

$$\therefore \boxed{R_1 = \frac{R_0^2}{R_2+R_0}} \longrightarrow (3)$$

from (2) and (3).

$$R_1 = \frac{R_0^2}{\frac{R_0}{N-1}+R_0} = \frac{R_0^2}{\frac{R_0+R_0[N-1]}{N-1}} = \frac{R_0^2[N-1]}{R_0 N}$$

$$\therefore \boxed{R_1 = \frac{R_0[N-1]}{N}} \longrightarrow (4)$$

Equations (2) & (4) are the design equations of L-type attenuator.

→ Attenuation 'N' of the network can be varied by varying the values of 'R_1 and R_2'.

(iv) Bridged-T Attenuator:

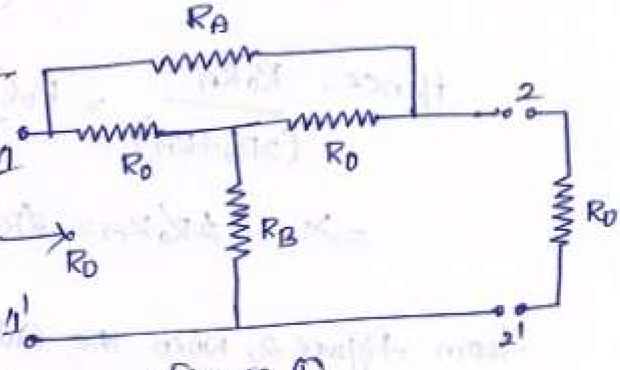

figure (1)

A bridged T-attenuator is shown in figure (1). Since the attenuator is formed by resistors only, $z_0 = R_0$ and $\gamma = \alpha$.

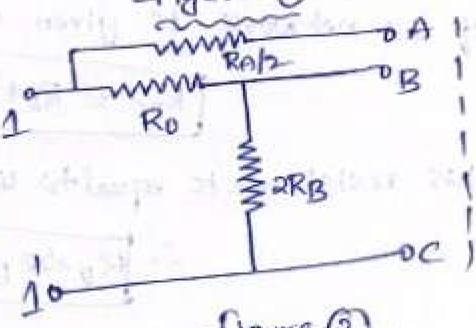

figure (2)

The bridged-T network may be designed to have any characteristic resistance 'R_0' and desired attenuation by making $R_A R_B = R_0^2$. where 'R_A' and 'R_B' are variable resistances.

We can obtain the design equations of the bridged-T attenuator by bisection theorem. A bisected half-section is shown in figure ②. According to the bisection theorem, a network having mirror image symmetry can be reduced to an equivalent structure.

The series arm of the equivalent structure is found by bisecting the given network into two parts, short circuiting all the cut-wires and equating the series impedance of the equivalent structure to the input impedance of the bisected network; the diagonal arm is equal to the input impedance of the bisected network when cut-wires are open circuited.

From figure ②, when the cut-wires A, B, C are shorted, the input resistance of the network is given by

$$R_{SC} = \frac{R_0 \times R_A/2}{R_0 + R_A/2} = \frac{R_0 R_A}{2R_0 + R_A} \longrightarrow (1)$$

This resistance is equal to the series-arm resistance of the equivalent network.

i.e. $\frac{R_0 R_A}{2R_0 + R_A} = R_1$

Since we have, $R_1 = R_0 \frac{(N-1)}{(N+1)}$

Hence, $\frac{R_0 R_A}{(2R_0 + R_A)} = R_0 \frac{(N-1)}{N+1} \Rightarrow R_0 R_A N + R_0 R_A = 2R_0^2 N - 2R_0^2 + R_0 R_A N - R_A R_0$

$$\Rightarrow \cancel{2} R_0 R_A = \cancel{2} R_0^{\cancel{2}} [N-1] \Rightarrow \boxed{R_A = R_0 (N-1)} \longrightarrow (2)$$

From figure 2, when the cut-wires A, B, C are open, the input resistance of the network is given by

$$\boxed{R_{OC} = R_0 + 2R_B} \longrightarrow (3)$$

This resistance is equal to the diagonal arm resistance.

$$\therefore \boxed{R_0 + 2R_B = R_2} \longrightarrow (4)$$

Since we have, $R_2 = R_0 \frac{(N+1)}{N-1} \longrightarrow (5)$

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Hence, $R_0 + 2R_B = R_0\left[\frac{N+1}{N-1}\right]$

$\Rightarrow \cancel{R_0}N - R_0 + 2R_BN - 2R_B = \cancel{R_0}N + R_0$

$\Rightarrow \cancel{2}R_B[N-1] = \cancel{2}R_0 \Rightarrow \boxed{R_B = \frac{R_0}{N-1}} \longrightarrow (6)$

Equations (2) and (6) are the design equations of Bridged-T attenuator

(V) Lattice Attenuator :-

The series and the diagonal arm of the network can be specified in terms of Z_0 and γ.

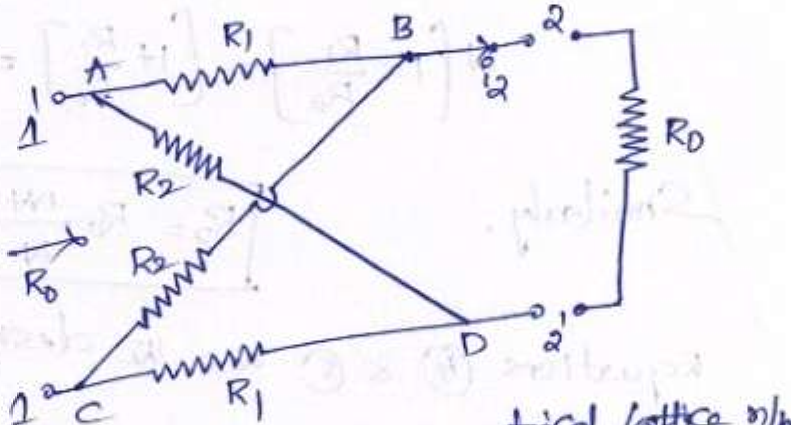

figure (1): symmetrical lattice n/w

Since the Z_0 of symmetrical network is the geometric mean of the open and short circuit impedance.

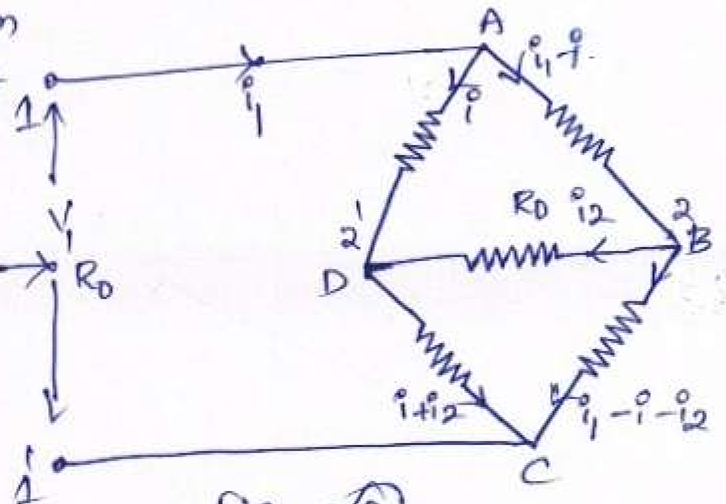

figure (2)

from figure (2),

$$Z_{SC} = \frac{2R_1R_2}{R_1+R_2}$$

$$Z_{OC} = \frac{R_1+R_2}{2}$$

Hence $Z_0 = R_0 = \sqrt{Z_{SC} * Z_{OC}} \Rightarrow R_0 = \sqrt{R_1R_2}$.

from figure (2), the input impedance at 1-1' is R_0. when the network is terminated in R_0 at 2-2'. By KVL,

$$V_1 = i_1R_0 = (i_1 - i)R_1 + i_2R_0 + (i+i_2)R_1$$

$$\Rightarrow i_1R_0 = R_1[i_1+i_2] + i_2R_0 \Rightarrow i_1[R_0 - R_1] = i_2[R_1+R_0]$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{R_1+R_0}{R_0-R_1} = \frac{1+R_1/R_0}{1-R_1/R_0} \longrightarrow (1)$$

$$N = e^{\alpha} = \frac{I_1}{I_2} = \frac{1+R_1/R_0}{1-R_1/R_0} \Rightarrow \boxed{e^{\alpha} = \frac{1+\sqrt{R_1/R_0}}{1-\sqrt{R_1/R_0}}} \longrightarrow (2)$$

The propagation constant, $\alpha = \log\left[\frac{1+\sqrt{R_1/R_2}}{1-\sqrt{R_1/R_2}}\right] \longrightarrow (3)$

from equation (2).

$$N\left[1-\frac{R_1}{R_0}\right] = \left[1+\frac{R_1}{R_0}\right] \Rightarrow \boxed{R_1 = R_0\left[\frac{N-1}{N+1}\right]} \longrightarrow (4)$$

Similarly,

$$\boxed{R_2 = R_0\frac{N+1}{N-1}} \longrightarrow (5)$$

Equations (4) & (5) are the design equations for lattice attenuator.

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Equalizers

Equalizers are networks designed to provide compensation against distortions that occur in a signal while passing through an electrical netwo[rk]

→ Distortions are two types. They are Attenuation distortion and phase distortion.

→ Attenuation distortion occurs due to non-uniform attenuation against frequency characteristics.

→ phase distortion occurs due to phase delay against frequency characteristics.

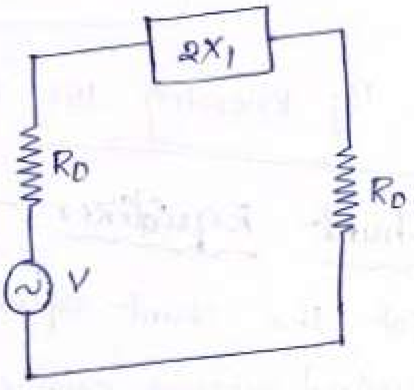
→ Attenuation equalizer is used to compensate attenuation distortio[n] in any network.

→ A phase equilizer is used to compensate phase distortion in any network.

→ the equilizers are used in medium to high frequency-carrier telephone systems, amplifiers, transmission lines, and speech reproduction, etc.

→ Various types of equilizers are series, shunt, Constant resistance, bridge T-attenuation, bridge T-phase, Lattice attenuation, lattice phase equilizers.

(i) Series Equalizer:



The series equalizer is a two-terminal network connected in series with a network to be corrected.

Let

N = Input to output power ratio of the load

D = Attenuation in decibels

R_0 = Resistance of the load as well as source,

P_i = Input power,

P_l = Load power,

$2X_1$ = Reactance of the equilizer,

V_{max} = Maximum voltage applied to the network.

Attenuation $\boxed{D = \log_{10} N}$, (or) $\boxed{N = \text{antilog}\left[D/10\right]}$ //

$$N = \frac{\text{Maximum power delivered to the load when equilizer is not present}}{\text{power delivered to the load when equalizer is present}}$$

$$N = \frac{P_i}{P_l} \quad \text{where } P_i = \left[\frac{V_{max}}{2R_0}\right]^2 R_0 = \frac{V_{max}^2}{4R_0} //$$

When the equilizer is connected.

$$I_l = \frac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}} \quad \text{and}$$

hence,

$$P_l = \left[\frac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}}\right]^2 R_0 = \frac{V_{max}^2}{4\left[R_0^2 + X_1^2\right]} * R_0$$

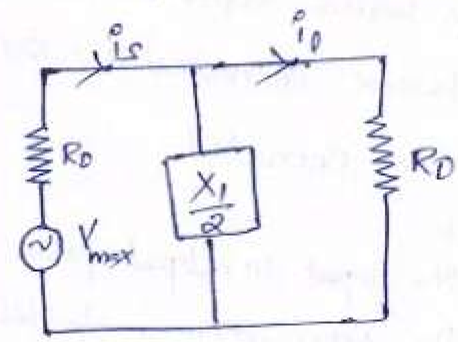
Therefore,

$$N = \frac{P_i}{P_l} = \frac{V_{max}^2 / 4R_0}{V_{max}^2 R_0 / 4\left[R_0^2 + X_1^2\right]} = \frac{4\left[R_0^2 + X_1^2\right]}{4R_0^2}$$

$$\therefore \boxed{N = \frac{R_0^2 + X_1^2}{R_0^2} = 1 + \left(\frac{X_1}{R_0}\right)^2}$$

By knowing the values of R_0 and N, X_1 can be determined.

ii) Shunt Equilizers:



The shunt equilizer is a two-terminal network connected in shunt with a network to be corrected.

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Let N = Input to output power ratio

D = Attenuation in decibels

R_0 = Source resistance / Load resistance

I_s = Source Current

I_l = Load Current

P_i = Input power

P_l = Load power

$\frac{X_1}{2}$ = Reactance of shunt equalizer

$$\text{Source Current } I_s = \frac{V_{max}}{R_0 + \left[R_0 \,//\, \frac{jX_1}{2}\right]} = \frac{V_{max}}{R_0 + \left[\frac{jX_1 R_0}{2R_0 + jX_1}\right]}$$

$$I_s = \frac{V_{max}\left[2R_0 + jX_1\right]}{2R_0\left[R_0 + jX_1\right]}$$

$$\text{Load Current, } I_l = I_s * \frac{jX_1/2}{R_0 + \frac{jX_1}{2}} = I_s * \frac{jX_1}{2R_0 + jX_1}$$

Substituting I_s in the above equation.

$$I_l = \frac{V_{max} * jX_1}{2R_0\left(R_0 + jX_1\right)}$$

Power delivered to the load.

$$P_l = |I_l|^2 R_0 = \frac{V_{max}^2 X_1^2}{4R_0\left[R_0^2 + X_1^2\right]}$$

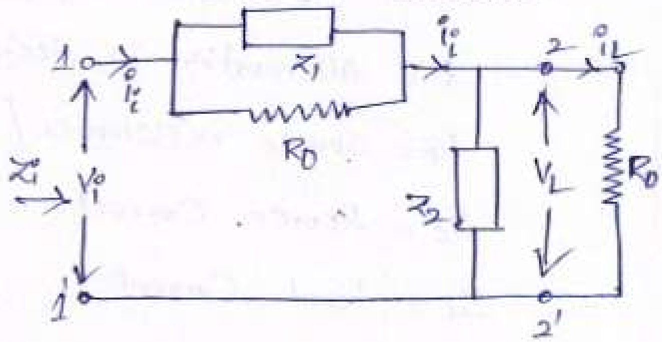
and $\quad P_i = V_{max}^2 / 4R_0$

Therefore, $$N = P_i / P_l = \frac{V_{max}^2/4R_0}{V_{max}^2 X_1^2 / 4R_0(R_0^2 + X_1^2)}$$

$$\therefore N = 1 + \left[\frac{R_0}{X_L}\right]^2$$

By knowing the values of R_0 and N, X_1 can be determined.

Full-Series Equalizer: —— (or) Constant Resistance Equalizer:



The circuit is a constant resistance equalizer that satisfies the relation $Z_1 Z_2 = R_D^2$. The input impedance is given by

$$Z_i = [R_D \| Z_1] + [R_D \| Z_2]$$

$$= \frac{R_D Z_1}{R_D + Z_1} + \frac{R_D Z_2}{R_D + Z_2} = \frac{R_D[2Z_1Z_2 + R_D(Z_1+Z_2)]}{R_D^2 + R_D[Z_1+Z_2] + Z_1Z_2}$$

Since $Z_1 Z_2 = R_D^2$ and $Z_i = R_D$

Also,

$$|V_i| = I_i Z_i = I_i R_D$$

and $|V_l| = I_i[R_D \| Z_2] = I_i * \frac{R_D Z_2}{R_D + Z_2}$ $\left\{ \text{Here } V_l = i_L * R_D \text{ and } i_L = I_i * \frac{Z_2}{R_D + Z_2} \right\}$

$$\therefore N = \left|\frac{V_i}{V_l}\right|^2 = \left|\frac{R_D + Z_2}{Z_2}\right|^2 = 1 + \left[\frac{R_D}{X_2}\right]^2$$

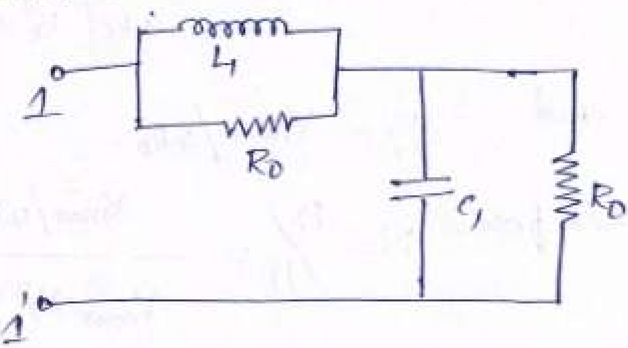
(or) $\boxed{N = 1 + \left[\frac{X_1}{R_D}\right]^2}$

Since Z_1 and Z_2 are pure reactances and $X_1 X_2 = R_D^2$.

(a) when $X_1 = \omega L$,

$X_2 = \frac{1}{\omega C_1}$ Since both are inverse.

The full-series equalizer is shown in figure



where $\frac{L_1}{C_1} = R_D^2$ //

from the equation,

$$N = 1 + \frac{X_1^2}{R_D^2} = 1 + \frac{\omega^2 L_1^2}{R_D^2} //$$

By knowing the values of N and R_D, the elemental values of L_1, C_1 may be obtained.

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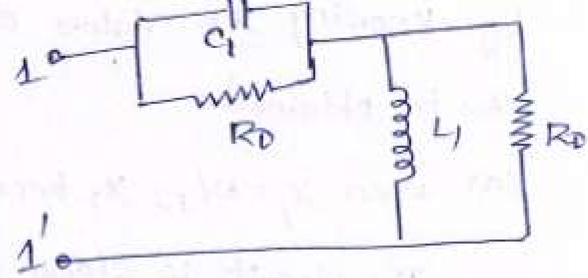
(b) when $X_1 = \frac{1}{\omega C_1}$,

$X_2 = \omega L_1$

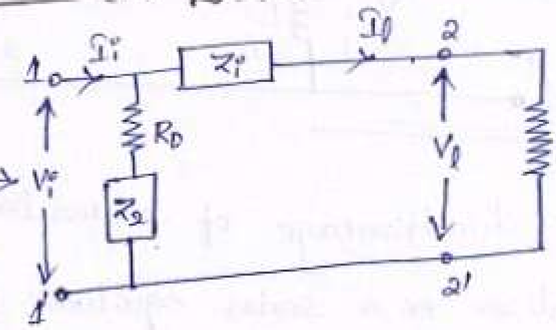


The full series equalizer is shown in figure. Here, $L_1/C_1 = R_0^2$.

from the equation, $N = 1 + \frac{R_0^2}{X_2^2} = 1 + \frac{R_0^2}{\omega^2 L_1^2}$

By knowing the values of N and R_0, the values of L_1, C_1 may be obtained.

(iii)(b) Full-Shunt Equalizer: —— (or) Constant-Resistance Equilizer



The given constant-resistance equilizer satisfies the equation $Z_1 Z_2 = R_0^2$.

The input impedance is given by

$$Z_i = \frac{(R_0+Z_2)(R_0+Z_1)}{2R_0+Z_1+Z_2} = \frac{Z_1Z_2 + R_0^2 + R_0(Z_1+Z_2)}{2R_0+Z_1+Z_2}$$

Since $Z_i = R_0$ and $Z_1 Z_2 = R_0^2$

Also, $V_i = I_i Z_i = I_i R_0$, $V_l = I_l R_0$

$$\therefore \frac{V_i}{V_l} = \frac{I_i}{I_l}$$

But $I_l = I_i \dfrac{R_0+Z_2}{2R_0+Z_1+Z_2} \Rightarrow \dfrac{I_i}{I_l} = \dfrac{Z_1+Z_2+2R_0}{R_0+Z_2}$

Multiplying both numerator and denominator by Z_1, we get

$$\frac{I_i}{I_l} = \frac{Z_1^2 + Z_1Z_2 + 2R_0Z_1}{Z_1R_0 + Z_1Z_2} = \frac{(Z_1+R_0)^2}{R_0[Z_1+R_0]} \qquad \text{Since } Z_1Z_2 = R_0^2$$

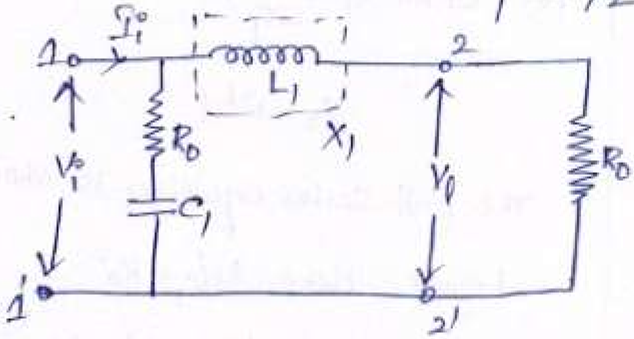
Therefore, $N = \left|\frac{V_i}{V_l}\right|^2 = \left|\frac{I_i}{I_l}\right|^2 = \left|\frac{R_0+Z_1}{R_0}\right|^2 = 1 + \frac{X_1^2}{R_0^2} = 1 + \frac{R_0^2}{X_2^2}$

Since Z_1 and Z_2 are pure reactances and are equal to X_1 and X_2, respectively.

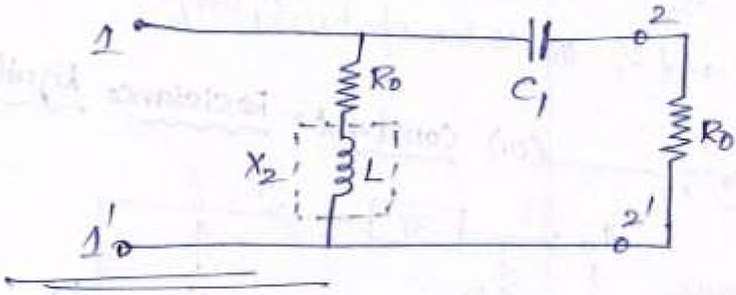
By knowing the values of R_0 and N, the elemental values X_1 and Y_2 can be obtained.

(a) When $X_1 = \omega L_1$, X_2 becomes $\frac{1}{\omega C_1}$.

The circuit is shown in figure.

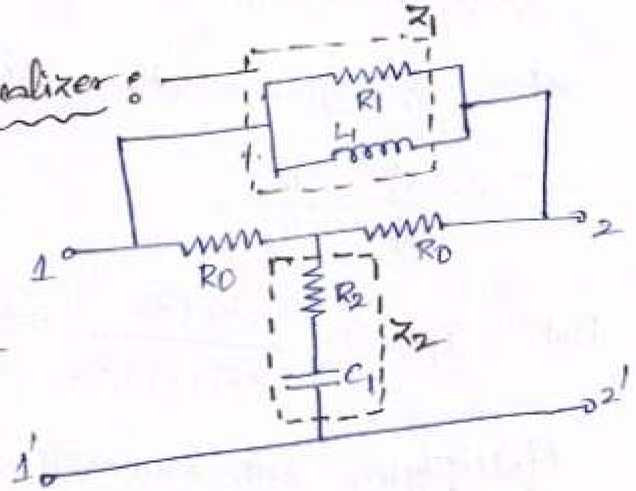


(b) When $X_1 = \frac{1}{\omega C_1}$, X_2 becomes ωL_1.



*> The disadvantage of a reactance equalizer either in a shunt equalizer or a series equilizer is that; the variation of impedance with frequency causes impedance mismatch which results in reflection losses. A four terminal equalizer which offers a constant resistance at all frequencies avoids reflection loss when terminated in its design impedance.

(iv) Bridged-T Attenuation Equalizer:



Let Z_1 be a parallel combination of resistor 'R_1' and Inductance 'L_1'.

To provide a constant resistance, the impedance Z_2 must be an inverse of Z_1 which is a series combination of 'R_2' and a capacitor 'C_1'. Let R_0 be the design resistance. Then, $Z_1 Z_2 = R_0^2$.

The propagation constant for a bridged-T network is given by

$$\gamma = \ln\left[1 + \frac{Z_1}{Z_0}\right] = \ln\left[1 + \frac{Z_0}{Z_2}\right]$$

But $Z_0 = R_0$, and $Z_1 = \frac{jR_1 \omega L_1}{R_1 + \omega L_1 j}$

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Therefore, the propagation constant

$$\gamma = \ln\left[1+\frac{jR_1\omega L_1}{R_0(R_1+j\omega L_1)}\right]$$

$$\alpha+j\beta = \ln\left[\frac{R_0R_1+j\omega L_1(R_0+R_1)}{R_0R_1+j\omega L_1R_0}\right]$$

By equating real parts on both sides,

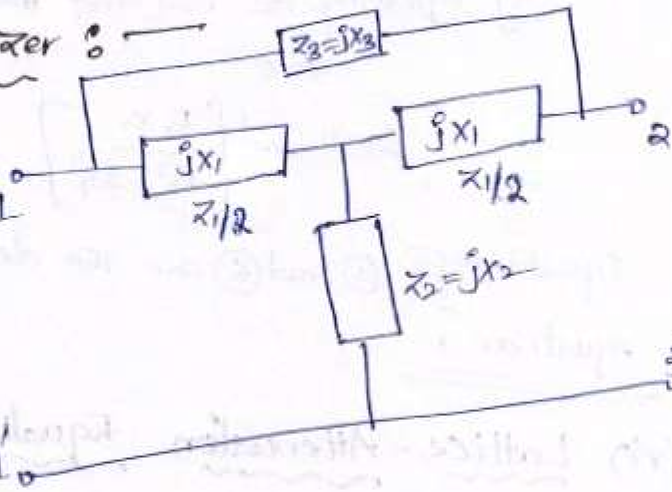
$$\alpha = \ln\left[\frac{(R_0R_1)^2+\omega^2L_1^2R_0^2+\omega^2L_1^2R_1^2+2\omega^2L_1^2R_0R_1}{R_0^2R_1^2+\omega^2L_1^2R_0^2}\right]^{1/2}$$

$$\alpha = \frac{1}{2}\ln\left[1+\frac{\omega^2L_1^2R_1(2R_0+R_1)}{R_0^2(R_1^2+\omega^2L_1^2)}\right]$$

and $R_1R_2 = R_0^2 = L_1/C_1$

The elements may be calculated from the above equations.

(V) Bridged-T phase Equalizer :-



The network consists of pure reactances. The characteristics impedance is given by

$$Z_0 = \left[\frac{Z_1Z_3(Z_1+4Z_2)}{4(Z_1+Z_3)}\right]^{1/2} \longrightarrow (1)$$

from figure, $Z_3=jX_3$, $\frac{Z_1}{2}=jX_1$, $Z_2=jX_2$ and $Z_0=R_0$.

$$R_0^2 = \frac{2jX_1\cdot jX_3(2jX_1+4jX_2)}{4(2jX_1+4jX_3)} = \frac{-X_1X_3(X_1+2X_2)}{2X_1+X_3} \longrightarrow (2)$$

Let X_1 and X_3 are made inverse.

$$jX_1\cdot jX_3 = R_0^2 \Rightarrow -X_1X_3 = R_0^2 \longrightarrow (3)$$

from equations (2) and (3),

$$R_0^2 = \frac{\cancel{R_0^2}[X_1+2X_2]}{2X_1+X_3} \Rightarrow X_1+2X_2 = 2X_1+X_3$$

$$\Rightarrow \boxed{X_2 = \frac{X_1+X_3}{2}} \longrightarrow (4)$$

The propagation constant is given by

$$e^{\gamma}=\frac{Z_0(Z_1+Z_3)+Z_1Z_3/2}{Z_0(Z_1+Z_3)-Z_1Z_3/2} \Rightarrow e^{\gamma}-1=\frac{Z_1Z_3}{Z_0(Z_1+Z_3)-Z_1Z_3/2}$$

and similarly, $$e^{\gamma}+1=\frac{2Z_0(Z_1+Z_3)}{Z_0(Z_1+Z_3)-Z_1Z_3/2}$$

from the above equations,

$$\frac{e^{\gamma}-1}{e^{\gamma}+1}=\tanh\gamma/2=\frac{Z_1Z_3}{2Z_0(Z_1+Z_3)}=\frac{2jX_1\cdot jX_3}{2R_0(2jX_1+jX_3)}=\frac{2R_0^2}{2R_0j[2X_1+X_3]} \longrightarrow (5)$$

$$\therefore \tanh\gamma/2=\frac{2R_0^2}{2R_0j\left[2X_1-\frac{R_0^2}{X_1}\right]} \Rightarrow \gamma/2=\tanh^{-1}\left[\frac{jR_0X_1}{R_0^2-2X_1^2}\right]$$

$$\therefore \alpha+j\beta=2j\tanh^{-1}\left[\frac{jR_0X_1}{R_0^2-2X_1^2}\right]=2j\tan^{-1}\left[\frac{R_0X_1}{R_0^2-2X_1^2}\right]$$

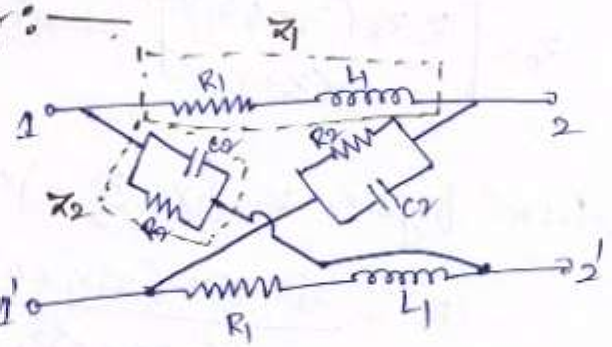
By equating the real and imaginary parts, we get

$$\alpha=0$$

$$\beta=2\tan^{-1}\left[\frac{R_0X_1}{R_0^2-2X_1^2}\right] \longrightarrow (6)$$

Equations (2), (3) and (6) are the design equations of a bridged-T phase equalizer.

(vi) Lattice-Attenuation Equalizer:—



The element Z_1 represents series arm and Z_2 represent diagonal arm as shown in figure. The equalizer is a constant-resistance equalizer such that Z_1 must be inverse of Z_2 to the designed resistance R_0.

$$Z_1Z_2=R_0^2$$

$$R_1R_2=\frac{L_1}{C_1}R_0^2 \longrightarrow (1)$$

The propagation constant of a lattice network is given by

$$\gamma=\ln\left[\frac{1+Z_1/R_0}{1-Z_1/R_0}\right]=\ln\left[\frac{1+Z_2/R_0}{Z_2/R_0-1}\right] \longrightarrow (2)$$

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$$\alpha + j\beta = \ln\left[\frac{1+\frac{R_1+j\omega L_1}{R_0}}{1-\frac{R_1+j\omega L_1}{R_0}}\right] \Rightarrow \alpha + j\beta = \ln\left[\frac{(R_0+R_1)+j\omega L_1}{(R_0-R_1)-j\omega L_1}\right]$$

Equating real parts on both sides

$$\alpha = \ln\left[\frac{(R_0+R_1)^2+\omega^2L_1^2}{(R_0-R_1)^2+\omega^2L_1^2}\right]^{1/2}$$

$$\Rightarrow N = e^{\alpha} = \left[\frac{(R_0+R_1)^2+\omega^2L_1^2}{(R_0-R_1)^2+\omega^2L_1^2}\right]^{1/2} \longrightarrow (3)$$

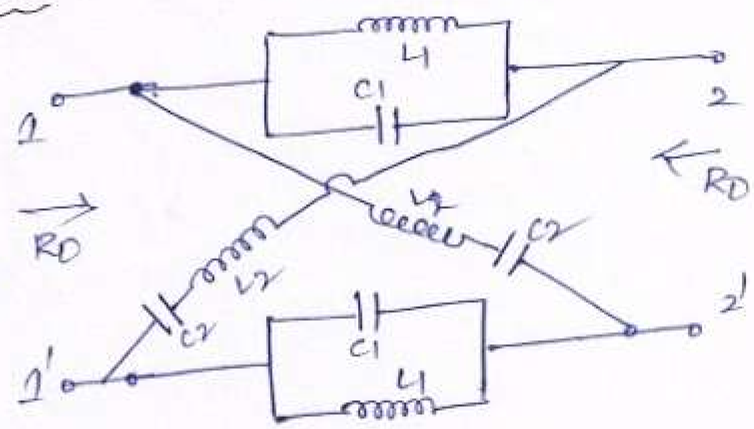
On the other hand, if $X_1 = \frac{1}{\omega C_1}$

$$N = e^{\alpha} = \left[\frac{(R_0+R_1)^2+\frac{1}{\omega^2C_1^2}}{(R_0-R_1)^2+\frac{1}{\omega^2C_1^2}}\right]^{1/2} \longrightarrow (4)$$

Equations (3) & (4) are the design equations for the lattice-attenuat equalizer network.

(vii) Lattice-phase Equalizer:

The circuit consists of only reactive components. This is also a constant-resistance equalizer which satisfies the equation $Z_1Z_2 = R_0^2$.



Z_1 is the series-arm impedance and

Z_2 is the shunt-arm impedance.

The propagation constant is given by

$$\tanh[\gamma/2] = \left[Z_1/R_0\right] = \sqrt{Z_1/Z_2}$$

$$\therefore \tanh(\gamma/2) = \frac{j\omega L_1 / j\omega C_1}{R_0\left[j\omega L_1 + 1/j\omega C_1\right]} = \frac{j\omega L_1}{R_0\left[1-\omega^2 L_1C_1\right]}$$

$$\gamma = 2\tanh^{-1}\left[\frac{j\omega L_1}{R_0\left[1-\omega^2 L_1 C_1\right]}\right]$$

$$\gamma = 2j\tan^{-1}\left[\frac{\omega L_1}{R_0\left(1-\omega^2 L_1 C_1\right)}\right]$$

Since $\gamma = \alpha + j\beta$

$$\therefore \alpha + j\beta = 2j\tan^{-1}\left[\frac{\omega L_1}{R_0\left[1-\omega^2 L_1 C_1\right]}\right]$$

By comparison,

$$\alpha = 0$$

$$\beta = 2\tan^{-1}\left[\frac{\omega L_1}{R_0\left(1-\omega^2 L_1 C_1\right)}\right]$$

This expression gives the phase delay in a lattice phase equalizer.